Key

6.1A – Linear Graphing Review Part 1

A linear relation is an equation that relates two variables together (usually x and y) where the variables are of degree 1. Graphing a linear relation creates a LINE.

There are typically three ways to graph a line using its linear equation:

1) Using a table of values
2) Using slope / y-intercept form
3) Using general or standard form

PART 1 – USING A TABLE OF VALUES

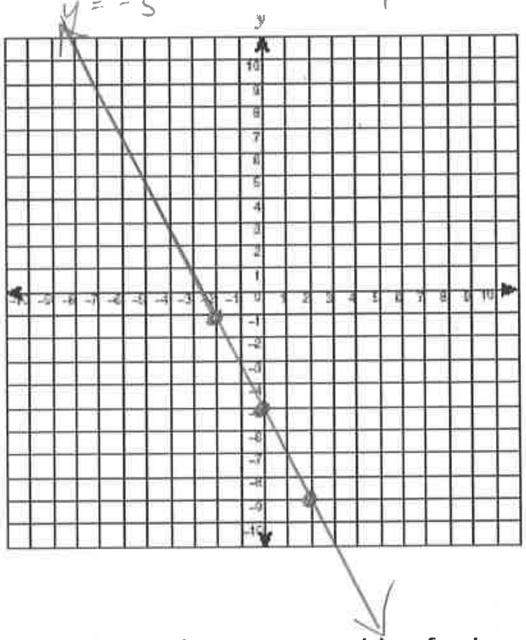
Example 1 - Graph the following linear relations using a table of values

a) $y = -2x - 5$

$y = -2(0) - 5$
$y = -5$

$y = -2(2) - 5$
$= -4 - 5$
$= -9$

x	y
0	-5
2	-9
-2	-1



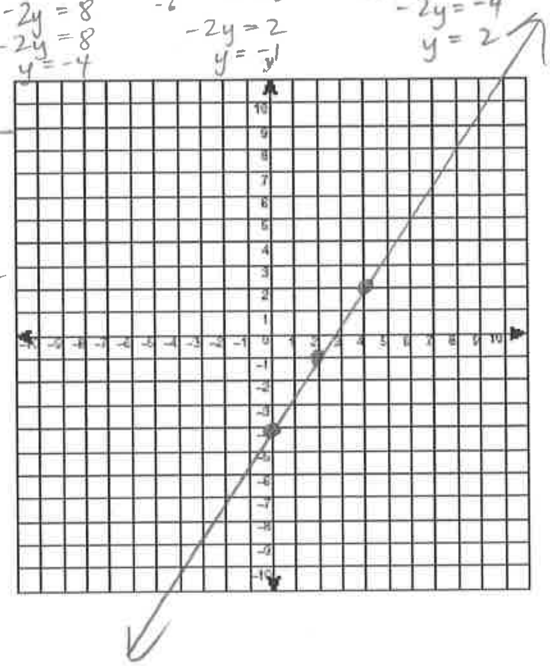
b) $3x - 2y = 8$

$3(0) - 2y = 8$
$-2y = 8$
$y = -4$

$3(2) - 2y = 8$
$6 - 2y = 8$
$-6 \quad -6$
$-2y = 2$
$y = -1$

$3(4) - 2y = 8$
$12 - 2y = 8$
$-12 \quad -12$
$-2y = -4$
$y = 2$

x	y
0	-4
2	-1
4	2



Graphing a line using a table of values is too time consuming, but a great backup method!

PART 2 – USING SLOPE / *Y*-INTERCEPT FORM

One of the most effective ways to graph linear equations is to get it into the form $y = mx \pm b$, which is known as slope / y-intercept form.

m is the SLOPE and can be represented as $\frac{\text{rise}}{\text{run}}$.

b is the y-intercept and tells you where the line crosses the y axis.

A slope of 1, or $\frac{1}{1}$ makes a 45® line that rises as you go to the right. Slopes larger than 1 make a line steeper than 45®, and slopes smaller than 1 make a line smaller than 45®

Example 2 – State the slope and y-intercept of $y = -\frac{1}{2}x + 4$. Then graph it.

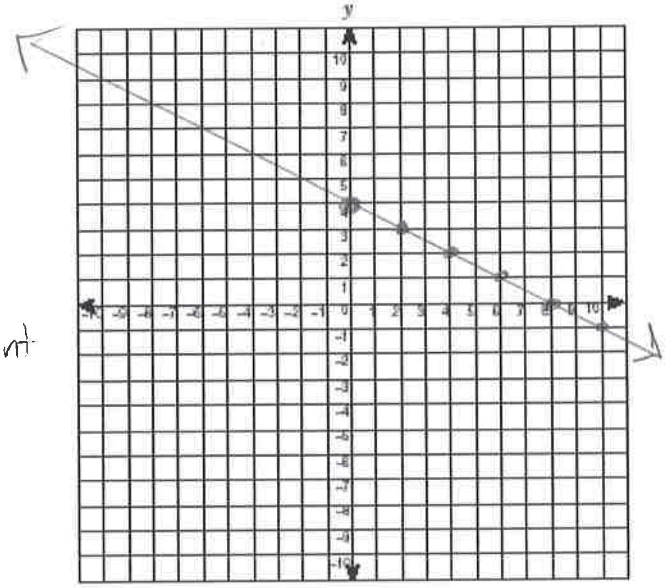
m = slope = $\frac{-1}{2}$

b = y-int = 4

① Plot y-int first

② Slope count from the y-int

$\frac{-1 \leftarrow \text{down}}{2 \leftarrow \text{right}}$



Another observation: The line will <u>RISE</u> as you go right for a positive slope, and <u>DROP</u> as you go right for a negative slope.

Example 3 – Graph $2x - y = 6$

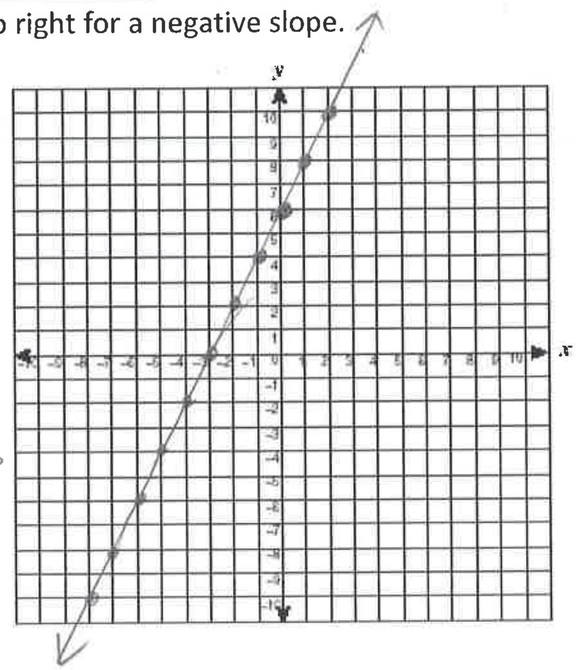
$2x - y = 6$
$\quad +y \quad +y$

$2x = \cancel{6} + y$
$-6 \quad -\cancel{6}$

$2x - 6 = y$ or $y = 2x - 6$

$b = 6$

$m = \frac{2 \leftarrow \text{up}}{1 \leftarrow \text{right}}$



What is a common trait of each point on the line?

The coordinates of any point on the line will satisfy the equation (the left side will equal the right side)

Example 4 – Is (4, -2) on the line a) $y = -2x + 6$? b) $3x + 4y = -1$?

a) $y = -2x + 6$
$-2 = -2(4) + 6$
$-2 = -8 + 6$
$-2 = -2$ ✓

(4, -2) is on the line $y = -2x + 6$

b) $3x + 4y = -1$
$3(-2) + 4(4) = -1$
$-6 + 16 = -1$
$10 \neq -1$

(4, -2) is NOT on the line $3x + 4y = -1$

Example 5 – Graph each equation using slope / y-intercept form

a) $\frac{-5y}{-5} = \frac{5}{-5} - \frac{5x}{-5}$

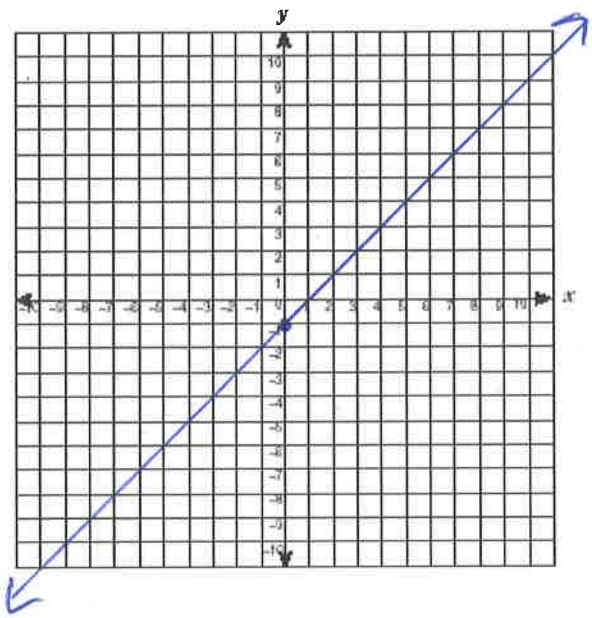
$y = -1 + x$

$y = x - 1$

$y = 1x - 1$

$b = -1$

$m = \frac{1 \leftarrow \text{up}}{1 \leftarrow \text{right}}$



b) $8x - 4y = 2$

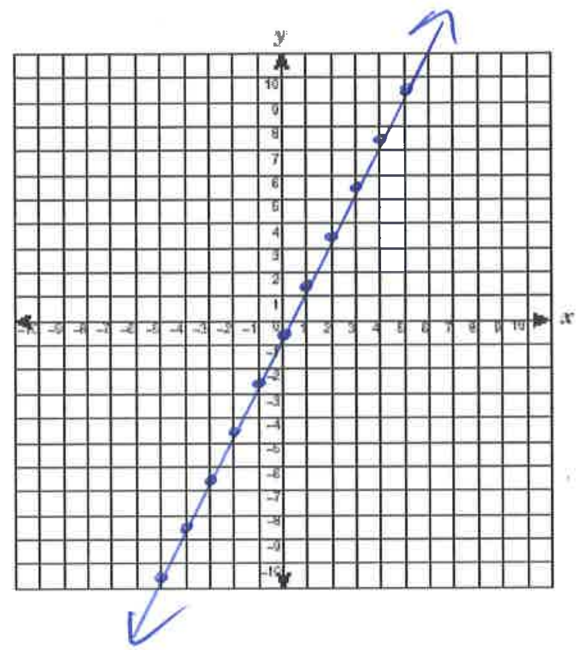
$-8x \qquad -8x$

$\frac{-4y}{-4} = \frac{-8x}{-4} + \frac{2}{-4}$

$y = 2x - \frac{1}{2}$

$b = -\frac{1}{2}$

$m = \frac{2 \leftarrow \text{up}}{1 \leftarrow \text{right}}$



PART 3 – USING GENERAL / STANDARD FORM

General Form: $Ax \pm By \pm C = 0$ Standard Form: $Ax \pm By = \pm C$

The quickest way to graph in general and standard form is to:

1) Put the equation into standard form.
2) Get the x-intercept by covering the y term, and then graph.
3) Get the y-intercept by covering the x term, and then graph.
4) Use the handy slope rule as an extra piece of useful information
   *the slope of a line in general or standard form is always:
   'A over B, switch the sign'

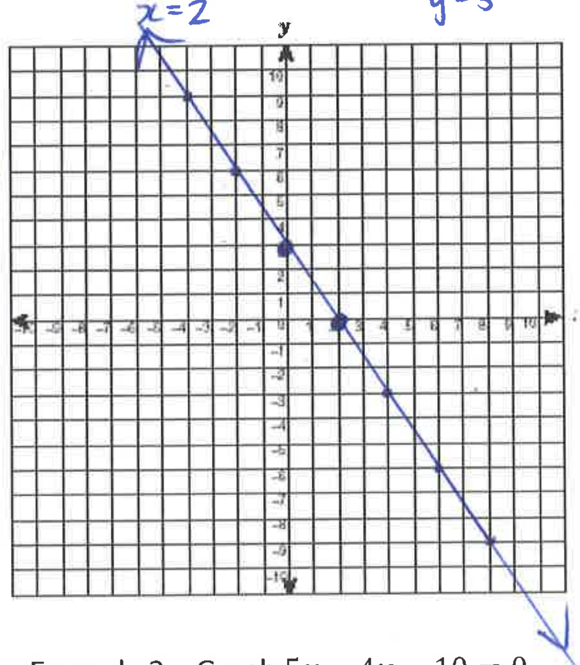
Example 1 – Graph the following linear relations:

a) $3x + 2y = 6$

x-int: $3x+2(0)=6$
$3x=6$
$x=2$

y-int: $3(0)+2y=6$
$2y=6$
$y=3$

slope $\frac{-3 \text{ down}}{2 \text{ right}}$



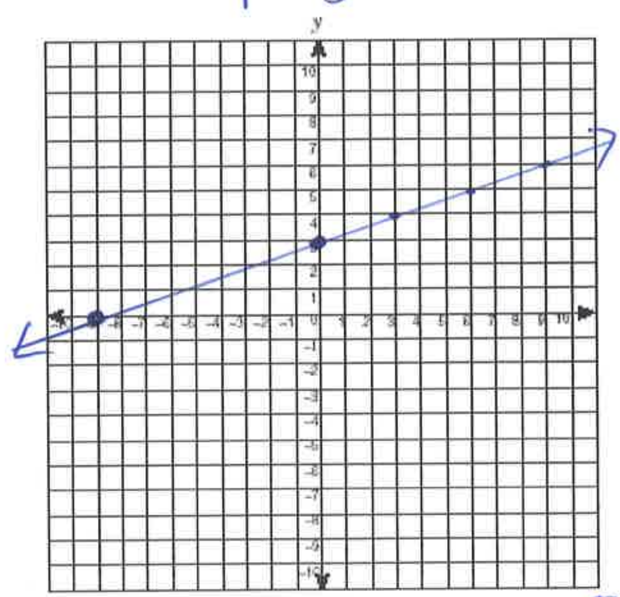
b) $x - 3y + 9 = 0$

$x-3y=-9$

x-int: $x=-9$

y-int: $-3y=-9$
$y=3$

slope $= \frac{1}{3}$



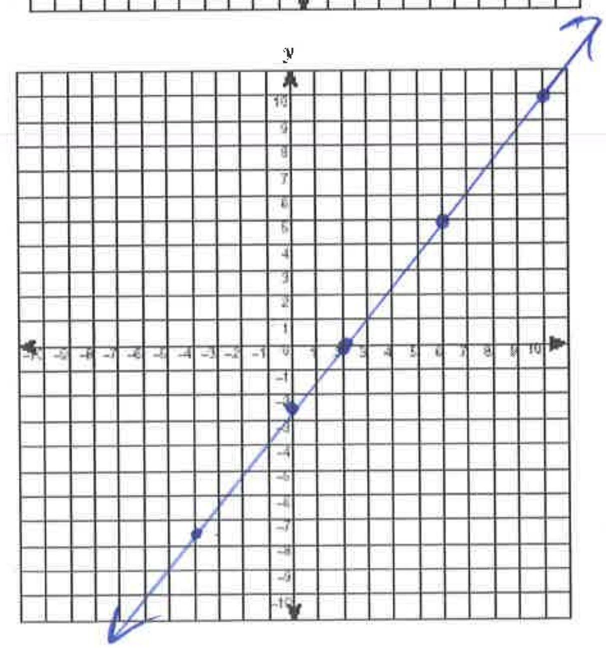
Example 2 – Graph $5x - 4y - 10 = 0$

$5x-4y=10$

x-int:
$5x=10$
$x=2$

y-int:
$-4y=10$
$y=\frac{-10}{4}=\frac{-5}{2}=-2.5$

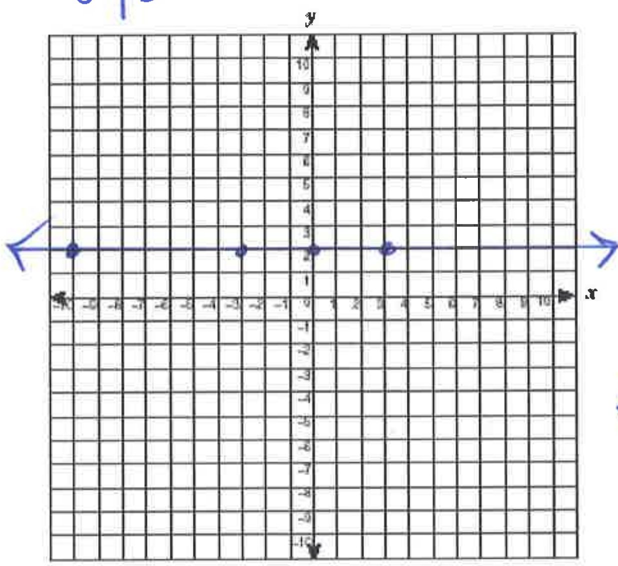
Slope $= \frac{5}{4}$



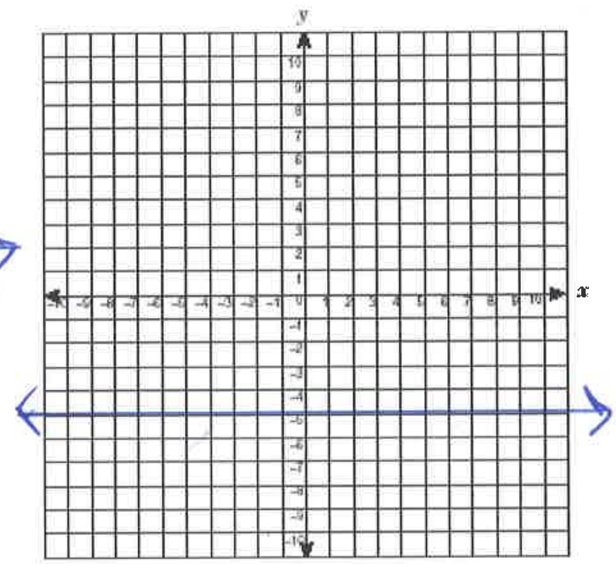
Special Cases

Example 3 – Graph (a) $y = 2$

x	y
-10	2
-3	2
0	2
3	2

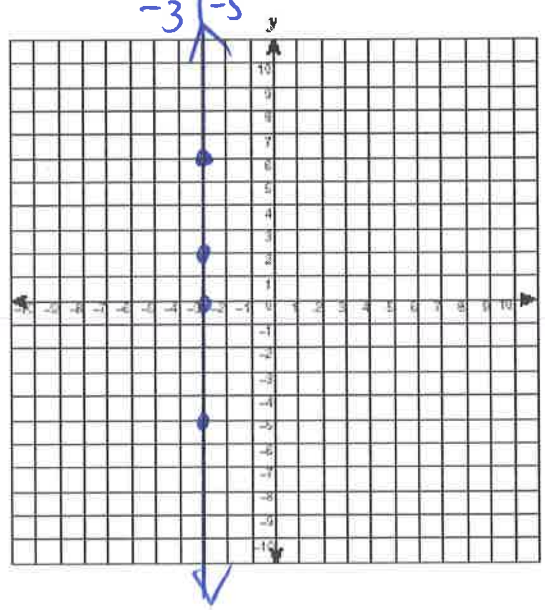


(b) $y = -5$

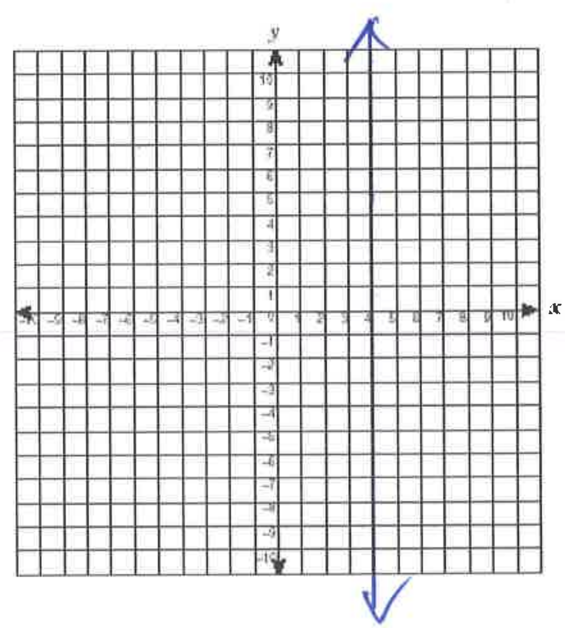


Example 4 – Graph (a) $x = -3$

x	y
-3	6
-3	2
-3	0
-3	-5



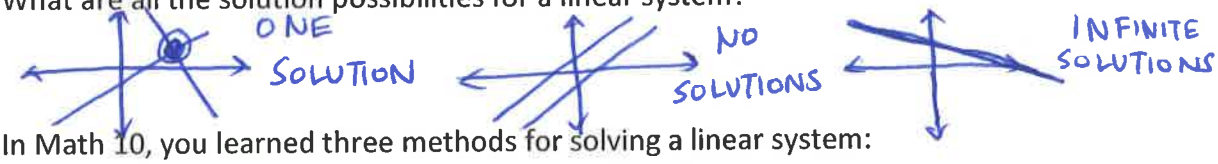
(b) $x = 4$



What does it mean to '**solve a linear system**'?

A linear system is two (or more) linear relations. To solve a linear system, you must find the intersection point(s) of the linear relations.

What are all the solution possibilities for a linear system?



In Math 10, you learned three methods for solving a linear system:

① Graphing ② Substitution ③ Elimination

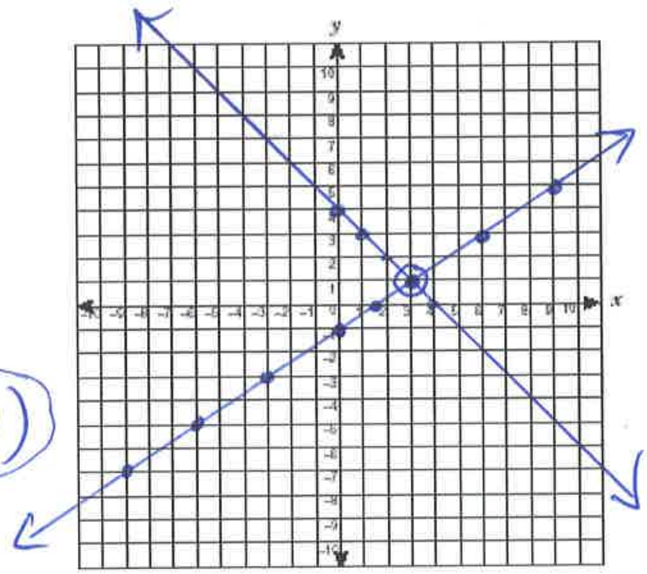
Example 1 – Solve the linear system by graphing

1) $y = 4 - x$
2) $2x - 3y = 3$

① $y = 4 - x$
$y = -x + 4$
$y = -1x + 4$
$b = 4$
$m = \frac{-1}{1}$

② $2x - 3y = 3$
x-int: $2x = 3$, $x = 1.5$
y-int: $-3y = 3$, $y = -1$
$\text{slope} = \frac{2}{3}$

Solution: (3, 1)



Example 2 – Solve the following system using substitution

1) $3x + y = 3$ ← get y by itself (easiest choice)
2) $7x - 2y = 20$

① $3x + y = 3$
$-3x \quad -3x$
$y = -3x + 3$

② $7x - 2y = 20$
$7x - 2(-3x + 3) = 20$
$7x + 6x - 6 = 20$
$13x - 6 = 20$

$13x - 6 = 20$
$+6 \quad +6$
$\frac{13x}{13} = \frac{26}{13}$
$x = 2$

① $3x + y = 3$
$3(2) + y = 3$
$6 + y = 3$
$-6 \quad -6$
$y = -3$

Steps:
1) Get a variable by itself.
2) Substitute into the other equation.
3) Solve for the remaining variable.
4) Substitute the solved variable value back into one of the original equations to determine the other variable value.

Solution
(2, -3)

Example 3 – Solve the system using substitution

1) 2x + 3y = 1

2) 3x – y = 7 get by itself

② $3x - y = 7$
$+y \quad +y$
$3x = 7 + y$
$-7 \quad -7$

② $y = 3x - 7$

① $2x + 3y = 1$
$2x + 3(3x - 7) = 1$
$2x + 9x - 21 = 1$
$11x - 21 = 1$
$+21 \quad +21$

$\frac{11x}{11} = \frac{22}{11}$
$x = 2$

② $3x - y = 7$
$3(2) - y = 7$
$6 - y = 7$
$y = -1$

Solution
$(2, -1)$

Example 4 – Solve the system using elimination (text calls it the 'addition method')

1) (2x + 5y = 11) ×3

2) (3x – 2y = 7) × 2

$6x + 15y = 33$
$-(6x - 4y = 14)$
$\frac{19y}{19} = \frac{19}{19}$
$y = 1$

① $2x + 5y = 11$
$2x + 5(1) = 11$
$2x + 5 = 11$
$-5 \quad -5$
$\frac{2x}{2} = \frac{6}{2}$
$x = 3$

Steps:
1) Line up the equations by like terms.
2) Make sure either the coefficients for ***x*** or the coefficients for ***y*** have the same magnitude.
3) Add or subtract to eliminate a variable
4) Do Steps 3 & 4 described in the substitution method.

Solution
$(3, 1)$

Example 5 – Solve the system using elimination

1) (4x – y = 2) ×3

2) x – 3y = -5

① $12x - 3y = 6$
② $-(x - 3y = -5)$
$\frac{11x}{11} = \frac{11}{11}$
$x = 1$

② $x - 3y = -5$
$1 - 3y = -5$
$-1 \quad -1$
$\frac{-3y}{-3} = \frac{-6}{-3}$
$y = 2$

Solution
$(1, 2)$

*Look over Example 3 on the bottom of p.194, and Example 4 on the top of p.195.

These show examples of what the situation will look like if there are infinite solutions and no solutions to a system

6.1D – Solving Linear Systems Part 2 (Word Problems)

Solving word problems for linear systems can be challenging. Here are some steps to aid in the process:

1) Read the problem over very carefully.
2) Let the two variable equal the two things you are being asked to solve.
3) If possible, make a table to help organize the data.
4) Build your two equations using your organized information and variables.
5) Use elimination or substitution to solve.
6) Check by substituting solutions back into each equation.

Example 1 – Two shirts and one sweater costs $60. Three shirts and two sweaters costs $104. What is the cost of one shirt and what is the cost of one sweater?

Let x = cost of one shirt
Let y = cost of one sweater

① $(2x + y = 60) \times 2$
② $3x + 2y = 104$

① $4x + 2y = 120$
② $-(3x + 2y = 104)$
$x = 16$

① $2x + y = 60$
$2(16) + y = 60$
$32 + y = 60$
$-32 \quad -32$
$y = 28$
$(16, 28)$

One shirt costs $16 and one sweater costs $28.

Example 2 – Adult tickets for the school play are $12.00 and children's tickets are $8.00. If a theatre holds 300 seats and the sold out performance brings in $3280.00, how many children and adults attended the play?

Let x = # of adults who attended the play
Let y = " " children " " "

	Adults	children	Total
Tickets	x	y	300
Revenue	$12x$	$8y$	3280

① $(x + y = 300) \times 8$
② $12x + 8y = 3280$

① $8x + 8y = 2400$
② $-(12x + 8y = 3280)$
$-4x = -880$
$\frac{-4x}{-4} = \frac{-880}{-4}$
$x = 220$

① $x + y = 300$
$220 + y = 300$
$y = 80$

220 adults attendended and 80 chidren attended.

Example 3 – Isaac borrowed \$2100 for his college tuition. Part of it he borrowed from the government at 5% annual interest. The rest he borrowed from a bank at 6.5% annual interest. If the total annual interest is \$114, how much did he borrow from each source?

Let x = amount borrowed from govt $5\% = 0.05$
Let y = amount borrowed from bank $6.5\% = 0.065$

	Govt	Bank	Total
\$ Borrowed	x	y	2100
Interest	$0.05x$	$0.065y$	114

① $x + y = 2100$
② $0.05x + 0.065y = 114$

Substitution:

① $x = 2100 - y$

② $0.05(2100 - y) + 0.065y = 114$

$105 - 0.05y + 0.065y = 114$

$105 + 0.015y = 114$
$-105 \qquad -105$

$\frac{0.015y}{0.015} = \frac{9}{0.015}$

$y = 600$

① $x + y = 2100$

$x + 600 = 2100$

$x = 1500$

\$1500 was borrowed from the gov't and \$600 was borrowed from the bank.