Chapter 1 Summary
(Square Roots, Powers, and Exponent Laws)

<u>1.1 The Real Number Systems</u>

Systems of Numbers:
Natural Numbers: The “counting numbers.” Starts at 1 and goes up one at a time.
Examples: {1, 2, 3, 4, 5, …}

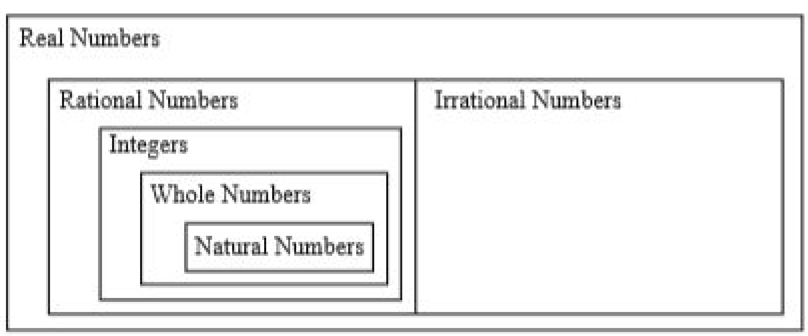
Whole Numbers: Includes all of the natural numbers. Starts at 0 and goes up one at a time.
Examples: {0, 1, 2, 3, 4, 5, …}

Integers: All of the whole numbers from negative infinity to positive infinity.
Examples: {...-3,-2, -1, 0, 1, 2, 3 ...}

Rational Numbers: Numbers that can be written as a fraction. They terminate or repeat.
Examples: $\{-4,\ 0,\ \frac{1}{2}, 0.547,\ 0.\overline{63}\}$

Irrationals: Numbers that can not be written as a fraction. They don't repeat or end.
Examples: $\{2.14243..., \Pi, \sqrt{10}\}$

Real Numbers: Any number you can think of. Do not include imaginary numbers or infinity.
Examples: $\{-10000,\ 5.47363...,\ \sqrt{44},\ \frac{-3}{4}\}$



Chapter 1.2 Square Roots

$\sqrt{\ }$ known as a "radical sign," denotes the square root action on a number.

Perfect squares: Numbers that have whole number square roots. Eg $\sqrt{64} = 8$
(Square Roots, Powers, and Exponent Laws)
Non perfect squares: Numbers that do not have whole number solutions as their roots.
Eg $\sqrt{2} = 1.4142...$

BEDMAS and Negatives:

a) When there is a negative sign on the outside of the radical, solve the root first, then multiply by the negative

Eg. $-\sqrt{9} = -(3) = -3$

b) When the negative is within the radical we have a problem as you can not multiply two identical things and get a negative value. The answer is said to be undefined.

Eg. $\sqrt{-16} = \oslash$ as only $-4 \times +4 = -16$ and $-4 \neq +4$

GEMA and radicals:

a) Any value that is within a radical is understood to be within the "radical group." Thus, some steps will need to be simplified within the radical before you can solve for it.

Eg. $\sqrt{\frac{8}{32}} = \sqrt{\frac{4}{16}} = \frac{\sqrt{4}}{\sqrt{16}} = \frac{2}{8} = \frac{1}{4}$

Eg. $-\sqrt{70+11} = -\sqrt{81} = -9$

b) As you can see from GEMA (BEDMAS), the Exponent operation is high on the hierarchy. Complete the radical operation before moving on to lower steps.

Eg. $\sqrt{16} + \sqrt{25} = 4 + 5 = 9$

Radicals and decimals:

a) Decimals are the hardest way to interpret information. For radical problems, convert decimals to fractions before trying to solve.

Eg. $\sqrt{.0121} = \sqrt{\frac{121}{10000}} = \frac{\sqrt{121}}{\sqrt{10000}} = \frac{11}{100} = .11$

1.3 Square Roots of Non-Perfect Square

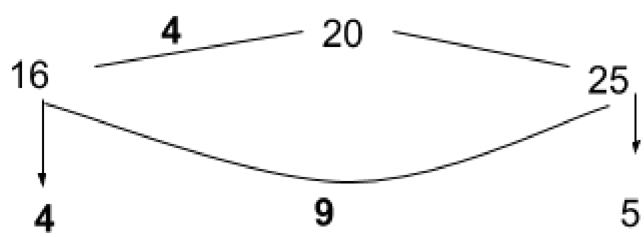
Square roots of non-perfect squares will end up as irrational numbers that are impossible to define exactly. For this reason, the best we can do is either leave non-perfect squares in radical form, or to approximate their values.

Eg 1) Approximate $\sqrt{20}$ to one decimal place

Steps: 1) Ask yourself what perfect squares are just less and more than 20: 16 & 25

2) As the $\sqrt{16} = 4$ and the $\sqrt{25} = 5$ we know that $\sqrt{20}$ sits somewhere between 4 and 5.

3) Next we must approximate the decimal place: To do this, we first see how far away our given root is from its lesser perfect square (this will be our numerator), and how far away the two perfect squares are from each other (this will be our denominator).



Therefore the fraction is $\frac{4}{9} \simeq 0.45$
The approximate solution to $\sqrt{20} \simeq 4.45$

The pythagorean theorem: $a^2 + b^2 = c^2$. a and b are interchangeable, c is the hypotenuse, the longest side.

1.4 Defining a Power

-3^2 : Has a *base* of 3, an *exponent* of 2, and an *answer* of -9
$(-3)^4$: Has a *base* of -3, an *exponent* of 4, and an *answer* of 81
$(-2)^{-3}$: Has a *base* of -2 an *exponent* of -3 and an *answer* of $\frac{-1}{8}$

When working with exponents it is important to know:

1) Whether the base is positive or negative

2) Whether the exponent is even or odd

Given $x > 0$
$(-x)^{even} = positive$ $(-x)^{odd} = negative$ $-x^{odd\ or\ even}$ = negative

$a^1 = a$, for any number a
$a^0 = 1$, for any non-zero a
$a^{-1} = \frac{1}{a}$

Note: $0^0 = undefined$

1.5 Order of Operations

Brackets

Exponents | **G**roupings

Division | **E**xponents

Multiplication | **M**ultiplicative

Addition | **A**dditive

Subtraction

With the following rules:

1) Work starting from the innermost brackets
2) When there is a tie in hierarchy, move from left to right.

Remember that Division and Multiplication are tied on the hierarchy and that Addition and Subtraction are also tied.

Show your work and take your time when solving these problems.

<u>1.6 Exponent Laws & 1.7 Power Rules</u>

Rule	Example	Notes
$a^n \cdot a^m = a^{n+m}$	$2^2 \cdot 2^3 = 2^{2+3} = 2^5$	The bases, which are the a's in this case, must be the same.
$\frac{a^n}{a^m} = a^{n-m}$	$\frac{2^5}{2^3} = 2^{5-3} = 2^2 = 4$	The bases, which are the a's in this case, must be the same.
$a^n \cdot b^n = (a \cdot b)^n$	$2^2 \cdot 3^2 = (2 \cdot 3)^2 = (6)^2 = 36$	The exponents, which are the n's in this case, must be the same.
$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$	$\frac{2^2}{3^2} = \left(\frac{2}{3}\right)^2 = \frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9}$	The exponents, the n's in this case, must be the same.
$(b^n)^m = b^{n \cdot m}$	$(2^3)^2 = 2^{3 \cdot 2} = 2^6 = 64$	Alternatively, we can use the first rule: $(2^3)^2 = 2^3 \cdot 2^3 = 2^{3+3} = 2^6$
$a^{-m} = \frac{1}{a^m}$	$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$	In order to solve, we must change negative exponents into positives with this method.
$\sqrt[m]{a^n} = a^{n/m}$	$\sqrt[3]{2^2} = 2^{2/3}$	In some situations, we must change radicals into exponents with this method.
$a^0 = 1$	$\left(\frac{1}{2}\right)^0 = 1, \quad 3^0 = 1, \ldots$	Any number raised to the power of zero is equal to 1.
$a^1 = a$	$\left(\frac{1}{4}\right)^1 = \frac{1}{4}, \quad 6^1 = 6, \ldots$	Any number raised to the power of 1 is equal to itself.