Key

Draw ***acute***, ***obtuse***, ***right***, and ***straight*** angles:

acute
less than 90°

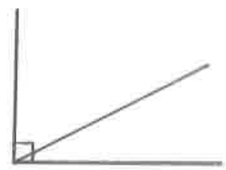
obtuse
between 90° and 180°

right
90°

straight
180°

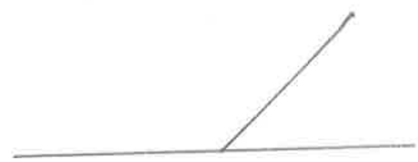
What are ***complementary angles***? Draw an example:

Angles that sum
to 90°



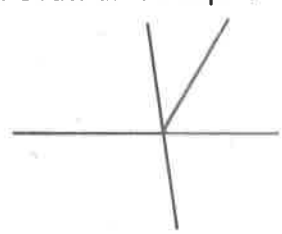
What are ***supplementary angles***? Draw an example:

Angles that sum to
180°



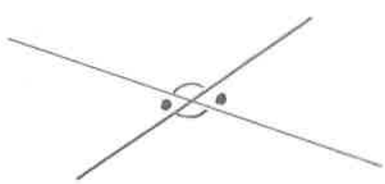
What do ***angles at a point*** sum to? Draw an example:

360°



all 5 angles
sum to 360°

What are ***vertically opposite angles***, & what are their relationship to one another?

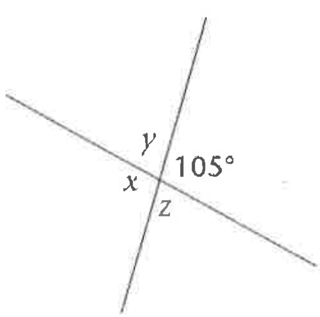


caused by two straight lines intersecting

• and • are vertically opposite and EQUAL (congruent)

same with the other two angles

Example 1: Find missing angles ***x***, ***y***, & ***z***:

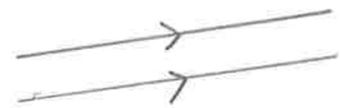


Statement	Reason
$x = 105°$	vertically opposite angles
$y = 75°$	supplementary angles
$z = 75°$	vertically opposite OR supplementary OR angles at a point = 360°

What are ***parallel lines***? How do we signify them?

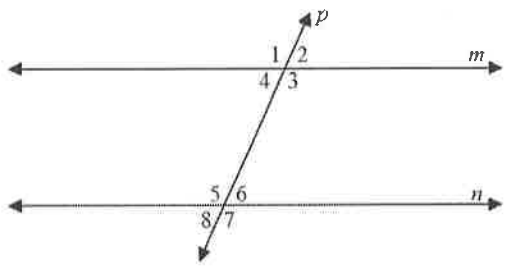
Two lines that will never intersect

signified with matching arrows



What is a ***transversal***? a line that cuts through a set of lines

Let's look at ***parallel lines*** cut by a ***transversal***:



Do you see any relationships between the angles we've already learned above?

$\angle 1 = \angle 3$, $\angle 2 = \angle 4$, $\angle 5 = \angle 7$, $\angle 6 = \angle 8$
vertically opposite angles

$\angle 1 + \angle 4 = 180°$, $\angle 1 + \angle 2 = 180°$, etc.
supplementary angles

Do you notice any other relationships between different angles?

$\angle 1 = \angle 5$ $\angle 2 = \angle 6$ | $\angle 4 = \angle 6$
$\angle 4 = \angle 8$ $\angle 3 = \angle 7$ | $\angle 3 = \angle 5$

List all sets of ***corresponding angles***:

$\angle 1 = \angle 5$, $\angle 2 = \angle 6$,
$\angle 4 = \angle 8$, $\angle 3 = \angle 7$

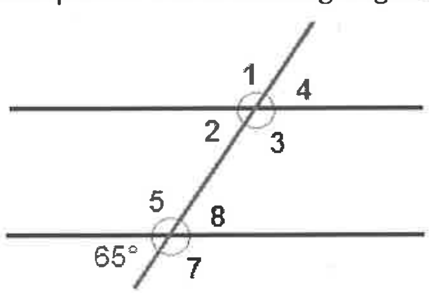
List all sets of ***alternate interior angles***:

$\angle 4 = \angle 6$
$\angle 3 = \angle 5$

What relationship exists for ***co-interior angles***? List all sets of ***co-interior angles***:

they sum to 180°
$\angle 4 + \angle 5 = 180°$
$\angle 3 + \angle 6 = 180°$

Example 2: Find all missing angles:



Statement	Reason
$\angle 8 = 65°$	vertically opposite
$\angle 5 = 115°$	supplementary
$\angle 7 = 115°$	supplementary
$\angle 2 = 65°$	corresponding
$\angle 3 = 115°$	alternate interior to $\angle 5$
$\angle 4 = 65°$	corresponding to $\angle 8$
$\angle 1 = 115°$	vertically opposite to $\angle 3$

What is the relationship of the three angles in any triangle? angles sum to 180°

What is an ***equilateral triangle***? An ***isosceles triangle***? A ***scalene triangle***?

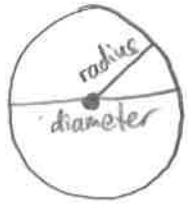
3 angles equal
3 sides equal

angles = $\frac{180}{3}$ = 60°

2 angles equal
2 sides equal

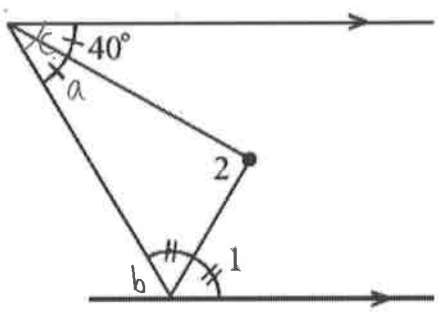
no angles equal
no sides equal

What are some useful ***circle properties*** to know in geometry?



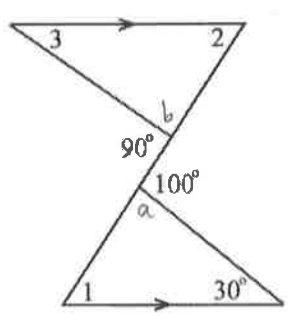
any radii for the same circle are equal

Example 3: Find missing angles 1 & 2:



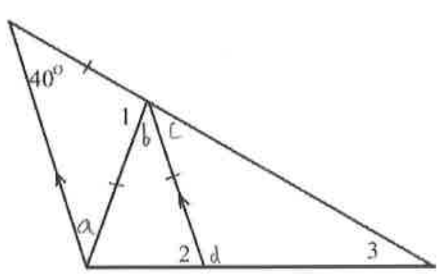
Statement	Reason
<a = 40°	congruent
<b = 80°	alternate interior with c
<1 = 50°	supplementary with b and other congruent angle
<2 = 90°	3 angles in Δ = 180°

Example 4: Find angles 1, 2, & 3:



Statement	Reason
<a = 80°	supplementary
<1 = 70°	3 angles in Δ = 180°
<2 = 70°	alternate interior
<b = 90°	supplementary
<3 = 20°	3 angles in Δ = 90°

Example 5: Find angles 1, 2, & 3:



Statement	Reason
<a = 40°	isosceles Δ
<1 = 100°	3 angles in Δ = 180°
<b = 40°	alternate interior w/< a
<2 = 70°	isosceles Δ
<c = 40°	supplementary w/<1 and <b
<d = 110°	supplementary w/ <2
<3 = 30°	3 angles in Δ = 180°

2.2 – Geometry Proofs

Terminology

Parallel lines: $AB \parallel CD$ means parallel

Perpendicular: $AB \perp XY$ means perpendicular (90° to each other)

Bisects: splits perfectly in half

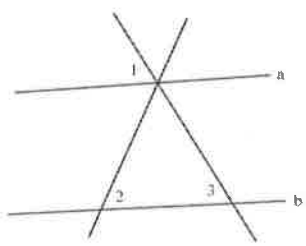
Perpendicular Bisector: a line that splits another line perfectly in half, and is at a 90° angle to the line.

Proofs

Each geometric proof is different. The information you start with is different, and what you're asked to prove using that information is different. Therefore, it cannot be taught in a procedural manner (ie teaching a 'set of steps'). Geometry is about have a 'set of tools' at your disposal, and being able to know when to use the right tool at the right time. We will do several examples for assistance.

Example 1:

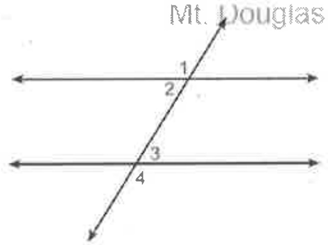
Given: $a \parallel b$
$\angle 2 = \angle 3$
Prove: $\angle 1 = \angle 2$



Statement	Reason
$a \parallel b$	given
$\angle 1 = \angle 3$	corresponding angles
$\angle 2 = \angle 3$	given
$\angle 1 = \angle 2$	substitution

Example 2:

Given: $\angle 2 = \angle 3$
Prove: $\angle 1 = \angle 4$

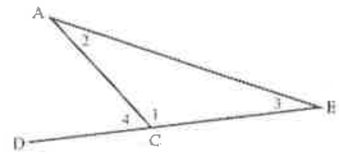


Statement	Reason
$\angle 2 = \angle 3$	given
$\angle 1 + \angle 2 = 180°$	supplementary
$\angle 3 + \angle 4 = 180°$	Supplementary
$\angle 1 + \angle 2 = \angle 3 + \angle 4$	both equal to 180°
$\angle 1 + \angle 2 = \angle 2 + \angle 4$	substitution
$\angle 1 = \angle 4$	subtraction

Example 3:

Given: $\triangle ABC$ with $\overleftrightarrow{DCB}$

Prove: $\angle 4 = \angle 2 + \angle 3$



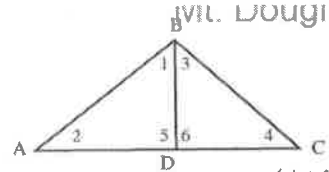
Statement	Reason
$\angle 1 + \angle 2 + \angle 3 = 180°$	sum of angles in Δ
$\angle 1 + \angle 4 = 180°$	supplementary
$\angle 1 + \angle 2 + \angle 3 = \angle 1 + \angle 4$	both equal to 180°
$\angle 2 + \angle 3 = \angle 4$	subtraction

Example 4:

Given: $BD \perp AC$

$\angle 1 = \angle 3$

Prove: $\angle 2 = \angle 4$

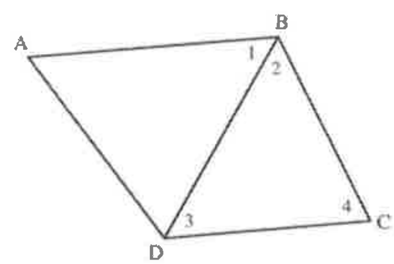


Statement	Reason
$BD \perp AC$	given
$\angle 5 = \angle 6$	⊥ angles equal (90°)
$\angle 1 = \angle 3$	given
$\angle 1 + \angle 2 + \angle 5 = \angle 3 + \angle 4 + \angle 6$	sum of angles of Δ = 180°
$\angle 1 + \angle 2 + \angle 5 = \angle 1 + \angle 4 + \angle 5$	substitution
$\angle 2 = \angle 4$	subtraction

Example 5:

Given: $AB \parallel CD$ in ABCD

Prove: $\angle ABC$ is supplementary to $\angle 4$



Statement	Reason
$AB \parallel CD$	given
$\angle 1 = \angle 3$	alternate interior
$\angle 2 + \angle 3 + \angle 4 = 180°$	sum of angles in Δ
$\angle 2 + \angle 1 + \angle 4 = 180°$	substitution
$\angle 2 + \angle 1 = \angle ABC$	addition
$\angle ABC + \angle 4 = 180°$	substitution

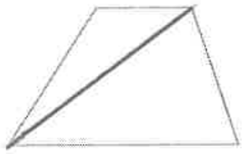
2.3 - Polygons

definition

A **polygon** is a closed geometric shape formed by a series of line segments. Each line segment is called a **side**, and each corner is called a **vertex**.

Let's devise a way to find the sum of all the interior angles of a polygon:

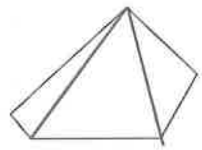
4 sides



- made 2 triangles
- each triangle has angle sum 180°

$2(180^\circ) = 360^\circ$

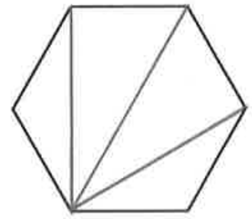
5 sides



3 triangles

$3(180^\circ) = 540^\circ$

6 sides



4 triangles

$4(180^\circ) = 720^\circ$

formula

The sum of the interior angles of a polygon with *n* sides is: $(n-2)180^\circ$

Example 1: If a polygon has 10 sides (called a **decagon**), what is the sum of the interior angles?

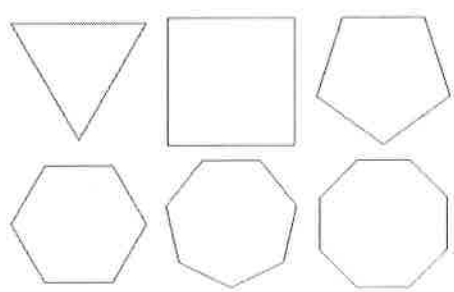
$$(10-2)180^\circ$$
$$= 8(180^\circ)$$
$$= 1440^\circ$$

What are the names of polygons?

Sides	Name
3	triangle
4	quadrilateral
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon
11	undecagon
12	dodecagon

What is a **regular polygon**?

a polygon with equal sides and equal interior angles



formula

How could we build off of the formula for the sum of the interior angles of a polygon to create a formula that gives the measure of each interior angle in a regular polygon?

$$\text{Interior Angle in Regular Polygon} = \frac{(n-2)180^\circ}{n}$$

Example 2: What is the interior angle measure for a regular heptagon?

7 sides

$$\text{Interior Angle Measure} = \frac{(7-2)(180)}{7} = 128.57^\circ$$

Example 3: If an interior angle of a regular polygon is 150°, how many sides does it have?

$$150^\circ = \frac{(n-2)180^\circ}{n}$$

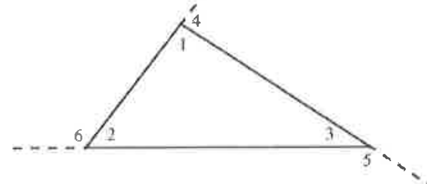
$$150n = 180n - 360^\circ$$
$$-180n \quad -180n$$

$$-30n = -360^\circ$$

$$n = 12$$

12 sides (dodecagon)

A polygon also has **exterior angles**:



Angles 1, 2, 3 are interior angles.
Angles 4, 5, 6 are exterior angles.

At each vertex of a polygon, the sum of
The interior & exterior angles is 180°.
<1 + <4 = 180°, etc.

If there are ***n*** sides to the polygon, then the sum of all of the interior and exterior angles is $\underline{180n}$.

Sum of interior angles + Sum of exterior angles = $\underline{180n}$

Sum of exterior angles = $\underline{180n}$ - Sum of interior angles

Sum of exterior angles = $\underline{180n}$ - $\underline{(n-2)180}$

$$= 180n - (180n - 360)$$
$$= 180n - 180n + 360^\circ$$
$$= 360^\circ$$

formula

Sum of Exterior Angles in a Polygon = $\underline{360^\circ}$
Each Exterior Angle of a Regular Polygon = $\frac{360^\circ}{n}$

Example 4: The sum of the interior angles of a regular polygon is 2520°. Find the measure of each exterior angle.

$$2520 = (n-2)180^\circ$$
$$2520 = 180n - 360$$
$$+360 \quad +360$$

$$2880 = 180n$$
$$n = 16$$

$$\frac{360}{n} = \frac{360}{16} = 22.5^\circ$$

Example 5: What type of regular polygon has an interior angle 3 times the exterior angle?

interior angle + exterior angle = 180°

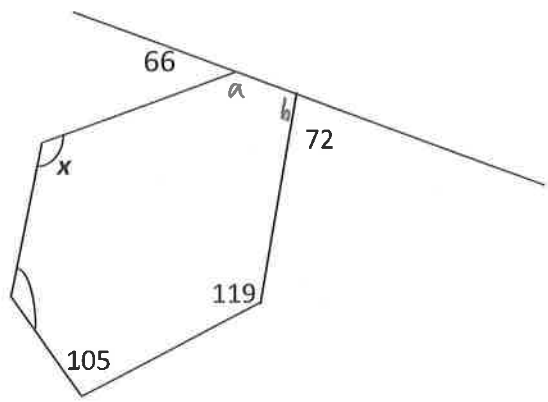
$$3x + x = 180^\circ$$
$$4x = 180^\circ$$
$$x = 45^\circ$$

$$\frac{360}{n} = 45^\circ$$
$$45n = 360$$
$$n = \frac{360}{45} = 8$$

octagon!

Example 6: Find <*x*

S	R
$<a = 114^\circ$	supplementary
$<b = 108^\circ$	Supplementary
sum of interior angles = 720°	Polygon formula
$<x = 137^\circ$	$\left(\frac{720-105-119-114-108}{2}\right)$



6 sided polygon

Sum of interior angles $= (n-2)180^\circ$

$= (6-2)180^\circ$

$= 4(180) = 720^\circ$

2.4 – Compass and Straightedge Construction

triangle

Construction 1: Construct a triangle with the following three lengths:

Draw line with straightedge.

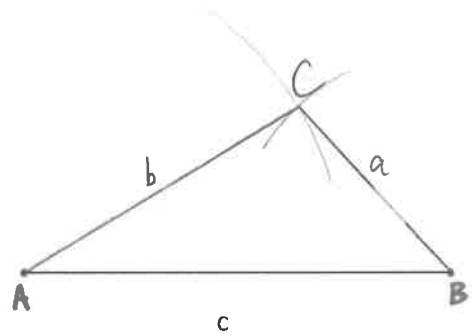
Use compass to construct one line segment equal to c, the longest line segment.

Label section AB.

At A, scribe arc of length b.

At B, scribe arc of length a to intersect arc b at C.

Draw line segments AC and BC



Construction 2: Construct the perpendicular bisector of a line segment AB.

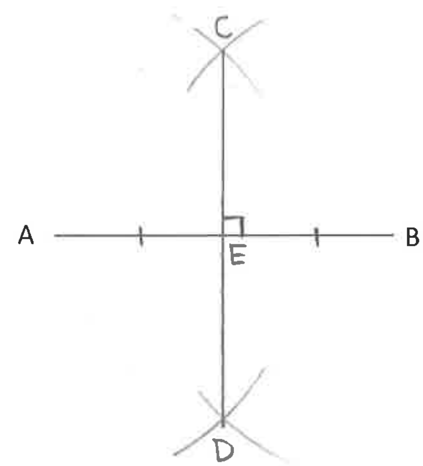
Adjust compass length such that it is more than half the length of AB.

At A, construct arcs on each side of AB.

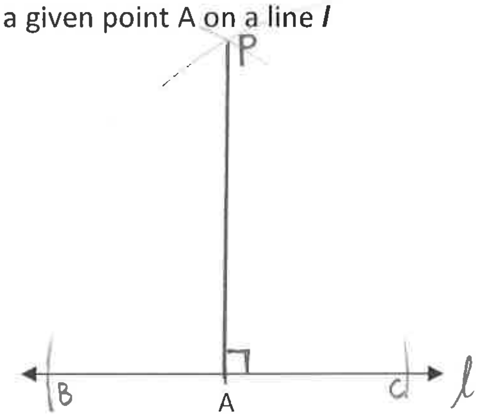
At B, of the same radii on each side, construct arcs intersecting arc of AB from A, marking intersection points C and D.

Draw line CD intersecting AB at E.

Thus, AB $\perp$ CD and AE = EB.



Construction 3: Construct a line perpendicular to a given point A on a line ***l***



At A, construct arcs of equal radius on each side of point A.

Label these points B and C.

At B, adjust compass greater than length AB and scribe arc on one side of BC.

Keeping compass at same width, place point at C, and scribe arc intersecting arc from B; label this point P.

Draw line AP.

Thus, AP $\perp$ *l*.

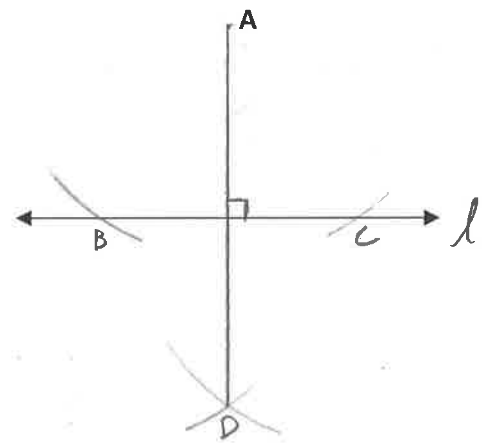
Construction 4: Construct a line perpendicular to a given line ***l*** from a Point A not on the line.

From A, draw arc intersecting line l at B and C.

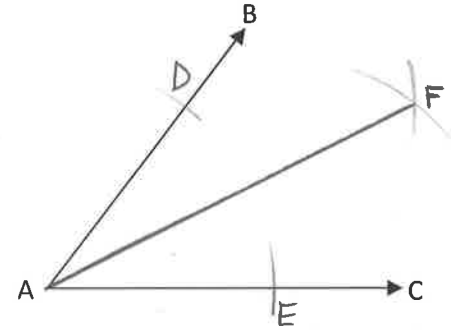
From B and C, scribe arcs of equal radius on other side of A, intersecting at D.

Draw line AD.

Thus, $AD \perp l$.



Construction 5: Bisect a given angle <BAC.



With A as centre, draw arc intersecting AB at D and AC at E, thus AD = AE.

With centres D and E, draw arcs of equal radius intersecting at F.

Draw AF.

Thus, $\angle BAF = \angle CAF$.