2.1 – Properties of Rational Expressions

A rational expression is …
an algebraic fraction with a numerator and a denominator that are polynomials

e.g. $\frac{1}{x}$, $\frac{y}{y-2}$, $\frac{x+2}{x^2+4x+4}$, $\frac{m^2-9}{3}$

not rational $\frac{\sqrt{x}}{x}$

how zero affects division

Evaluate $\frac{0}{3} = 0$ Think: If you have 0 items and split them into 3 groups, how many items are in each group? 0

When zero is divided by any non-zero real number, …

The resut is 0.

Evaluate $\frac{7}{0}$ = undefined

gives error

Division by zero is undefined because…

…. If you have 7 items, you cannot put them into zero groups

restricted values

For the expression $\frac{3}{x-2}$, what value of x is restricted? $x \neq 2$ (x cannot equal 2)

What is a **restricted value**?

Any value that will make the denominator zero, as this will cause the expression to be undefined.

Write a rule that explains how to determine restricted values:

Set the denominator equal to zero and solve. The results are the restricted values.

Example 1 – Determine the restrictions for each rational expression:

Top is allowed to be 0

a) $\frac{4a}{3b}$

$\frac{3b}{3} = \frac{0}{3}$

$b = 0$

restriction

$\boxed{b \neq 0}$

b) $\frac{x-1}{(x+2)(x-3)}$

already factored

find zero of the bracket

$(x+2)(x-3) = 0$

$x = -2$ $x = 3$

$\boxed{x \neq -2,\ x \neq 3}$

c) $\frac{2y^2}{y^2-4}$

factor first

$= \frac{2y^2}{(y+2)(y-2)}$

$y \neq -2, +2$

or

$\boxed{y \neq \pm 2}$

simplifying rational expressions

When simplifying rational expressions:

1) Factor as much as possible. State restrictions

2) Reduce/cancel common factors.

Example 2 – Simplify the rational expressions. Keep a running list of restrictions.

a) $\frac{3x-3}{6x-6}$ - factor - remove common factor

$\frac{3\cancel{(x-1)}}{6\cancel{(x-1)}}$ identical factors must be multiplying top/bottom

restriction $\boxed{x \neq 1}$

$= \frac{3 \div 3}{6 \div 3}$

$= \boxed{\frac{1}{2}, \; x \neq 1}$

b) $\frac{x-2}{x^2-4}$ → perfect square $\sqrt{x^2} = x$, $\sqrt{4} = 2$ $\quad x \neq \pm 2$

$\frac{\cancel{(x-2)}}{(x+2)\cancel{(x-2)}}$

list restrictions before canceling

$= \boxed{\frac{1}{(x+2)}, \; x \neq \pm 2}$

c) $\frac{3x-6}{2x^2+x-10}$ → $\underline{a}x^2 + bx + c$ "ac" method

$\cancel{(1,20)(2,10)}$ $(4,5)$

$②x^2 + x - \cancel{10}$

$x^2 + x - 20$

$\frac{5}{} \times \frac{-4}{} = -20$

$\frac{5}{} + \frac{-4}{} = 1$

$(x + \frac{5}{2})(x - \frac{4}{2})$

$(2x+5)(x-2)$

$\frac{3\cancel{(x-2)}}{(2x+5)\cancel{(x-2)}}$

restrictions $x \neq 2, \frac{-5}{2}$

$= \boxed{\frac{3}{(2x+5)}, \; x \neq 2, \frac{-5}{2}}$

(1,10)(2,5)

d) $\frac{2y^2+y-10}{y^2+3y-10}$ → use "ac" method → same as last example

$\frac{5}{} \times \frac{-2}{} = -10$

$\frac{5}{} + \frac{-2}{} = 3$

$\frac{\cancel{(y-2)}(2y+5)}{(y+5)\cancel{(y-2)}}$

$\boxed{\frac{(2y+5)}{(y+5)} \quad y \neq -5, 2}$

e) $\frac{6-2m}{m^2-9}$ ← Re-order!

$\frac{-2m+6}{m^2-9}$

$\frac{-2\cancel{(m-3)}}{(m+3)\cancel{(m-3)}}$

$\boxed{\frac{-2}{m+3}, \; m \neq \pm 3}$

note: $\frac{3-m}{m-3}$ doesn't cancel

but you can reorder and factor a neg

$= \frac{-m+3}{m-3} \rightarrow \frac{-\cancel{(m-3)}}{\cancel{m-3}} = -1$

f) $\frac{x^2y+xy^2}{xy+y^2}$

$= \frac{(x)\cancel{(y)}\cancel{(x+y)}}{\cancel{(y)}\cancel{(x+y)}}$ $\quad \boxed{y \neq 0, -x}$

$= \boxed{x, \; y \neq 0, -x}$

*See the bottom of page 71 for *Common Errors***

2.2 – Multiplying & Dividing Rational Expressions

multiplication & division review

Warmup – Simplify

a) $\left(\frac{3}{-4}\right)\times\left(\frac{1}{2}\right)$

$\frac{3}{-8} = \boxed{\frac{-3}{8}}$

b) $\left(\frac{5}{8}\right)\left(\frac{-4}{15}\right)$ or cross reduce

$\frac{-20 \div 20}{120 \div 20}$

$= \boxed{\frac{-1}{6}}$

c) $\frac{2}{3} \div \frac{3}{4}$ flip + multiply

$= \frac{2}{3} \times \frac{4}{3}$

$= \boxed{\frac{8}{9}}$

d) $\frac{\frac{2}{5}}{\frac{-1}{10}}$ ← rewrite

$\frac{2}{5} \div \frac{-1}{10}$

$\frac{2}{5} \times \frac{-10}{1}$

$\frac{-20}{5} = \boxed{-4}$

Explain how to multiply fractions:
- look to reduce early by cross reducing (optional)
- multiply across the top and bottom
- look to reduce to write in simplest form

Explain how to divide fractions:
- flip second fraction and then multiply (use steps above)

Example 1 – Simplify and keep a running list of restrictions.

a) $\left(\frac{x+3}{2}\right)\left(\frac{x+1}{4}\right)$ no restrictions

$\boxed{= \frac{(x+3)(x+1)}{8}}$

factored is better, could expand top if wanted

$\frac{x^2+4x+3}{8}$

b) $\left(\frac{4x^2}{3xy}\right)\left(\frac{y^2}{8x}\right)$ $x \neq 0$ $y \neq 0$

$\frac{4x^2y^{2\,1}}{24x^2y}$ ÷4

$\frac{y^2}{y} = \frac{y \cdot y}{y} = y$ or $y^{2-1} = y^1$

$\boxed{= \frac{y}{6},\ x \neq 0\ y \neq 0}$

c) $\left(\frac{d}{2\pi r}\right)\left(\frac{2\pi rh}{d-2}\right)$ $d \neq 2$ $r \neq 0$

$= \frac{2\pi rhd}{2\pi r(d-2)}$

$= \boxed{\frac{hd}{(d-2)},\ d \neq 2\ r \neq 0}$

Example 2 – Simplify and keep a running list of restrictions. → can't do restrictions until factored

updated steps
1) factor
2) state restrictions
3) Change any ÷ to × (flip 2nd fraction)
*3b) state any new restrictions
4) reduce (cancel identical factors top and bottom)

a) $\frac{y^2-9}{r^3-r} \times \frac{r^2-r}{y+3}$ diff of squ

$= \frac{(y+3)(y-3)}{r(r^2-1)} \times \frac{r(r-1)}{(y+3)}$ diff of squares

$= \frac{\cancel{(y+3)}(y-3)}{\cancel{(r)}(r+1)\cancel{(r-1)}} \cdot \frac{\cancel{(r)}\cancel{(r-1)}}{\cancel{(y+3)}}$

$= \boxed{\frac{(y-3)}{(r+1)}\quad r \neq 0, \pm 1\quad y \neq -3}$

b) $\left(\frac{x^2-x-12}{x^2-9}\right)\cdot\left(\frac{x^2-4x+3}{x^2-4x}\right)$

$= \frac{\cancel{(x-4)}\cancel{(x+3)}}{\cancel{(x+3)}\cancel{(x-3)}} \cdot \frac{\cancel{(x-3)}(x-1)}{(x)\cancel{(x-4)}}$ $x \neq \pm 3, 0, 4$

$= \boxed{\frac{(x-1)}{(x)},\ x \neq \pm 3, 0, 4}$

Example 3 – Simplify and keep a running list of restrictions.

a) $\frac{m^2-6m-7}{m^2-49} \div \frac{m^2+8m+7}{m^2+7m}$

$= \frac{(m-7)(m+1)}{(m+7)(m-7)} \div \frac{(m+1)(m+7)}{m(m+7)}$ $m \neq \pm 7, 0$

$= \frac{\cancel{(m-7)}\cancel{(m+1)}}{\cancel{(m+7)}\cancel{(m-7)}} \times \frac{(m)\cancel{(m+7)}}{\cancel{(m+1)}(m+7)}$ $m \neq -1$

$\boxed{= \frac{(m)}{(m+7)}, \quad m \neq -1, 0, \pm 7}$

b) $\frac{3x+12}{3x^2-5x-12} \div \frac{12}{3x+4} \times \frac{2x-6}{x+4}$

"ac" method

$3x^2 - 5x - 12$

$x^2 - 5x - 36$

$-9 \times 4 = -36$

$-9 + 4 = -5$

$(x - \frac{9}{3})(x + \frac{4}{3})$

$(x-3)(3x+4)$

$= \frac{(3)(x+4)}{(x-3)(3x+4)} \div \frac{(12)}{(3x+4)} \times \frac{(2)(x-3)}{(x+4)}$ $x \neq 3, \frac{-4}{3}, -4$

$= \frac{(3)\cancel{(x+4)}}{\cancel{(x-3)}\cancel{(3x+4)}} \times \frac{\cancel{(3x+4)}}{(12)} \times \frac{(2)\cancel{(x-3)}}{\cancel{(x+4)}}$ → no new restrictions

$= \frac{6 \div 6}{12 \div 6} = \boxed{\frac{1}{2}, \quad x \neq 3, \frac{-4}{3}, -4}$

2.3 – Adding & Subtracting Rational Expressions

adding & subtracting review

Warmup – Simplify each expression

a) $\frac{5}{6}-\frac{3}{8}$

$\frac{20}{24}-\frac{9}{24}$

$=\boxed{\frac{11}{24}}$

b) $-\frac{2}{3}+\frac{4}{5}$

$\frac{-10}{15}+\frac{12}{15}$

$=\boxed{\frac{2}{15}}$

c) $\frac{7x+1}{x}+\frac{5x-2}{x}$

$=\boxed{\frac{12x-1}{x}}$!

$x\neq 0$

d) $\frac{7}{6x^2}-\frac{3}{8x^3}$

$\frac{28x}{24x^3}-\frac{9}{24x^3}$

$\boxed{\frac{28x-9}{24x^3}}$ $x\neq 0$

Write the steps to adding/subtracting fractions:

1. Get common denominators
2. Add / subtract numerators (leave denom the same)
3. Reduce

Example 1 – Simplify and identify all restrictions.

a) $\frac{(10y-1)}{4y-3}-\frac{(8-2y)}{4y-3}$

$4y-3\neq 0$
$4y\neq 3$
$y\neq\frac{3}{4}$

$=\frac{10y-1-(8-2y)}{4y-3}$

$=\frac{10y-1-8+2y}{4y-3}$

$=\frac{12y-9}{4y-3}$ factor

$=\frac{3\cancel{(4y-3)}}{\cancel{(4y-3)}}$

$=3, y\neq\frac{3}{4}$

b) $\frac{2x}{xy}+\frac{4}{x^2}-3$ $\quad x\neq 0, y\neq 0$

$\frac{2x^2}{x^2y}+\frac{4y}{x^2y}-\frac{3x^2y}{x^2y}$

$\boxed{\frac{2x^2+4y-3x^2y}{x^2y}, x\neq 0\ y\neq 0}$

c) $\frac{3}{3x+6}+\frac{1}{x+2}$

$\frac{3}{3(x+2)}+\frac{1}{(x+2)}$ $\quad x\neq -2$

$\frac{3}{3(x+2)}+\frac{3}{3(x+2)}$

$=\frac{\cancel{6}^{2}}{\cancel{3}(x+2)}$

$=\boxed{\frac{2}{(x+2)}, x\neq -2}$

Steps: 1) Factor as much as possible.
2) List restrictions. Do any relevant reducing.
3) Get common denominators.
4) Add or subtract numerators.
5) Do any further factoring and/or reducing.

Example 2 – Simplify and identify all restrictions.

a) $\frac{4}{x^2-1}+\frac{3}{1-x}$ → $-x+1$

$\frac{4}{(x+1)(x-1)} + \frac{3}{-(x-1)} \cdot \frac{(x+1)}{(x+1)}$ $x \neq \pm 1$

$\frac{4}{(x+1)(x-1)} + \frac{-3(x+1)}{(x-1)(x+1)}$

$\frac{4-3x-3}{(x+1)(x-1)}$

$\boxed{\frac{-3x+1}{(x+1)(x-1)},\ x \neq \pm 1}$

b) $\frac{x-2}{x^2+x-6}-\frac{x^2+6x+5}{x^2+4x+3}$

$\frac{\cancel{(x-2)}}{(x+3)\cancel{(x-2)}} - \frac{\cancel{(x+1)}(x+5)}{(x+3)\cancel{(x+1)}}$ $x \neq -3, 2, -1$

$\frac{1}{(x+3)} - \frac{(x+5)}{(x+3)}$

$\frac{1-x-5}{(x+3)}$

$\boxed{\frac{-x-4}{(x+3)},\ x \neq -3, 2, -1}$

c) $\frac{1}{x^2-1}-\frac{2}{x^2+x}$

$\frac{1}{(x+1)(x-1)} \cdot \frac{x}{x} - \frac{2}{(x)(x+1)} \cdot \frac{(x-1)}{(x-1)}$ $x \neq \pm 1, 0$

$= \frac{x-2(x-1)}{x(x+1)(x-1)}$

$= \frac{x-2x+2}{x(x+1)(x-1)}$

$= \boxed{\frac{-x+2}{x(x+1)(x-1)},\ x \neq \pm 1, 0}$

d) $\frac{3x+9}{x^2+7x+10}+\frac{14}{x^2+3x-10}$

$\frac{3(x+3)}{(x+5)(x+2)} \cdot \frac{(x-2)}{(x-2)} + \frac{14}{(x+5)(x-2)} \cdot \frac{(x+2)}{(x+2)}$ $x \neq -5, \pm 2$

$\frac{3(x+3)(x-2) + 14(x+2)}{(x+5)(x+2)(x-2)}$ → $3(x^2+x-6) + 14x + 28$

$\frac{3x^2+3x-18+14x+28}{(x+5)(x+2)(x-2)}$

$\frac{3x^2+17x+10}{(x+5)(x+2)(x-2)}$

side trip factor

$3x^2+17x+10$

$x^2+17x+30$

$\frac{15}{} \times \frac{2}{} = 30$

$\frac{15}{} + \frac{2}{} = 17$

$(x+\frac{15}{3})(x+\frac{2}{3})$

$(x+5)(3x+2)$

$\frac{\cancel{(x+5)}(3x+2)}{\cancel{(x+5)}(x+2)(x-2)}$

$\boxed{\frac{3x+2}{(x+2)(x-2)},\ x \neq \pm 2, -5}$

2.4 – Mixed Operations in Rational Expressions

When simplifying rational expressions with mixed operations, ORDER OF OPERATIONS is to be followed (**BEDMAS**).

Example 1 – Simplify & identify all restrictions.

a) $\frac{x+5}{x+6}+\underbrace{\frac{1}{x+4}\div\frac{x+6}{x^2-x-20}}_{\text{Divide first before add}}$

$\frac{(x+5)}{(x+6)}+\frac{1}{(x+4)}\div\frac{(x+6)}{(x-5)(x+4)}$ $\quad x\neq -6,-4,+5$

$\frac{(x+5)}{(x+6)}+\frac{1}{\cancel{(x+4)}}\times\frac{(x-5)\cancel{(x+4)}}{(x+6)}$

$\frac{(x+5)}{(x+6)}+\frac{(x-5)}{(x+6)}$

$\frac{x+5+x-5}{(x+6)}$

$\boxed{\frac{2x}{(x+6)},\ x\neq -6,-4,+5}$

b) $\left(\underbrace{\frac{x-3}{x^2-9}+\frac{x+3}{x^2+6x+9}}_{\text{bracket, start here}}\right)\left(\frac{x+3}{x+1}\right)$

$\left(\frac{\cancel{(x-3)}}{(x+3)\cancel{(x-3)}}+\frac{\cancel{(x+3)}}{\cancel{(x+3)}(x+3)}\right)\left(\frac{(x+3)}{(x+1)}\right)$ $\quad x\neq \pm 3,-1$

$\frac{2}{\cancel{(x+3)}}\times\frac{\cancel{(x+3)}}{(x+1)}$

$\boxed{\frac{2}{x+1},\ x\neq \pm 3,-1}$

Complex Fractions – Rational Expressions that contain fractions in the numerators and/or denominators.

Example 2 – Simplify and identify all restrictions.

$$\frac{2-\frac{4}{y}}{y-\frac{4}{y}}$$

$\left(\frac{2\cdot y}{1\cdot y}-\frac{4}{y}\right)\div\left(\frac{y\cdot y}{1\cdot y}-\frac{4}{y}\right)$

$\frac{2y-4}{y}\div\frac{y^2-4}{y}$ $\quad y\neq 0$

$\frac{2\cancel{(y-2)}}{\cancel{y}}\times\frac{\cancel{y}}{\cancel{(y-2)}(y+2)}$ $\quad y\neq \pm 2,0$

$\boxed{\frac{2}{(y+2)},\ y\neq \pm 2,0}$

Steps:

1) Get a common denominator for the numerator and then the denominator of the complex fraction.

2) Write each as one fraction.

3) Rewrite the division in a side-by-side manner and simplify.

Example 3 – Simplify and identify all restrictions.

a) $\dfrac{\left(\frac{2}{5x} - \frac{3}{x^2}\right)}{\left(\frac{7}{2x} + \frac{3}{4x^2}\right)}$

$\left(\frac{2 \cdot x}{5x \cdot x} - \frac{3 \cdot 5}{x^2 \cdot 5}\right) \div \left(\frac{7 \cdot 2x}{2x \cdot 2x} + \frac{3}{4x^2}\right)$

$\left(\frac{2x}{5x^2} - \frac{15}{5x^2}\right) \div \left(\frac{14x}{4x^2} + \frac{3}{4x^2}\right)$

$\frac{2x-15}{5x^2} \div \frac{14x+3}{4x^2}$ $\quad x \neq 0$

$\frac{2x-15}{5\cancel{x^2}} \times \frac{4\cancel{x^2}}{14x+3}$ $\quad x \neq 0, \frac{-3}{14}$

$\boxed{\frac{4(2x-15)}{5(14x+3)}, \; x \neq 0, \frac{-3}{14}}$

b) $\dfrac{\frac{1}{x-1} + \frac{2}{x+2}}{\frac{2}{x+2} - \frac{1}{x-3}}$

$\left(\frac{1\,(x+2)}{(x-1)(x+2)} + \frac{2\,(x-1)}{(x+2)(x-1)}\right) \div \left(\frac{2\,(x-3)}{(x+2)(x-3)} - \frac{1\,(x+2)}{(x-3)(x+2)}\right)$

$\frac{x+2+2x-2}{(x-1)(x+2)} \div \frac{2x-6-x-2}{(x+2)(x-3)}$

$\frac{3x}{(x-1)(x+2)} \div \frac{x-8}{(x+2)(x-3)}$ $\quad x \neq 1, -2, 3, 8$

$\frac{3x}{(x-1)\cancel{(x+2)}} \times \frac{\cancel{(x+2)}(x-3)}{(x-8)}$ $\quad x \neq 8$

$\boxed{\frac{3x(x-3)}{(x-1)(x-8)}, \; x \neq 1, -2, 3, 8}$

A rational equation is an equation containing at least one rational expression. Remember, when working with an equation, whatever you do to one side, you do to the other side.

Steps to solving rational equations:

1) Factor each denominator if possible.
2) Identify any restrictions (and do this throughout).
3) Multiply both sides of the equation by what would be the lowest common denominator in order to eliminate the fractions. → clearing fractions
4) Solve the equation.
5) Check your solutions. ☆ check restrictions

Example 1 – Solve

a) $\frac{x}{2}+\frac{7}{3}=\frac{5}{6}$ no Restrictions

LCD: 6

$\frac{x}{2}\cdot\frac{6}{1}+\frac{7}{3}\cdot\frac{6}{1}=\frac{5}{6}\cdot\frac{6}{1}$

$3x+14=5$

$-14 \quad -14$

$\frac{3x}{3}=\frac{-9}{3}$

$\boxed{x=-3}$ No R ✓

b) $\frac{5}{3x}-\frac{1}{9}=\frac{4}{x}$ LCD = $9x$

$\frac{5}{3x}\cdot\frac{9x}{1}-\frac{1}{9}\cdot\frac{9x}{1}=\frac{4}{x}\cdot\frac{9x}{1}$ $x\neq 0$

$15-x=36$

$-15 \quad -15$

$\frac{-x}{-1}=\frac{21}{-1}$

$\boxed{x=-21}$ $x\neq 0$ ✓

Example 2 – Solve

a) $\frac{2x}{(x-4)}=\frac{8}{(x-4)}+1$ $x\neq 4$

LCD: $(x-4)$

$\frac{2x}{(x-4)}\cdot(x-4)=\frac{8}{(x-4)}\cdot(x-4)+\frac{1}{1}\cdot(x-4)$

$2x=8+x-4$

$-x \quad -x$

$x=4$ ⚠ $x\neq 4$ ✗

$\boxed{\text{No solution}}$

b) $\frac{9}{y-3}-\frac{4}{y-6}=\frac{18}{y^2-9y+18}$ LCD: $(y-6)(y-3)$ $y\neq 3,6$

$\frac{9}{(y-3)}\cdot(y-6)(y-3)-\frac{4}{(y-6)}\cdot(y-6)(y-3)=\frac{18}{(y-6)(y-3)}\cdot(y-6)(y-3)$

$9(y-6)-4(y-3)=18$

$9y-54-4y+12=18$

$5y-42=18$

$+42 \quad +42$

$\frac{5y}{5}=\frac{60}{5}$

$\boxed{y=12}$ $y\neq 3,6$ ✓

When a solution is the same as a restricted value, it is called an EXTRANEOUS solution.

Quadratics $\rightarrow$ factor? $\rightarrow$ quad formula

Example 3 – Solve

a) $\frac{x}{x-5} - \frac{3}{x+1} = \frac{30}{x^2-4x-5}$

$\frac{x}{(x-5)}(x-5)(x+1) - \frac{3}{(x+1)}(x-5)(x+1) = \frac{30}{(x-5)(x+1)}(x-5)(x+1)$ $x \neq 5, -1$ LCD $= (x-5)(x+1)$

$x(x+1) - 3(x-5) = 30$

$x^2 + x - 3x + 15 = 30$
$\quad\quad\quad -30 \quad -30$

$x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

sol is zero's of brackets

$\cancel{x=5}$ $\quad x=-3 \quad\quad x \neq 5, -1$

sol $\boxed{x=-3}$

b) $\frac{3x}{x+2} - \frac{5}{x-3} = \frac{-25}{x^2-x-6}$

$\frac{3x}{(x+2)}(x-3)(x+2) - \frac{5}{(x-3)}(x-3)(x+2) = \frac{-25}{(x-3)(x+2)}(x-3)(x+2)$ $x \neq -2, 3$ LCD $= (x-3)(x+2)$

$3x(x-3) - 5(x+2) = -25$

$3x^2 - 9x - 5x - 10 = -25$
$\quad\quad\quad +25 \quad +25$

factor "ac" method

$3x^2 - 14x + 15 = 0$

$x^2 - 14x + 45 = 0$

$(x - \frac{9}{3})(x - \frac{5}{3}) = 0$

$(x-3)(3x-5) = 0$

$\cancel{x=3}$ $\quad x = \frac{5}{3} \quad\quad x \neq -2, 3$

Sol $\boxed{x = \frac{5}{3}}$

There is no fool-proof way to solve a word problem. You should try to read the problem carefully, create a *'Let'* statement for your variable, build your equation (sometimes using a table or diagram for assistance), and solve the equation. Then do a check.

Shared work

Example 1 – Stella takes 4 hours to paint a room. It takes Jose 3 hours to paint the same area. How long will the paint job take if they work together?

Let x = time it takes together

	Time to Paint (hours)	Fraction of Work Done in 1 hour	Fraction of Work Done in *x* hours
Stella	4	$\frac{1}{4}$ job/h	$\frac{1}{4}x = \frac{x}{4}$
Jose	3	$\frac{1}{3}$ job/h	$\frac{1}{3}x = \frac{x}{3}$
Together	X	$\frac{1}{x}$ job/h	$\frac{1}{x}x = \frac{x}{x} = 1$

$$\frac{\text{total time}}{\text{stella's time}} + \frac{\text{total time}}{\text{jose's time}} = 1 \text{ job}$$

$$\frac{x}{\cancel{4}}(4)(3) + \frac{x}{\cancel{3}}(4)(3) = 1\,(4)(3)$$

LCD = (4)(3) or 12

$3x + 4x = 12$

$\frac{7x}{7} = \frac{12}{7}$

$x = \frac{12}{7} = 1\frac{5}{7} \rightarrow \frac{5}{7} \times 60\text{min} = 43\text{min}$

$\boxed{\text{1h and 43 min}}$

Example 2 – Jenny takes 5 hours to install laminate flooring in the kitchen by herself. Mike can do the job alone in 6 hours. How long would it take them if they did it together? Let x = time together

$$\frac{\text{total time}}{\text{jenny's T}} + \frac{\text{total time}}{\text{mike's T}} = 1$$

$$\frac{x}{\cancel{5}}(5)(6) + \frac{x}{\cancel{6}}(5)(6) = 1\,(5)(6)$$

$6x + 5x = 30$

$\frac{11x}{11} = \frac{30}{11}$

$x = 30/11 \rightarrow 2\frac{8}{11}$

$\frac{8}{11} \times 60 = 44\text{min}$

$\boxed{\text{2 hours 44 min}}$

Example 3 – Evan works twice as fast as JJ. If it takes them 13 minutes & 20 seconds together to shovel snow from the driveway, how long would it take JJ by himself?

Let x = evans time / 2x = JJ's time

	Time	Fraction 1 min	Fraction of work in $\frac{40}{3}$
evan	x	$\frac{1}{x}$	$\frac{1}{x} = \frac{40/3}{x}$
JJ	2x	$\frac{1}{2x}$	$\frac{1}{2x} = \frac{40/3}{2x}$
together	$\frac{40}{3}$	$\frac{1}{40/3}$	1

$$\frac{\text{total time}}{\text{evans time}} + \frac{\text{total time}}{\text{jj's time}} = 1 \text{ job}$$

Total time $= 13 + \frac{20}{60}$

$= 13 + \frac{1}{3} = \frac{40}{3}$

$$\frac{40/3}{\cancel{x}} \cdot 2x + \frac{40/3}{\cancel{2x}} \cdot 2x = 1 \cdot 2x$$

$\frac{80}{3} + \frac{40}{3} = 2x$

$\frac{120}{3} = 2x$

$\frac{40}{2} = \frac{2x}{2}$

$20 = x$

Evan = 20 min

JJ = 2(20)

= 40 min

$\boxed{\text{Take JJ 40 min}}$

Distance

Example 4 – A speedboat can travel 108km downstream in the same time it can travel 78km upstream. If the current of the river is 10km/h, what is the speed of the boat in still water?

d
t | v

$$s = \frac{d}{t} \qquad d = st \qquad t = \frac{d}{s}$$

	d (km) given in Q	s (km/h) variable	t (h) use formula $t = \frac{d}{s}$	Equation
downstream →	108	$x+10$	$\frac{108}{x+10}$	"Same time" $\frac{108}{x+10} = \frac{78}{x-10}$
upstream ←	78	$x-10$	$\frac{78}{x-10}$	

let x = speed in still water

$$\frac{108}{\cancel{x+10}}(\cancel{x+10})(x-10) = \frac{78}{\cancel{x-10}}(x+10)(\cancel{x-10})$$

$$108(x-10) = 78(x+10)$$

$$108x - 1080 = 78x + 780$$

$$-78x + 1080 \qquad -78x + 1080$$

$$\frac{30x}{30} = \frac{1860}{30}$$

$$x = 62$$

$\boxed{\text{Speed in still water is 62 km/h}}$

Example 5 – Rob and Alissa ride a skateboard a distance of 4km. It takes Rob one more minute (hint: $\frac{1}{60}$ of an hour) than it takes Alissa, and Alissa travels 1km/h faster than Rob. At what speed is each traveling?

	d	s	$t = d/s$	equation
Rob	4	x	$\frac{4}{x}$	Rob's time is 1min more than Alissa's $\frac{4}{x} = \frac{1}{60} + \frac{4}{x+1}$
Alissa	4	$x+1$	$\frac{4}{x+1}$	

let x = Rob's speed (Alissa's $= x+1$)

Solve: $$\frac{4}{\cancel{x}} \cdot (60)(\cancel{x})(x+1) = \frac{1}{\cancel{60}} \cdot (\cancel{60})(x)(x+1) + \frac{4}{\cancel{(x+1)}} \cdot (60)(x)(\cancel{x+1})$$

$$\overset{240}{4(60)}(x+1) = x(x+1) + 4(60)(x)$$

$$240x + 240 = x^2 + x + 240x$$

$$-240x - 240$$

$$0 = x^2 + x - 240$$

$$= (x-15)(x+16)$$

$$x = 15 \qquad \cancel{x = -16}$$

reject doesn't make sense

$\therefore$ $x = 15$ (Rob)

$x+1 = 16$ (Alissa)

$\boxed{\text{Rob travels at 15 km/h; Alissa travels at 16 km/h}}$

Reminder

Can use quad formula if it doesn't factor