Chapter 5 - Linear Equations

5.1 - Different Forms of Linear Equations

We will continue our investigation of solving and graphing linear equations.

Standard Form of a Linear Equation: *Ax+By=C.*

If A, B, and C are real numbers, the equation $Ax+By=C$ is called the standard form equation - it is often used when solving problems algebraically and when finding axis intercepts.

General Form: *Ax+By+C=0*

The only difference here is that the constant 'C' has been moved to the other side of the equal sign; putting every value on one side, and zero on the other.

Slope-Intercept Form of a Linear Equation: *y=mx+b*

m = slope
b = y-intercept
y & x = points on the line

We can convert our equations from one form to another easily:

$$Ax+By=C \;->\; By=-Ax+C \;->\; y=-\tfrac{A}{B}x+\tfrac{C}{B}$$

This means from our standard form $(Ax+By=C)$ that:

The slope = $-\frac{A}{B}$

The y-intercept = $\frac{C}{B}$ $(0,\frac{C}{B})$

Consider the example: $3x-4y=12$

Slope: $-\frac{A}{B}=-\frac{(3)}{(-4)}=\frac{3}{4}$

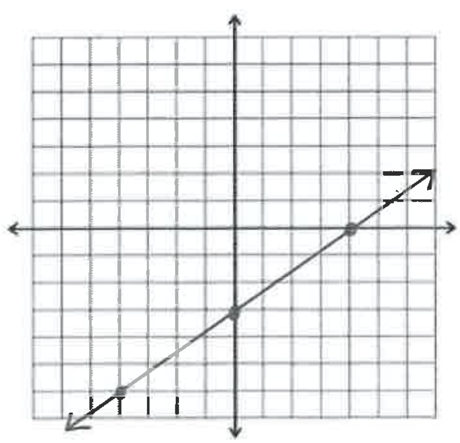
y-int: $\frac{C}{B}=\frac{12}{-4}=-3 \Rightarrow (0,-3)$

Conversion:

$3x-4y=12$

$-4y=-3x+12$

$y=\frac{3}{4}x-3$



Graphing a Line Using the <u>Slope</u> and <u>y-intercept</u>

Steps:

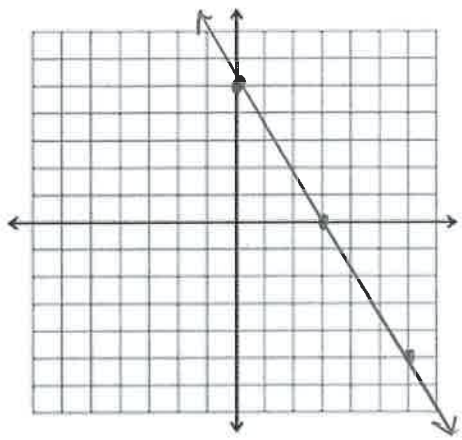
1) Rewrite the equation so it is in the form y=mx+b
2) Identify the y-intercept (b) and plot it on the graph
3) Starting from the y-int, use the slope to plot two more points
4) Draw your line

Graph: $5x + 3y = 15$

$5x+3y=15$

$3y=-5x+15$

$y=\frac{-5}{3}x+5$



Graphing a Line Using the <u>Slope</u> and a <u>Point</u>

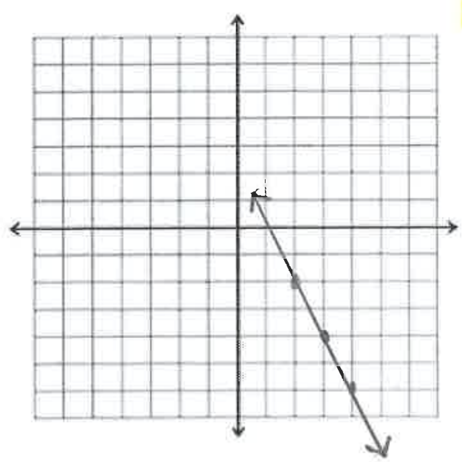
Steps:

1) Locate and graph the provided point
2) Starting from that plotted-point, use the slope to plot two more points
3) Draw your line

Graph the line through (3, -4) with slope -2

Point = $(\overset{x}{3},\overset{y}{-4})$

slope = m = $\frac{-2}{1}$ ↓ →



Writing an Equation of a Line Using a Slope and a Point

Just as important as being able to graph your data is the ability to write it into an equation.

Remember: we are fitting our values into our overall equation of $y=mx+b$

Write the equation of the line with slope -1 that runs through (6,4)

$y=mx+b$

$y=4$ $m=-1$

$x=6$ $b=?$

$4=(-1)(6)+b$

$4=-6+b$

$10=b$ *Answer* : $y=-1x+10$

Point-Slope Form of a Linear Equation

NEW FORMULA: $y-y_1=m(x-x_1)$

is known as Point-Slope form and it is derived from re-arranging the definition of slope, $m=\frac{y_2-y_1}{x_2-x_1}$.

$m=$ slope $x_1 \& y_1=$ *provided points*

$x \& y=$ placeholders for future points

Write the equation of a line in point-slope form for a line with slope 3 that passes through (5, 2).

$y-y_1=m(x-x_1)$

$m=3$

$x_1=5$

$y_1=2$

$y-2=3(x-5)$

$y-2=3x-15$

$+2$ $+2$

$\boxed{y=3x-13}$

Write the equation of a line in general form for a line with slope -¾ that passes through (-1, 2).

from $y-y_1=m(x-x_1) \rightarrow Ax+By+C=0$

$y-2=-\frac{3}{4}(x+1)$

$4y-8=-3(x+1)$

$4y-8=-3x-3$

$\boxed{3x+4y-5=0}$

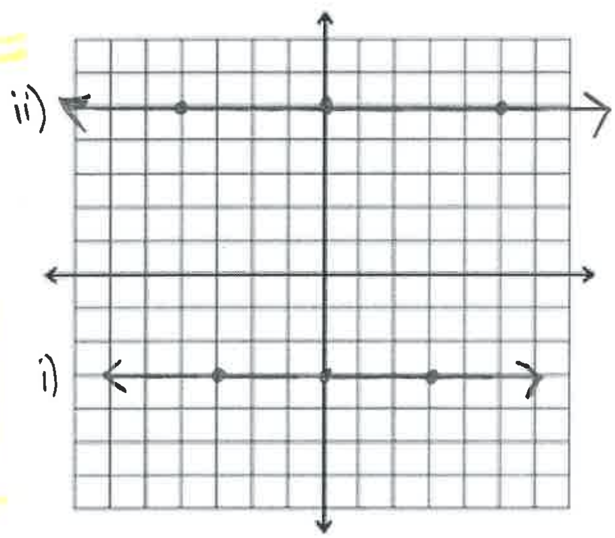
5.2 - Special Cases of Linear Equations

Horizontal Lines *(y=k)*

A horizontal line can be thought as all the points on a graph where *y* has the same value throughout. Thus, if our line does not go *up* or *down*, it remains flat, and thus has a slope of zero (0).

Graph the lines:

i) $y=-3$ ii) $y=5$

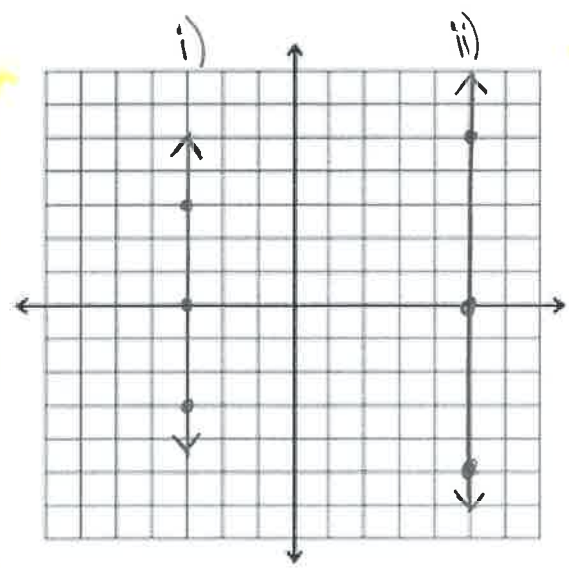


Vertical Lines *(x=k)*

A vertical line can be thought as all the points on a graph where *x* has the same value throughout. Thus, if our line does not go *left* or *right*, it remains standing, and thus has a slope that is undefined (∅).

Graph the lines:

i) $x=-3$ ii) $x=5$



Writing the Equation of a Lines Through Two Points

Steps:

1) Find the slope of your equation using the full formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$.
2) Use either of the points to fill in the remainder of your point-slope formula: $y - y_1 = m(x - x_1)$. Remember to leave x and y blank.

Write the equation of the line passing through the points A(-5,3) and B(2,4)

x_1, y_1 x_2 y_2

① $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(4)-(3)}{(2)-(-5)} = \frac{1}{7} = m$

② $y-(3) = \frac{1}{7}(x-(-5)) \Rightarrow y-3 = \frac{1}{7}(x+5) \Rightarrow y-3 = \frac{1}{7}x + \frac{5}{7}$

$\Rightarrow y = \frac{1}{7}x + \frac{5}{7} + 3 \Rightarrow y = \frac{1}{7}x + \frac{5}{7} + \frac{21}{7} \Rightarrow \boxed{y = \frac{1}{7}x + \frac{26}{7}}$

Parallel and Perpendicular Lines

As we continue to build on our knowledge, we can solve further problems around parallel and perpendicular lines.

Recall: Parallel lines are two lines that have the exact same slope.
Perpendicular lines have negative-reciprocal slopes.

Examples: Are the two lines related to one another?

a) $3x - 5y = 4$ *and* $5x - 3y = 4$

Option 1) Convert to $y=mx+b$

$3x - 5y = 4$

$-5y = -3x + 4$

$\boxed{y = \frac{3}{5}x - \frac{4}{5}}$ ① $\boxed{m = \frac{3}{5}}$

$5x - 3y = 4$

$-3y = -5x + 4$

$\boxed{y = \frac{5}{3}x - \frac{4}{3}}$ ② $\boxed{m = \frac{5}{3}}$

$\therefore$ Not related

Option 2) Read off of Standard Form

$3x - 5y = 4$

$Ax + By = C$

$m = -\frac{A}{B} = \frac{-3}{-5} = \boxed{\frac{3}{5}}$

$5x - 3y = 4$

$Ax + By = C$

$m = -\frac{A}{B} = \frac{-5}{-3} = \boxed{\frac{5}{3}}$

$\therefore$ Not related (Still :)

b) $2x - 5y = -3$ *and* $10x + 4y = 1$

Option 1) Convert to $y=mx+b$

$2x-5y=3$

$-5y=-2x+3$

$\boxed{y=\frac{2}{5}x-\frac{3}{5}}$ $\boxed{m=\frac{2}{5}}$

$10x+4y=1$

$4y=-10x+1$

$\boxed{y=-\frac{10}{4}x+\frac{1}{4}}$ $\boxed{m=-\frac{10}{4}=\frac{-5}{2}}$

$\therefore$ Slopes are negative reciprocal

$\therefore \perp$

Option 2) Read off of Standard Form

$2x-5y=-3$

$Ax+By=C$

$m=-\frac{A}{B}=-\frac{2}{-5}=\boxed{\frac{2}{5}}$

$10x+4y=1$

$Ax+By=C$

$m=-\frac{10}{4}=\boxed{\frac{-5}{2}}$

Slopes are neg. recip.

$\therefore \perp$

5.3 - Equations of Parallel and Perpendicular Lines

To write the equation of a line only a single point and slope are needed. That said, for some problems, the information provided may not be explicitly provided - it takes practice and definitional understanding to make the appropriate connections.

Examples:

a) Write an equation that is parallel to the line $2x - 3y = 6$, and has a y-intercept of 1.

$Ax+By=C$

① find the slope : $-\frac{A}{B}=m=\frac{-2}{-3}=\boxed{\frac{2}{3}}=m$

② locate and use your given point : $(0,1)$

③ use slope-point form : $y-y_1=m(x-x_1)$

$y-1=\frac{2}{3}(x-0)$

$\boxed{y=\frac{2}{3}x+1}$ (slope intercept form)

$y=\frac{2}{3}x+1$

$3y=2x+3 \rightarrow \boxed{-2x+3y=3}$ (standard form)

b) Write an equation that is perpendicular to the line $2x + 4y = 7$ going through the point (-4, 3).

$2x+4y=7 \Rightarrow 4y=-2x+7 \Rightarrow y=\frac{-2}{4}x+\frac{7}{4}$

① $\boxed{y=\frac{-1}{2}x+\frac{7}{4}} \Rightarrow m=\frac{-1}{2} \therefore m_{\perp}=\frac{2}{1}$

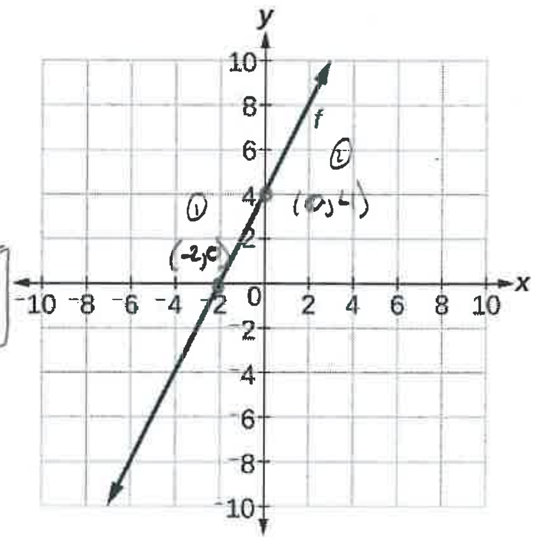
② $y-y_1=m(x-x_1) \Rightarrow y-(3)=2(x-(-4))$

$y-3=2x+8$

③ $\boxed{y=2x+11}$

c) Determine the equation of a line parallel and perpendicular to the graph going through the given point, in standard form.

Given point: (-4, 6)



① Slope of graph:

$m=\frac{y_2-y_1}{x_2-x_1} = \frac{4-0}{0-(-2)} = \frac{4}{2} = \boxed{\frac{2}{1}}$

Parallel: (same slope)

$m=2$

$y-y_1=m(x-x_1)$

$y-6=2(x-(-4))$

$y-6=2(x+4)$

$y-6=2x+8 \Rightarrow \boxed{y=2x+14}$

notice new y-int.

notice easy slope

Perpendicular (negative inverse slope)

$m=\frac{-1}{2}$

$y-y_1=m(x-x_1)$

$y-6=-\frac{1}{2}(x-(-4))$

$y-6=-\frac{1}{2}(x+4)$

$y-6=-\frac{1}{2}x-2 \Rightarrow \boxed{y=\frac{-1}{2}x+4}$

5.4 - Linear Applications and Modelling

Graphs are effective visual tools for representing data, they present data quickly and easily. Let's investigate some.

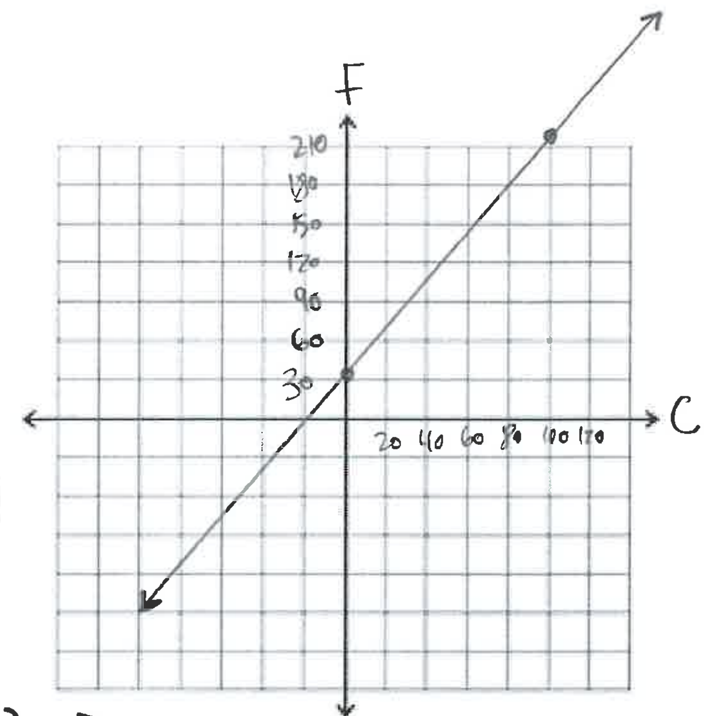
a) Celsius vs. Fahrenheit: Water freezes at 32°F, or 0°C. Water boils at 212°F, or 100°C. Graph the linear relation between °C and °F, and find a formula that converts between the two units.

$(x, y) \Rightarrow (C, F)$

freezing $(0, 32)$ Boiling $(100, 212)$

$\text{Slope} = m = \frac{212-32}{100-0} = \frac{180}{100} = \boxed{\frac{9}{5}}$

$\boxed{y = \frac{9}{5}x + 32} \Rightarrow F = \frac{9}{5}C + 32$



☆ Calculate 20°C

$F = \frac{9}{5}(20) + 32 \Rightarrow F = \frac{180}{5} + 32 \Rightarrow F = 36 + 32 = \boxed{68°F}$

b) It cost \$1500 to produce your first ever music album, and you make \$.25 for every download.

a) Create an equation that represents this problem.

$y = .25x - 1500$

b) How many downloads until you break even?

$0 = .25x - 1500$

$1500 = \frac{1}{4}x$ $\boxed{x = 6000}$

c) If a million people download your album, how much money did you make?

$y = 1000000(.25) - 1500$

$\boxed{y = \$248{,}500}$

5.5 - Function Notation

We are now going to add in new mathematical notation, but not change anything else. Everything we have learned in this unit will remain the same, but the way our equations look will now take on a more succinct shape.

The notation $f(x)$ which stands for "the function of x" or "f of x" is another way of writing y.

Note: $f(x)$ does NOT mean the variable f times x.

So, the shift we are making in this section is changing problems that once looked like this: $y = 3x + 4$ now will look like this: $f(x) = 3x + 4$.
This new notation allows us to more succinctly write out our problems.
What we used to write: Given $y = 3x + 4$, find y when $x=5$
What we can now write: Given $f(x) = 3x + 4$, find $f(5)$.

Lets solve this problem:
Given $f(x) = 3x + 4$, find $f(5)$.
$f(5) = 3(5) + 4$
$f(5) = 19$
Therefore, when $x = 5$, $y = 19$ OR for $x = 5$, $f(5) = 19$ OR $(5,19)$

This type of notation, known as "function notation" is a very powerful tool, one that you will use extensively in Math 12.

Let's do some examples that exercise our new notation:

a) Given $f(x) = 4x - 8$, determine the coordinates of one point on the line for $f(2)$.

$f(2) = 4(2) - 8$

$f(2) = 8 - 8 = 0$

$f(2) = 0$

$\therefore$ when $x = 2$, $y = 0$

$f(2) = 0$

$\therefore$ $(2, 0)$

b) Given $f(x) = 3x - 5$, determine the coordinates of the point where $f(x)$=13.

$\uparrow y$

$13 = 3x - 5$

$18 = 3x \rightarrow 6 = x$

$(6, 13)$

c) Determine the slope-intercept function for $f(1)$=8 and $f(-3)$=-4

$x_1 \; y_1 \quad x_2 \; y_2$

$$\frac{(-4)-(8)}{(-3)-(1)} = \frac{-12}{-4} = \boxed{3 = m}$$

$$y - y_1 = m(x - x_1) \rightarrow y - 8 = 3(x - 1)$$

$$y - 8 = 3x - 3 \rightarrow \boxed{y = 3x + 5}$$

d) If $f(x) = 3x + 2$, what is: *i)* $f(6x)$ *ii)* $f(x\text{-}3)$

plug in as x

i) $f(6x) = 3(6x) + 2$

$\boxed{f(6x) = 18x + 2}$

ii) $f(x-3) = 3(x-3) + 2$

$f(x-3) = 3x - 9 + 2$

$\boxed{f(x-3) = 3x - 7}$

e) If $f(x) = 2x + 1$, determine $\frac{f(x+h)-f(x)}{h}$

$$\frac{[2(x+h)+1] - [2(x)+1]}{h} \Rightarrow \frac{2x + 2h + 1 - 2x + 1}{h}$$

$$\Rightarrow \frac{2x - 2x + 2h + 1 - 1}{h} \Rightarrow \frac{2h}{h} = \boxed{2}$$

Notes: