Chapter 2 - Polynomials

2.1 - Classifying Polynomials

Term

A term is a number, or a product of a number with one or more variables, which can be raised to a power.

Consider the following algebraic expressions:

$$3x^5,\ 7a - 3b,\ 2y + 7,\ x^2 + x + y^2$$

Of these expressions, the distinct terms are:

$$3x^5,\ 7a,\ -3b,\ 2y,\ 7,\ x^2,\ x,\ y^2$$

In short, a term can be broken into three categories: coefficients, variables, and degrees.

Term	$3x^5$	$-3b$	$\frac{3}{4}xy^2z$	$-x$	-11
Coefficient	3	-3	3/4	-1	-11
Variables	x	b	x, y, z	x	None
Degree	5	1	4	1	0

Polynomial

A polynomial is a term, or a sum (+) or difference (−) of terms. These terms must have whole numbers as their exponents (0, 1, 2, 3, …), that appear only in the numerator. This means that any term that has a negative exponent, or an exponent that is a fraction does not classify as a polynomial.

Example	Polynomial	Why?
5	YES	$5 = 5x^0$ = whole number exponent
$\frac{1}{2}x$	YES	$\frac{1}{2}x = \frac{1}{2}x^1$ = whole number exponent
$\sqrt{x}$	NO	$\sqrt{x} = x^{\frac{1}{2}}$ = rational exponent
$\sqrt{3}y^2$	YES	Coefficient can be anything, & y^2 has whole # exponent
$\frac{5}{4}k^2 + 2x - 3$	YES	All exponents are whole numbers
$\frac{1}{4x} - 2$	NO	$\frac{1}{4x} - 2 = \frac{1}{4}(x^{-1}) - 2$ = negative exponent

In this unit, and mostly throughout your math education, polynomials will be a main focus. We will now classify them, determined by the numbers of terms they contain.

Note: terms are separated by addition (+) or subtraction (−) operators.

Classifying Polynomials

Monomial	One term polynomial	$10,\ 3r^6st^2,\ -\frac{1}{2}y,\ x + 2x = 3x$
Binomial	Two term polynomial	$3r^6st^2 + 10,\ 3x - \frac{1}{2}y$
Trinomial	Three term polynomial	$3r^6st^2 + 3x - 10,\ 8x^2 - 2x + \frac{1}{3}$
Polynomial	General term for all, and for >3 terms.	$x^4 + x^3 - 2x^2 - 11x + 5$

Degree of a Polynomial

The degree of a term in a polynomial is the sum of the exponents for the variables in that term ($5xy^3$ => has a degree of 4). Each term has a degree, but the biggest one is the most important. The term that has the biggest, most important degree, also stands for, and is known as the "degree of the polynomial" itself.

Polynomial	Degrees of terms	Degree of Polynomial
$4x^2 + 3x - 1$	$4x^2 = 2$ $3x^1 = 1$ $-1x^0 = 0$	As $4x^2$, which has degree 2, is the largest of the polynomial, the polynomial itself is said to have a degree of 2.
$5x^2y + 6x^3yz$	$5x^2y^1 = 3$ $6x^3y^1z^1 = 5$	Term with largest polynomial is $6x^3yz = 5$, so the degree of the entire polynomial is 5.

Leading Term

It may seem a bit redundant, but there is one last classification of polynomials we must discuss, the "leading term." The Leading Term is the term with the highest degree. For the examples in the box above, the polynomial $4x^2 + 3x - 1$ has a leading term of $4x^2$ and the polynomial $5x^2y + 6x^3yz$ has a leading term of $6x^3yz$.

So to recap: Polynomials are made up of terms. Each term has its own degree. The term that contains the highest degree is known as the leading term. The entire polynomial has a degree, which is equal to the degree of the leading term.

Putting it all together:

Polynomial	$-3x^2y - 5xyz^4 + y - \frac{1}{4}$
Terms	$-3x^2y,\ -5xyz^4,\ y,\ -\frac{1}{4}$
Coefficients	-3, -5, 1, $-\frac{1}{4}$
Degree of each term	3, 6, 1, 0
Leading Term	$-5xyz^4$
Degree of polynomial	6
ReWritten in Descending Order	$-5xyz^4 - 3x^2y + y - \frac{1}{4}$

Combining Like Terms (+ / –)

"Like terms" are terms that have the SAME VARIABLES raised to the SAME POWER (keep in mind that the order of the variables does not matter). If two or more terms are "like" one another, they can be "combined" or "simplified" via addition or subtraction. When the terms are combined, only the coefficients will add or subtract, the variables, and powers will stay the same!

Term	Like Terms? (Y/N)	Simplified
$4x^2 - 3x$	NO (different exponents)	$4x^2 - 3x$
$5y^3 + 4y^3$	YES (same V, same E)	$9y^3$
$3x^2y + 3xy^2$	NO (different E)	$3x^2y + 3xy^2$
$\frac{1}{4}zx + \frac{3}{4}xz$	YES (same V, same E)	$1xz = xz$
$3zx^2 - 2yx^2$	NO (different V)	$3zx^2 - 2yx^2$

All the examples in the box above are binomials, but we will often be dealing with larger polynomial problems:

Example) Simplify the expression $10x^2y - 5x^2 + 3x^2y + 7z^8 + 3x^2$

1) Rewrite the question: $10x^2y - 5x^2 + 3x^2y + 7z^8 + 3x^2$
2) Collect the like terms together: $10x^2y + 3x^2y - 5x^2 + 3x^2 + 7z^8$
3) Combine like terms: $13x^2y - 2x^2 + 7z^8$

Simplify $8 + 5t - 7ty - 4 - 6t + y + 3y + 1t - 2 + 10y$

1) $8 + 5t - 7ty - 4 - 6t + y + 3y + 1t - 2 + 10y$
2) $-7ty + 5t - 6t + 1t + y + 3y + 10y + 8 - 4 - 2$
3) $-7ty + 14y + 2$

<u>Evaluating Polynomials</u>

When a variable has a stated value (like $x = 2$), in a polynomial, the polynomial can be evaluated (or solved). We do this, by substituting the given constant in to our polynomial wherever that variable appears.

Eg. Evaluate $2y^3 - 5,\ when\ y = 2$

1) Rewrite the question: $2y^3 - 5,\ y = 2$
2) Substitute the constant, for the variable: $2(2)^3 - 5$
3) Simplify: $2(2)^3 - 5 = 2(8) - 5 = 16 - 5 = 11$

Eg. Evaluate $4x - 11 + 6x,\ when\ x = (-1)$

1) $4x - 11 + 6x,\ x = (-1)$

2) $10x - 11$

3) $10(-1) - 11 = -10 - 11 = -21$

<u>Multiplying a Monomial by a Monomial</u>

To multiply two monomials together, first multiply the constant factors (the coefficients) and then multiply the variable factors.

Eg. Find the product of the monomials:

a) $(3y^4) \times (5y^2)$
$= (3 \times 5)(y^4 \times y^2) = (15)(y^{4+2}) = (15)(y^6) = 15y^6$

b) $(-4xyz^2)(-3x^2yw)$
$= (-4)(-3)(x)(x^2)(y)(y)(z^2)(w) = 12(x^{1+2})(y^{1+1})(z^2)(w^1)$
$= (12)(x^3)(y^2)(z^2)(w) = (12wx^3y^2z^2)$

c) $(2x)(2y)(2xy)(2xyz)$
$= (2)(2)(2)(2)(x)(x)(x)(y)(y)(y)(z) = 16x^{1+1+1}y^{1+1+1}z^1 = 16x^3y^3z^1$

Multiplying a Polynomial by a Binomial

To multiply polynomials together, we must use the *distributive property* to remove the brackets, then simplify. The distributive property makes sure that all the terms from one polynomial get multiplied by all the terms in the other polynomial.

The Distributive Property

$$a(b+c) = (a \times b) + (a \times c) = ab + ac$$

Eg. Multiply the polynomials together.

a) $4(2y+3)$
$= (4 \times 2y) + (4 \times 3) = 8y + 12$

b) $6x^2(3x - 4y^2)$
$= (6x^2 \times 3x) - (6x^2 \times 4y^2) = 18x^3 - 24x^2y^2$

c) $5(2+3)$
$= (5)(2) + (5)(3) = 10 + 15 = 25$ OR $5(2+3) = 5\ (5) = 25$

2.2 -Multiplying Polynomials

Multiplying polynomials with more than one term requires use of the distributive property. Remember that every term in the first polynomial will multiply to each term in the second polynomial.

There are several ways to multiply polynomials together: The Distributive method (p. 71), The Vertical method (p. 72), The Horizontal Method (p. 73), The Rectangle Method (p. 74). Any of these can be used on assessments for compete marks, but we will observe the Horizontal Method (known as the FOIL method) in class.

The Horizontal (FOIL) Method for Multiplying Polynomials

FOIL stands for (First, Outside, Inside, Last) and is a model that can be used to guarantee that all the terms of the polynomial are accounted for, when finding products. The process that is being conveyed through this acronym is that each term is used to completion, before we move onto the next.

The following equation shows the FOIL method. Once the binomials have been expanded, the like-terms can be combined and simplified (often the middle two terms can be collected).

$$(a+b)(c+d)$$

$$= ac + ad + bc + bd$$

First - the first terms of the binomials are multiplied together : $(a \times c)$
Outside - then the outside terms of the binomials: $(a \times d)$
Inside - then the inside terms: $(b \times c)$
Last - then the last terms: $(b \times d)$

Eg. Find the product of the binomials

a) $(x - 3y)(2x^3 - 2y)$

First *Outside* *Inside* *Last*

$$= (x)(2x^3) + (x)(-2y) + (-3y)(2x^3) + (-3y)(-2y)$$

$$= 2x^4 - 2xy - 6x^3y + 6y^2$$

$$= 2x^4 - 6x^3y - 2xy + 6y^2$$

b) $(2x+3)(6x-4)$

First *Outside* *Inside* *Last*

$= (2x)(6x) + (2x)(-4) + (3)(6x) + (3)(-4)$

$= 12x^2 - 8x + 18x - 12$

$= 12x^2 + 10x - 12$

Some questions ask you to find the product of a binomial with trinomials (or other combinations of polynomials). The FOIL model won't exactly work, but the principle behind it still remains: multiply every term in one polynomial by every term in the other polynomial.

Consider: $(a+b)(c+d+e) = a \times c + a \times d + a \times e + b \times c + b \times d + b \times e$

Eg. $(2x+3)(x^3+4x-11)$

$= (2x)(x^3) + (2x)(4x) + (2x)(-11) + (3)(x^3) + (3)(4x) + (3)(-11)$

$= 2x^4 + 8x^2 - 22x + 3x^3 + 12x - 33$

$= 2x^4 + 3x^3 + 8x^2 - 22x + 12x - 33$

$= 2x^4 + 3x^3 + 8x^2 - 10x - 33$

Notice that we write this question in descending order of powers (biggest to smallest exponents)

Verification after Multiplying Polynomials

It is important to "check your work" when doing mathematics. This process is vital for not only catching any accidental mistakes, but also in showcasing that you know how a problem works - a sign of higher order thinking.

To verify an answer, simply substitute a chosen integer into your equation (it can be anything, but small numbers like 0, 1, and 2 are often easiest). Use the same integer in both the prestaged-question (the given problem) and in the poststaged-answer (your found product). After substituting in a constant for your variables (numbers for letters), simplify the equation. If the two answers come out the same, you have done the multiplication correctly. If they come out differently, you have made a mistake somewhere along the way (first see if it is a bookkeeping error, before doing the problem completely over again).

Multiply and then verify: $(5x-1)(2x-3)$

$$(5x-1)(2x-3)$$

$$=(5x)(2x)+(5x)(-3)+(-1)(2x)+(-1)(-3)$$

$$=(10x^2-15x-2x+3)=10x^2-17x+3$$

Now check, by substituting $x=1$ into both stages of the equation:

$(5x-1)(2x-3),\ x=1$ AND $10x^2-17x+3,\ x=1$

$=(5(1)-1)(2(1)-3)$	$=(10(1)^2)-17(1)+3$
$=(5-1)(2-3)$	$=(10\times 1)-17+3$
$=(4)(-1)=(-4)$	$=(10-17+3)=(-4)$

Since both sides of the equation result in -4, the answer is verified.

General Rules: Here are some further common examples that should be concretely understood.

Square of a Binomial:

$$(a+b)^2 = (a+b)(a+b) = (a^2+ab+ba+b^2) = (a^2+ab+ab+b^2) = a^2+2ab+b^2$$

$$(a-b)^2 = (a-b)(a-b) = (a^2-ab-ba+b^2) = (a^2-ab-ab+b^2) = a^2-2ab+b^2$$

Example: $(x-1)^2$

$$=(x-1)^2 = (x-1)(x-1)$$
$$=(x^2-1x-1x+1) = x^2-2x+1$$

Product of Sum and Difference:

$$(a-b)(a+b) = (a^2+ab-ba-b^2) = (a^2+ab-ab-b^2) = (a^2-b^2)$$

Example: (y+2)(y-2)

$$=(y+2)(y-2)$$
$$=(y^2-2y+2y-4) = y^2-4$$

Common Errors

Polynomial	Error Version	Correct Version
$(x+2)^2$	$(x+2)^2 \neq x^2+4$	$(x+2)^2 = (x+2)(x+2) = x^2+4x+4$

2.3 - Removing Common Factors

Factoring is very similar to dividing, but as they have different names, they have different processes and outcomes. In general, *dividing* is a final action and something you will do when working with *equations* (questions that have = signs in them). *Factoring* is a more intermediate step, one that you will use near the beginning of your solution, and is used when dealing with *expressions* or statements (problems that do not already contain an = sign).

Removing a factor is the most basic form of factoring: If every term in a polynomial consists of duplicate numbers and/or variables, then we can "factor out" the "common factors" to reduce the problem to its simplest form. Unlike in division where the factors are completely erased from the problem, when factoring, we *cast out* the "common factors" by simply placing them on the outside of the simplified statement's brackets.

Steps:

1) Look at each term, noting the numbers and variables

2) If every term shares a factor, cast it out from the statement, dividing each term by that value along the way.

3) Reassess. You may have to do step 2 more than once; if this is the case, multiply all your casted out factors together.

Note A: If an entire term is itself a common factor, a 1 is left in its place.
Note B: You can check your work by multiplying the factor back into your equation. The answer should be equal to the original question.
Note C: Not all factors are monomials, common factors can be any-sized polynomials (see example "e").

Factor:

a) $4x + 10$

$= 2(2x + 5)$

b) $10xy - 5y$

$= 5(2xy - 1y) = 5y(2x - 1)$

c) $60x^4 - 30x^3 + 45x^2$

$= 3(20 - 10x^3 + 15x^2) = 3 \cdot 5(4x^4 - 2x^3 + 3x^2) = 15x^2(4x^2 - 2x^1 + 3)$

d) $xyz + xz$

$= xz(y + 1)$

e) $k(k - 2) + 3(k - 2)$

$= (k - 2)k + 3 = (k - 2)(k + 3)$

<u>Factor by Grouping</u>

Polynomials of different sizes will have different factoring techniques. When a polynomial has four terms, we can use the "factor by grouping" method.

Steps:

1) Group the polynomial into two pairs of two: Place brackets around each pair of terms. Make sure you leave an addition operator between the two sets of brackets.
2) Remove the common factor from the two binomials (this will leave you with two identical binomials inside of your brackets, and two terms that have been factored out).
3) Collect the factored out terms (creating a new binomial), as well as one of the duplicate binomials. These two binomials is your successfully factored version of your original quadnomial.

Eg. Factor the following

a) $2x^3 - 8x^2 - 3x + 12$

$= (2x^3 - 8x^2) + (-3x + 12)$

$= 2x^2(x^1 - 4) - 3(x - 4)$

$= (2x^2 - 3)(x - 4) = (x - 4)(2x^2 - 3)$

b) $3x^2 + 6x + 4x + 8$

$= (3x^2 + 6x) + (4x + 8)$ OR $= (3x^2 + 4x) + (6x + 8)$

$= 3x(x + 2) + 4(x + 2)$ OR $= x(3x + 4) + 2(3x + 4)$

$= (3x + 4)(x + 2)$ OR $= (x + 2)(3x + 4)$ THEY ARE EQUAL!!

c) $3b^3 + a^2b - 3a - ab^4$

$= (3b^3 - 3a) + (a^2b - ab^4)$

$= 3(b^3 - a) + ab(a - b^3) \Rightarrow 3(b^3 - a) - ab(-a + b^3) \Rightarrow 3(b^3 - a) - ab(b^3 - a)$

$= (b^3 - a)(3 - ab)$

2.4-Factoring $1x^2 + bx + c$

Consider the example: $(x+m)(x+n) \Rightarrow x^2 + nx + mx + mn \Rightarrow x^2 + (n+m)x + mn$

Notice:

1) The product of two binomials (generally) is a trinomial.
2) The first and last terms of the trinomial are products of the first terms (x and $x = x^2$) and last terms (m and $n = mn$) of the binomial respectively.
3) The coefficient of the middle term of the trinomial is the sum of m and n $(m+n)$.

In general, to factor the polynomial $ax^2 + bx + c$, look for two numbers that when multiplied $= ac$ and when added $= b$.

This concept will be exactly the same for the next section of our notes (2.5), the only difference is that in this section we are encountering problems that have an a value equal to 1.

TIPS:

1) Before you start the process of factoring the trinomial, try to see if you can simplify the polynomial by casting out a common factor - this will make your numbers easier to deal with.
2) To find the trinomial factors, first list the product factors of ac before trying to think of potential sums of b. There are actually only a few factors of any product, but an infinite number of possibilities for sums, so this method will narrow your search immediately.

Factor:

a) $x^2 + x - 6$

×	+	We know that for ac to be negative, our terms must be $(+ \& -)$, and for b to be positive, the larger number must be the positive one.
$ac = -6$	$b = +1$	
+3 AND −2		

$(\_\_\_\_\_\_)(\_\_\_\_\_\_)$

$(\ x \qquad)(\ x \qquad)$

$(\quad x+3 \quad)(\quad x-2 \quad) = (x+3)(x-2)$

Check with FOIL:

$(x+3)(x-2) \Rightarrow (x^2 - 2x + 3x - 6) \Rightarrow (x^2 + 1x - 6)$

Because this gives us the same answer as our original problem, we know we are correct!

b) $-x^2 - 9x - 20$

$= -(x^2 + 9x + 20)$

×	+	We know that ac is positive, so our terms are either $(- \& -)$ or $(+ \& +)$. As b is positive, our terms must be $(+ \& +)$ to produce as a positive sum.
$ac = +20$	$b = +9$	
+4 AND +5		

$-(\_\_\_ + \_\_\_)(\_\_\_ + \_\_\_)$

$-(\ x \ + \_\_\_)(\ x \ + \_\_\_)$

$-(\quad x+4 \quad)(\quad x+5 \quad) = -(x+4)(x+5)$

Check with FOIL: $-(x+4)(x+5) \Rightarrow -(x^2 + 5x + 4x + 20) \Rightarrow -(x^2 + 9x + 20)$

$= -x^2 - 9x - 20$

c) $-3x^3+27x^2-54x$
$=-3x(x^2-9x+18)$

×	+	We know that ac is positive, so our terms are either $(-\&-)$ or $(+\&+)$. As b is negative, our terms must be $(-\ \&\ -)$ to produce as a negative sum.
$ac=+18$	$b=-9$	
-3 AND -6		

-3x(x ____)(x ____)

-3x($x-3$)($x-6$) $=-3x(x-3)(x-6)$
Check with FOIL: $-3x(x-3)(x-6) \Rightarrow -3x(x^2-6x-3x+18) \Rightarrow -3x(x^2-9x+18)$
$=(-3x^3+27x^2-54x)$

d) $x^2-xy-12y^2$

×	+	We know that ac is negative, so our terms are either $(-\&+)$ and for b to be negative, the larger number must be the negative one.
$ac=+12$	$b=-1$	
$+3$ AND -4		

(____+____)(____−____)

(x +____y)(x −____y)

($x+3y$)($x-4y$) $=(x+3y)(x-4y)$

Check with FOIL: $=(x+3y)(x-4y)=(x^2-4xy+3xy-12y^2)=(x^2-1xy-12y^2)$

2.5 - Factoring $ax^2 + bx + c$

In section 2.4 we factored polynomials that had an 'a-value' of 1. In this section we will observe polynomials that have rational a-values.

To solve these types of problems there are several methods: the trial and error FOIL method (p. 95), the ac-method (p.97), and the decomposition method (which will be the primary method used in our course - but not explained in our workbook).

Factoring by decomposition:

This method utilizes the tactics of grouping (in 2.3) and FOIL (in 2.4).

Steps:

1. Find two integers that when multiplied = ac and sum to b.
2. As the two integers sum to b, split the b value into these two addends - creating a quadnomial-like polynomial.
3. Use the factor by grouping method to complete your factor.

Steps for grouping:

4) Group the polynomial into two pairs of two: Place brackcts around each pair of terms. Make surc you leave an addition operator between the two sets of brackets.
5) Remove the common factor from the two binomials (this will leave you with two identical binomials inside of your brackets, and two terms that have been factored out).
6) Collect the factored out terms (creating a new binomial), as well as one of the duplicate binomials. These two binomials is your successfully factored version of your original quadnomial.

Eg. $2x^2 + 7x - 4$

×	+	$7x = 8x - 1x$
$ac = -8$	$b = 7$	
8 AND −1		

$2x^2 + 7x - 4$

$= 2x^2 + 8x - 1x - 4$

$= (2x^2 + 8x) + (-1x - 4)$

$= 2x(x + 4) - 1(x + 4)$

$= (2x - 1)(x + 4)$

Eg. $12x^2 - 5x - 2$

×	+	$-5x = -8x + 3x$
$ac = -24$	$b = -5$	
−8 AND +3		

$12x^2 - 5x - 2$

$= 12x^2 - 8x + 3x - 2$

$= (12x^2 - 8x) + (3x - 2)$

$= 4x(3x - 2) + 1(3x - 2)$

$= (3x - 2)(4x + 1)$

Eg. $-6x^3 + 4x^2 + 16x$
$= -2x(3x^2 - 2x - 8)$

$\times$	$+$	$-2x = -6x + 4x$
$ac = -24$	$b = -2$	
-6 AND $+4$		

$-2x(3x^2 - 2x - 8)$
$= -2x(3x^2 - 6x + 4x - 8)$
$= (-2x)(3x^2 - 6x) + (4x - 8)$
$= (-2x)3x(x - 2) + 4(x - 2)$
$= (-2x)(3x + 4)(x - 2)$

Check with FOIL: $(-2x)(3x+4)(x-2) \Rightarrow (-6x^2 - 8x)(x-2)$
$\Rightarrow (-6x^3 + 12x^2 - 8x^2 + 16x) \Rightarrow (-6x^3 + 4x^2 + 16x)$

2.6 - Special Factors

Some collections of polynomials are unique, as form expressions that have special factors. The two we will cover in this section are the "difference of squares" and "perfect square trinomials." After practicing, you should be able to recognize these special polynomials.

Difference of Squares

$(a^2 - b^2) = (a + b)(a - b) = a^2 - ab + ba - b^2 - a^2 - ab + ab - b^2 = a^2 - b^2$

$x^2 - 25 = (x - 5)(x + 5) = x^2 - 5x + 5x - 25 = x^2 - 25$

Notice: a and b are perfect squares, separated by a subtraction sign.
Common error: $x^2 + 16 \neq (x + 4)(x - 4)$

Examples:

1. $25x^2 - 9 = (5x - 3)(5x + 3)$

2. $x^8 - 16y^2 = (x^4 - 4y)(x^4 + 4y)$

   $-2y$ $+8y$

3. $8x^2 - 50y^2 = 2(4x^2 - 25y^2) = (2)(2x - 5y)(2x + 5y)$

Perfect Square Trinomials

A trinomial that is a square of a binomial is called a "perfect square trinomial."
For the polynomial $ax^2 + bx + c$ to be a perfect square:

a) The last term (c) must be positive, and a perfect square.
b) The first term (a) must be a perfect square.
c) The coefficient of the middle term is the product of the square root of a and c, doubled (this value can either be positive or negative).

Factoring Perfect Square Trinomials:

$(a+b)^2 = (a+b)(a+b) = (a^2 + ab + ba + b^2) = (a^2 + ab + ab + b^2) = (a^2 + 2ab + b^2)$
$= a^2 + 2ab + b^2 = (a+b)^2$

$(a-b)^2 = (a-b)(a-b) = (a^2 - ab - ba + b^2) = (a^2 - ab - ab + b^2) = (a^2 - 2ab + b^2)$
$= a^2 - 2ab + b^2 = (a-b)^2$

Eg. $x^2 + 8x + 16 = (x + 4)(x + 4) = (x + 4)^2$

Common error: $(x+4)^2 \neq (x^2 + 16)$. $(x+4)^2 = (x+4)(x+4)$ You must multiply the entire binomial by itself!

To solve, take the square root of the first term (a), the sign of the middle term, and square root of the last term (c). Bracket it all, and write the squared superscript.

Examples:

1. $x^2 + 18x + 81$

$$= (x+9)^2 = (x+9)(x+9) = (x^2 + 9x + 9x + 81) = (x^2 + 18x + 81)$$

2. $4q^2 - 16qp + 16p^2$

$$= (2q-4p)^2 = (2q-4p)(2q-4p) = (4q^2 - 8qp - 8qp + 16p^2)$$
$$= (4q^2 - 16qp + 16p^2)$$

Notes: