Chapter 8 - Trigonometry

8.1 - Sine, Cosine, and Tangent for Right Angles

We will now take on a very different domain in our math course: the concept of trigonometry. Trigonometry comes from the Greek words *trigon* (meaning triangle) and *metria* (meaning measurement). Trigonometry has been used for over 2500 years and has significant importance for an array of subjects such as physics, astronomy, engineering, and business.

Note: In our unit, we will only discuss right-angle triangles (triangles with a 90-degree angle).

Additionally, keep in mind that the size of any triangle does not matter, only the ratio of the lengths of the sides and angles.

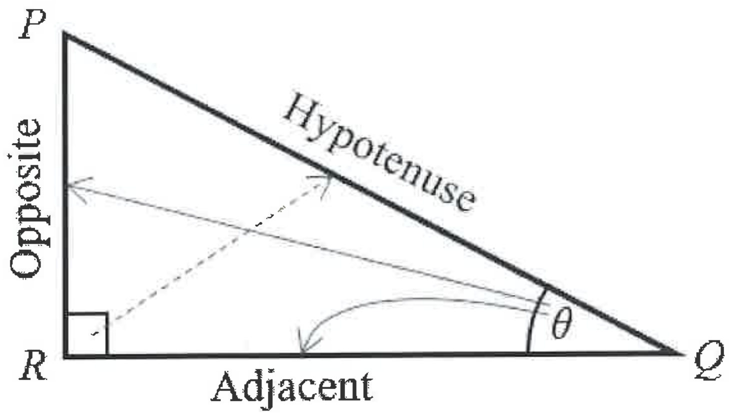
Naming the Side of a Right Triangle

A right triangle has one perpendicular angle measured at $90°s$. The side that is directly across from the right-angle is known as the hypotenuse. As the right angle is the largest angle, the hypotenuse is thus the longest side*.

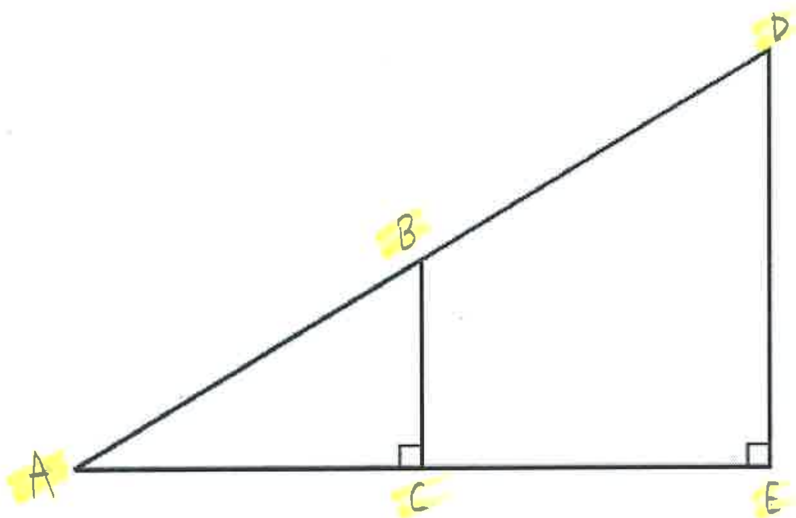
We then have two remaining acute angles (angles that are less than 90 degrees), we then must pick one angle that we will work from (or it will be given to you in the question) and call it "theta" (θ). The side that is directly opposite from this chosen angle (theta) is known as the "opposite side." The remaining side (the one that is touching theta but is not the hypotenuse) is known as the "adjacent side."

*Remember: All angles in a triangle sum to 180°s so if one of them is 90°s that leaves the other two adding up to 90° and since neither of them can be 0° that means they are both less than 90°s (this proves that the right angle is the biggest so the side across from it is the hypotenuse).

Let's take a look at some diagrams for this:



<u>Ratios and angles in right triangles.</u>



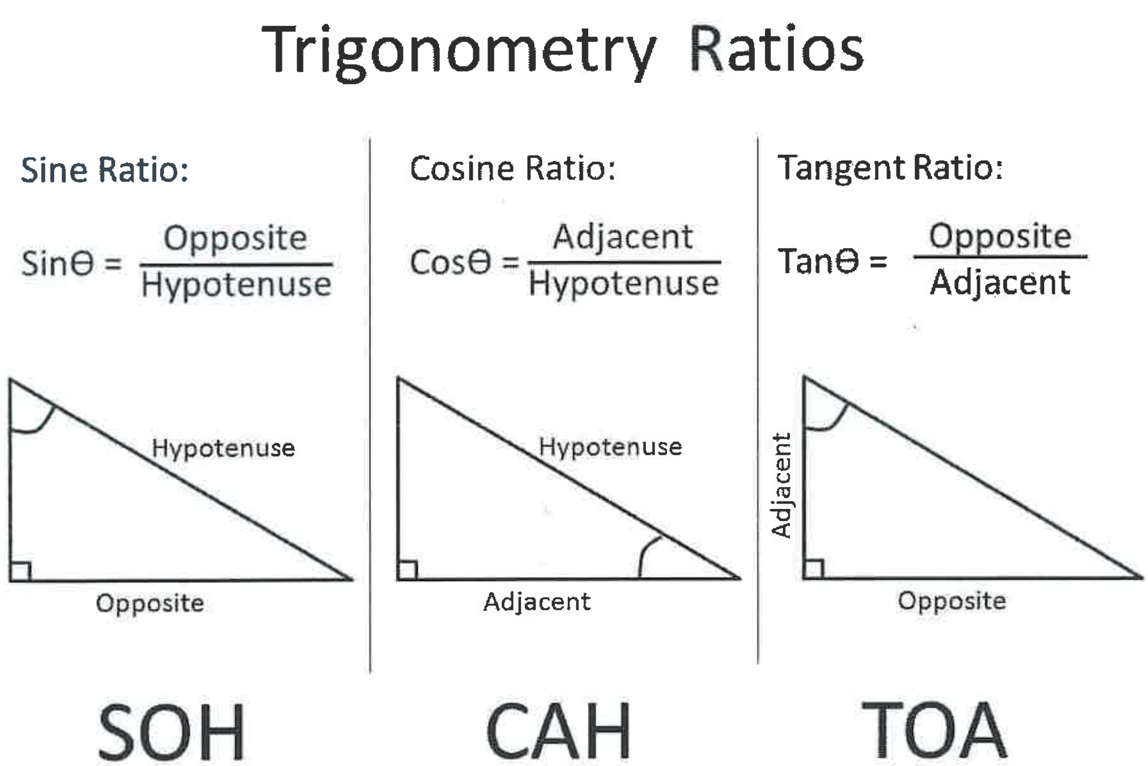
Take our above diagram, we can see that <A is shared by both triangles. <C and <E are the same at 90°, therefore, <B and <D are the same as all the angles in the respective triangles must sum to 180°.
This has implications on the side lengths as well as the ratio between the sides of the triangle, which are determined by the degrees, are all the same. This is regardless of the actual size of our triangles. So if (hypothetically) BC is half of AB, then DE is half the size of AD!

Using Theta as our reference point:
When we observe the length of the opposite side in comparison to that of the hypotenuse the ratio is $\frac{opposite}{hypotenuse}$ which is known as "sine of θ "(sin θ).

When we observe the length of the adjacent side in comparison to that of the hypotenuse the ratio is $\frac{adjacent}{hypotenuse}$ known as "cosine of θ "(cos θ)

When we observe the opposite side in comparison to the adjacent side the ratio is $\frac{opposite}{adjacent}$ known as "tangent of θ "(tan θ).

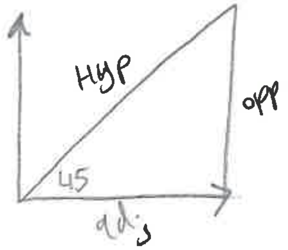
Remember: sine (sin), cosine (cos), and tangent (tan) are just ratios between the lengths of the sides of your triangle. Each ratio is side length is connected to some degree. So if we know either the ratio or the angle we solve between the two.



Example 1:

Draw a diagram, use a calculator, and explain what the answer means for the following values.

a) sin 45

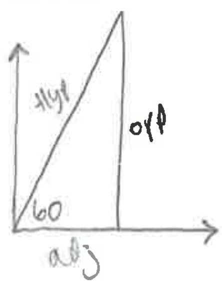


$\sin 45 = .707$

$\sin = \frac{opp}{Hyp} = \frac{.707}{1}$

The opposite side is ~71% the size of the hypotenuse.

b) cos 60

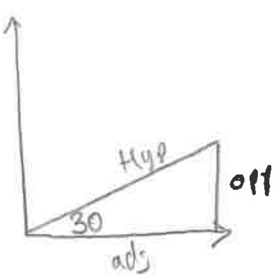


$\cos 60 = .5$

$\cos = \frac{adj}{hyp} = \frac{.5}{1} = \frac{1}{2}$

The adjacent side is half the length of the hypotenuse

c) tan 30



$\tan 30 = .577$

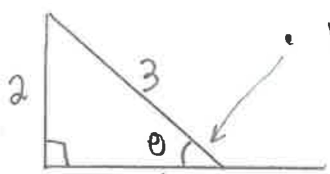
$\tan = \frac{opp}{adj} = \frac{.577}{1}$

The opposite side is ~58% the length of the adjacent side

Draw the triangle, explain what it means, estimate, then find θ (using your calculator) to one decimal place.

a) $\sin\theta = 0.6667 \approx \frac{2}{3}$ ← opp, ← Hyp

$\frac{\text{opp}}{\text{hyp}}$



• looks to be just less than 45°

I just want theta (angle)

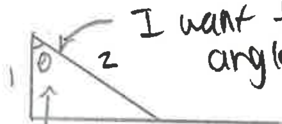
$\sin\theta = .6667$

$\sin^{-1}(\sin\theta) = (.6667)\sin^{-1}$

$\theta = \sin^{-1}(.6667) = \boxed{41.8^\circ = \theta}$

b) $\cos\theta = 0.5 = \frac{1}{2}$ ← adj, ← hyp

$\frac{\text{adj}}{\text{hyp}}$



I want this angle

$\cos\theta = \frac{1}{2}$

$\cos^{-1}(\cos\theta) = \frac{1}{2}(\cos^{-1})$

$\theta = \cos^{-1}\left(\frac{1}{2}\right)$

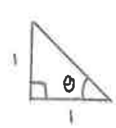
$\boxed{\theta = 60^\circ}$

looks like 30°, 60°, 90° Δ and its the 60° angle.

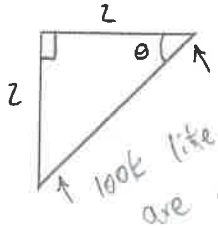
c) $\tan\theta = 1 = \frac{1}{1}$ ← opp, ← adj

$\frac{\text{opp}}{\text{adj}}$

$= \frac{2}{2}$



or



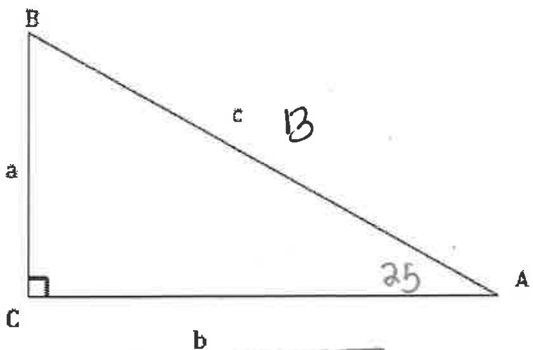
look like they are equal, so maybe 45° each

$\tan\theta = 1$

$\theta = \tan^{-1}(1)$

$\boxed{\theta = 45^\circ}$

Solve the right triangle:



$A = 25°$ $c = 13$

① Find missing angle

$\angle B = 180 - (25 + 90) = \boxed{65°} = B$

② we know hyp = 13
we want opp. use Sin

$\sin 25 = \frac{opp}{13}$

$13 \sin 25 = opp$

$5.49 = opp = a$

③ Pythag

$a^2 + b^2 = c^2$

$b^2 = c^2 - a^2$

$b^2 = (13)^2 - (5.49)^2$

$b^2 = 138.9$

$\boxed{b = 11.8}$

$A = 25°$ $\boxed{a = 5.49}$

$\boxed{B = 65°}$ $b = \boxed{11.8}$

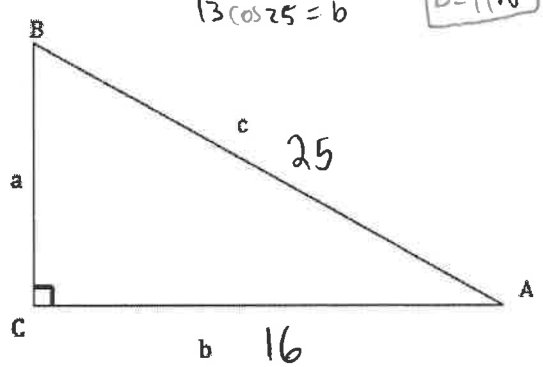
$C = 90°$ $c = 13$

③ Trig

adj/hyp

$\cos 25 = \frac{b}{13}$

$13 \cos 25 = b$ $\boxed{b = 11.8}$



$c = 25$ $b = 16$

① Pythag
$a^2 + b^2 = c^2$ $a^2 = c^2 - b^2$

$a^2 = 25^2 - 16^2$ $a^2 = 369$

$a = 19.2$

② $\angle A$ → I know hyp & adj ∴ cos

$\cos A = \frac{16}{25}$ $A = \cos^{-1}(16/25)$

$\boxed{A = 50°}$

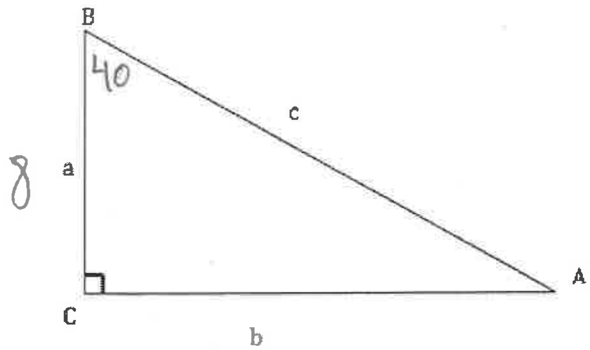
③ $\angle B = 180 - (50 + 90)$

$180 - 140 = \boxed{40°}$

$\boxed{A = 50°}$ $\boxed{a = 19.2}$

$\boxed{B = 40°}$ $b = 16$

$C = 90°$ $c = 25$



B = 40 a = 8

$\boxed{A = 50^\circ}$ $a = 8$

$B = 40^\circ$ $\boxed{b = 6.7}$

$C = 90^\circ$ $\boxed{c = 10.4}$

① $\angle A = 180 - (40+90)$

$180 - (130)$

$= 50^\circ$

② $b \Rightarrow \tan 40 = \frac{b}{8}$

$8 \tan 40 = b$

$\boxed{b = 6.7}$

Trig

③ $c \Rightarrow \cos 40 = \frac{8}{c} \rightarrow c \cos 40 = 8$

$c = \frac{8}{\cos 40}$

$\boxed{c = 10.4}$

③ Pythag

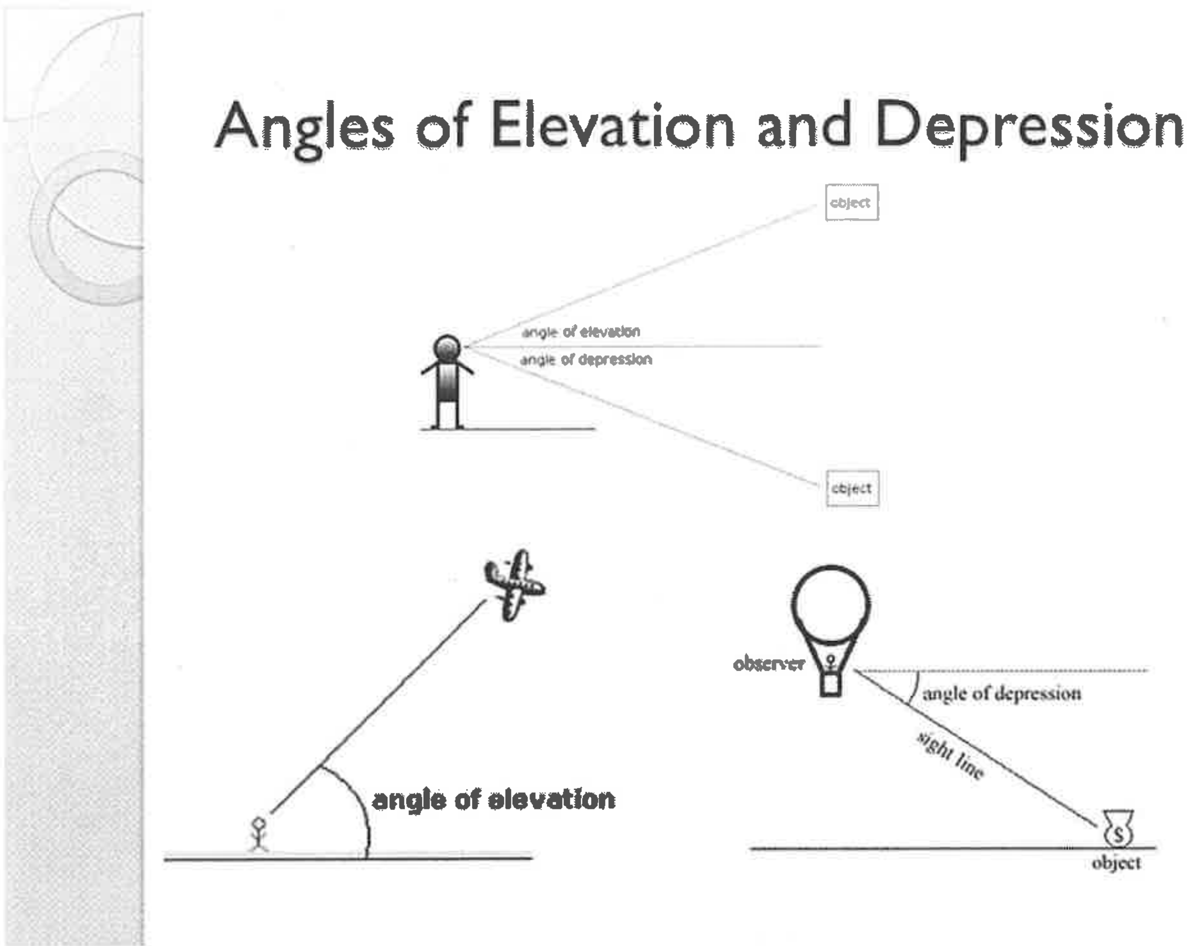
$c^2 = a^2 + b^2$

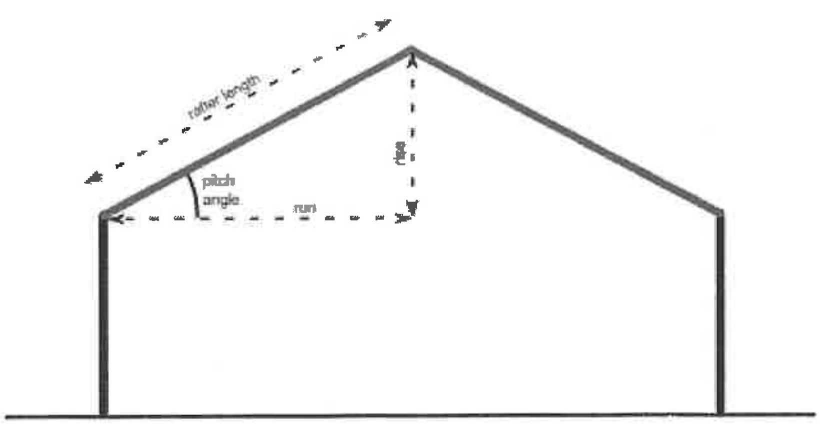
$c^2 = 8^2 + 6.7^2$

$c^2 = 108.89$

$\boxed{c = 10.4}$

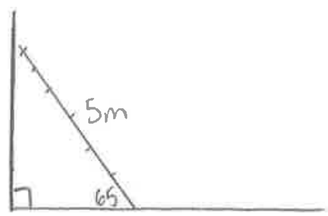
8.4 Applications of Trigonometry





Pitch = Rise/Run

1. A ladder 5 m long, leaning against a vertical wall makes an angle of 65° with the ground.
a) How high on the wall does the ladder reach?
b) How far is the foot of the ladder from the wall?
c) What angle does the ladder make with the wall?



a) I know hyp, I want opp $\therefore$ $\sin = \frac{opp}{hyp}$

$\sin 65 = \frac{opp}{5} \rightarrow 5\sin 65 = opp$

$opp = \boxed{4.5m = wall}$

b) I know hyp, I want adj $\therefore$ $\cos = \frac{adj}{hyp}$

$\cos 65 = \frac{adj}{5} \rightarrow 5\cos 65 = adj$

$adj = \boxed{2.1m = ground}$

c) $\theta = 180 - (90+65)$

$\boxed{\theta = 25^\circ}$

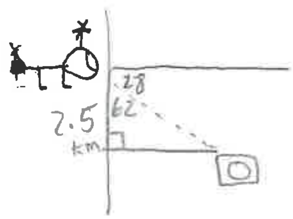
2. A helicopter pilot sights a life raft. The angle of depression is 28 degrees and the helicopter's altitude is 2.5 km

(a) Draw a figure to represent the situation.

(b) What is the horizontal distance from the helicopter to the raft?

(c) If the helicopter flies at a constant speed of 200 km/hr, how long will it be before it is directly above the raft? Assume that the altitude of the helicopter doesn't change.

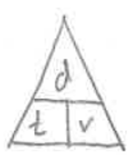
a)



b) I know adj, I want opp ∴ tan

$$\tan 62 = \frac{opp}{2.5} \rightarrow 2.5 \tan 62 = opp = \boxed{4.7\text{ km}}$$

c)



$d = 4.7$ km $V = 200$ km/h

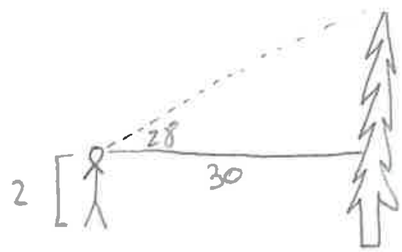
$t = ?$

$$t = \frac{d}{v} = \frac{4.7}{200} = 0.0235\text{ h}$$

$$.0235\text{h}\left(\frac{60\text{min}}{1\text{h}}\right) = \boxed{1.41\text{ min}}$$

$$\boxed{1\text{min } 25\text{sec}}$$

3. A man who is 2 m tall stands on horizontal ground 30 m from a tree. The angle of elevation of the top of the tree from his eyes is 28°. Estimate the height of the tree.



I know adj, I want opp ∴ tan

$$\tan 28 = \frac{opp}{30} \rightarrow 30\tan 28 = \boxed{15.95\ m}$$

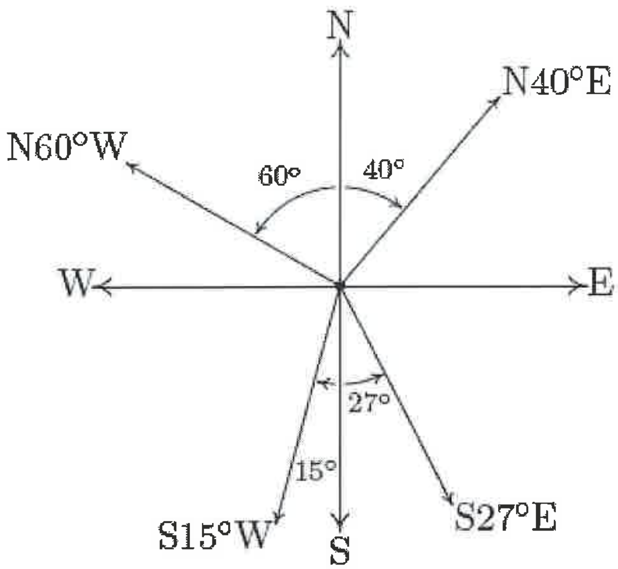
$+ 2m$ ← for height of person

$$= \boxed{17.95m}$$

8.5 - Compound Trigonometry

Navigation was one of the primary uses of trigonometry and represents a direction of travel. A bearing measures the acute angle displacement, and direction, from a fixed north-south (y-axis) line.

Take this diagram:



The bearing in quadrant I is read: "North, 40° East." We say the vertical direction, followed by the displacement off of it.

QII: North, 60° West. QIII: South, 15° West QIV: South, 27° East

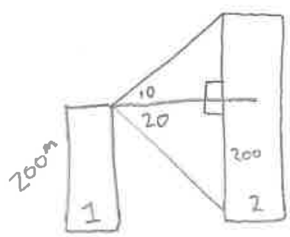
You may also see them written with the displacement first. We could rewrite our definitions like so:

QI: 40° East of North. QII: 60° West of North.

QIII: 15° West of South. QIV: 27° East of South.

Let's do some examples:

1. From the top of a 200 meters high building, the angle of depression to the bottom of a second building is 20 degrees. From the same point, the angle of elevation to the top of the second building is 10 degrees. Calculate the height of the second building.



① need distance between two building as it is the shared value

I have opp, I want adj ∴ $\tan = \frac{opp}{adj}$

$\tan \theta = \frac{opp}{adj}$

$\tan 20 = \frac{200}{adj} \rightarrow adj = \frac{200}{\tan 20}$

$\boxed{adj = 549.5m}$

② I know adj, I want opp ∴ tan

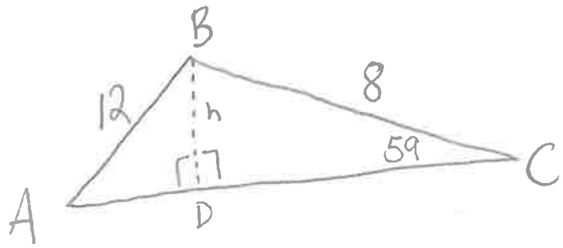
$\tan 10 = \frac{opp}{549.5} \Rightarrow 549.5 \tan 10 = opp$

$\boxed{opp = 96.9m}$

③ $\underset{\text{bottom}}{200} + \underset{\text{top}}{96.9} = \boxed{296.9m}$

all sides different

2. The lengths of side AB and side BC of a scalene triangle ABC are 12 cm and 8 cm respectively. The size of angle C is 59 degree. Find the length of side AC.



$h = BD$

(1) I know hyp, I want opp $\therefore$ sin

$$\sin 59 = \frac{h}{8} \Rightarrow 8 \sin 59 = h$$

$$\boxed{h = 6.86}$$

(2) DC: I know hyp, I want adj $\therefore$ $\cos = \frac{adj}{hyp}$

$$\cos 59 = \frac{DC}{8} \Rightarrow 8 \cos 59 = DC$$

$$\boxed{DC = 4.12}$$

(3) AD: I know two sides, use pythag:

$$a^2 + b^2 = c^2 \Rightarrow b^2 = c^2 - a^2$$

$$b^2 = (12)^2 - (6.86)^2$$

$$b^2 = 144 - 47.06$$

$$b^2 = 96.9 \Rightarrow b = \boxed{9.85 = AD}$$

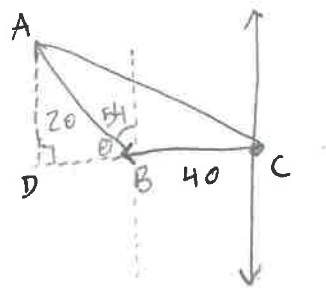
(4)

AC: $9.85 + 6.86 = \boxed{13.97 \text{ cm}}$

3. A ship leaves at noon and heads due west at 20 knots per hour. At 2 pm the ship changes course to N54°W. Find the ship's bearing and distance from the port of departure at 3 pm.

noon $\Rightarrow$ 2pm = 2 hrs @ $\frac{20k}{h}$

= 40 knots



① $\theta = 90 - 54$

$\theta = 36°$

② AD: I know hyp, I want opp $\therefore \sin = \frac{opp}{hyp}$

$\sin 36 = \frac{opp}{20} \Rightarrow 20 \sin 36 = opp = \boxed{11.8 = AD}$

③ DB: I know hyp, I want Adj $\therefore \cos = \frac{adj}{hyp}$

$\cos 36 = \frac{adj}{20} \Rightarrow 20 \cos 36 = adj$

$adj = \boxed{16.2 = DB}$

④ full bottom: $16.2 + 40 = \boxed{56.2 \text{ knots}} = DC$

⑤ AC: $a^2 + b^2 = c^2 \Rightarrow (AD)^2 + (DC)^2 = (AC)^2$

$(11.8)^2 + (56.2)^2 = (AC)^2$

$3297.7 = AC^2$

$\boxed{AC = 57.4 \text{ knots}}$

⑥ Bearing

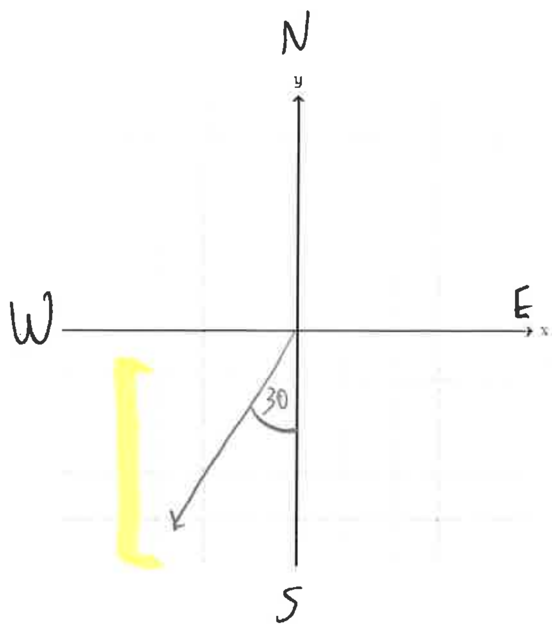
C? $\tan C = \frac{11.8}{56.2}$

$C = \tan^{-1}\left(\frac{11.8}{56.2}\right) = 11.9°$

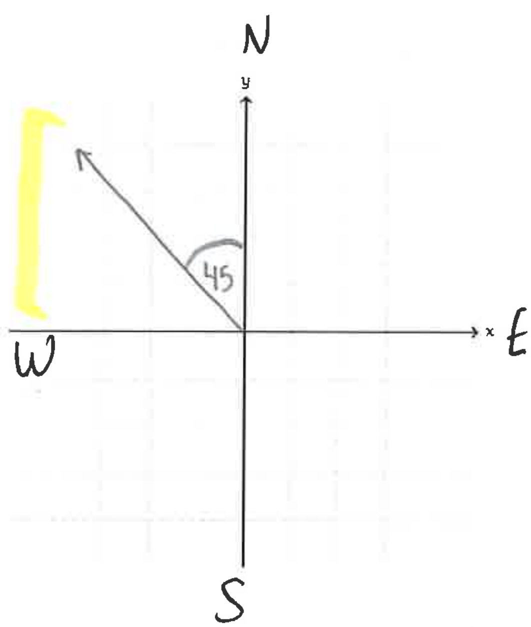
Bearing: $90° - 11.9° = \boxed{78.1°}$

$\boxed{N\ 78.1°\ W}$

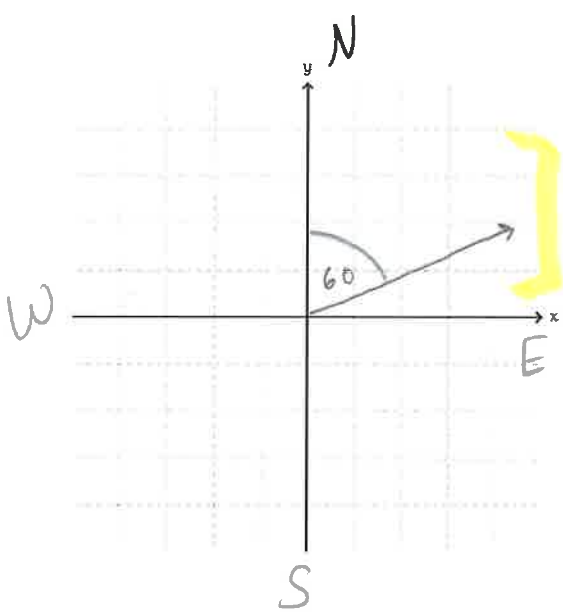
Bearings



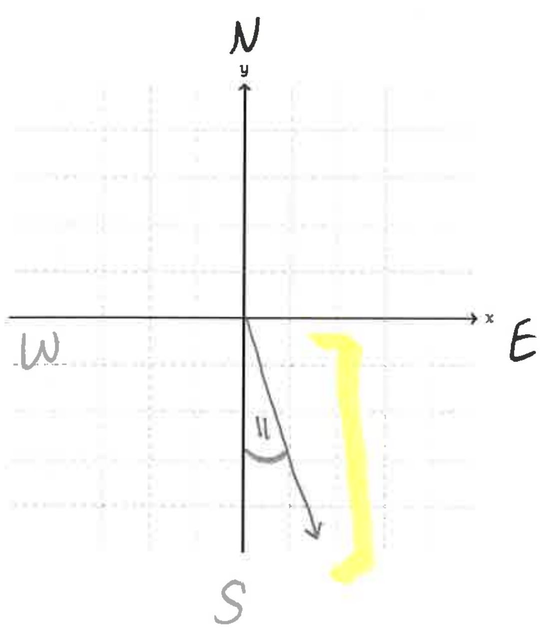
$S30°W$



$N45°W$



$N60°E$



$S11°E$

Notes: