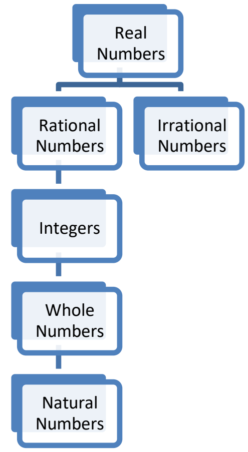
1.0 – Classifying Real Numbers and Rational Exponents Review

Classifying Real Numbers:



Natural Numbers – The counting numbers

Whole Numbers – Zero and the counting numbers

Integers – Neg counting numbers, 0, pos counting numbers

Rational Numbers – All numbers that can be written as a fraction (*if written as a decimal can it be converted to a fraction?)

Irrational Numbers – Everything else that can't be written as a fraction

Real Numbers – All numbers

Example 1 – Consider the list of numbers: -2, 0, 1, $\frac{4}{5}$, 0.777, -2.5, $\sqrt{17}$, $\sqrt[3]{8}$, 4.14562972....

List all:

a) Natural Numbers

b) Whole Numbers

c) Integers

d) Rational numbers

e) Irrational Numbers

f) Real numbers

Example 2 – State whether each statement is **true or false**.

a) Every integer is a natural number ________________

b) All whole numbers are integers ________________

c) Every real number is a rational number ________________

Rational Exponents:

When n is a natural number and x is a rational number, $x^{\frac{1}{n}} = \sqrt[n]{x}$

Example 3 – Write each power in radical form and evaluate without using a calculator:

a) $1000^{\frac{1}{3}}$ b) $25^{0.5}$ c) $(-8)^{\frac{1}{3}}$ d) $\left(\frac{16}{81}\right)^{\frac{1}{4}}$

When m and n are natural numbers, and x is a rational number, $x^{\frac{m}{n}} = \left(x^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{x}\right)^m$

Example 4 – Write the following in radical form:

a) $26^{\frac{2}{5}}$ b) $25^{\frac{3}{2}}$

Example 5 – Write the following in exponent form:

a) $\left(\sqrt{6}\right)^5$ b) $\left(\sqrt[4]{19}\right)^3$

To evaluate a power with a negative rational exponent,

1) Write with a positive exponent
2) Re-write into radical form
3) Work from the inside out
4) Write answer with no exponents

Example 6 – Simplify

a) $\left(\frac{9}{16}\right)^{-\frac{3}{2}}$ b) $16^{-\frac{5}{4}}$ c) $-25^{-1.5}$

1.4A – Simplifying Radicals Review and Preview

Example 1 – Identify and define all parts of the radical, then simplify:

$5\sqrt[3]{8}$

Radical Properties from Math 10:

1) $a^{\frac{1}{n}} = \sqrt[n]{a}$ as discussed in previous notes

2) $a^{\frac{m}{n}} = (a^{\frac{1}{n}})^m = (\sqrt[n]{a})^m$ Example: $16^{\frac{3}{4}}$

3) $a^{-\frac{m}{n}} = (a^{\frac{1}{n}})^{-m} = (\sqrt[n]{a})^{-m} = \frac{1}{(\sqrt[n]{a})^m}$ Example: $27^{-\frac{2}{3}}$

4) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ Example: $\sqrt[3]{\frac{27}{64}}$

5) $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ Example: $\sqrt{12}$

In this chapter, it is helpful to know the following:

Perfect squares up to 144:

Perfect cubes up to 125:

Perfect powers of 4 up to 81:

When a radical is simplified, an *entire radical* is changed to a *mixed radical* (or a mixed radical is further simplified).

1) Find the largest perfect **square** factor of the radicand ("under the root")
2) Rewrite the radicand as a product of perfect **square** x other factor
3) **Square** root the perfect **square** and write that new answer out front of the root
4) If there is already a number out front of the root, multiply them together
   *(Similar process for **cube** roots, etc.)*

Example 2 – Simplify:

a) $\sqrt{50}$ b) $\sqrt{20}$ c) $\frac{5}{6}\sqrt{18}$

Example 3 – Simplify:

a) $\sqrt[3]{40}$ b) $2\sqrt[3]{-54}$ c) $4\sqrt[4]{162}$

What if there are variables in the expression?

Roots of Positive Powers of x – Case 1: When $x \geq 0$ in $\sqrt{x^n}$ with n a positive integer.
The square roots of negative numbers are undefined in the set of real numbers. Therefore, if $x \geq 0$, simplification is easier to realize. We will work only with "Case 1" for most of the chapter. You will know it's case 1 because the question will say "assume all variables represent positive numbers" or "all variables represent non-negative real numbers."

For example:

$\sqrt{x} =$ $\sqrt[3]{x} =$

$\sqrt{x^2} =$ $\sqrt[3]{x^2} =$

$\sqrt{x^3} =$ $\sqrt[3]{x^3} =$

$\sqrt{x^4} =$ $\sqrt[3]{x^4} =$

$\sqrt{x^5} =$ $\sqrt[3]{x^5} =$

$\sqrt{x^6} =$ $\sqrt[3]{x^6} =$

A note about Case 2: When x is any real number in $\sqrt{x^n}$, with n a positive integer
This case introduces complications and restrictions which we will avoid by working with variables that represent non-negative values. See section 5.1 in your workbook for an explanation of Case 2.

Example 4 – Simplify. Assume all variables represent positive numbers.

a) $\sqrt{x^7y^3}$ b) $\sqrt[3]{x^{14}y^9}$ c) $\sqrt[4]{x^{11}}$

1.4B – Simplifying Radicals

Example 1 – Simplify

a) $\sqrt{8}$ b) $\sqrt{27}$ c) $3\sqrt{52}$ d) $\sqrt[3]{24}$ e) $5\sqrt[3]{-81}$ f) $\sqrt[4]{32}$

Example 2 – Simplify (assume variables are positive)

a) $\sqrt{18x^3y^6}$ b) $\sqrt{63n^7p^4}$ c) $\sqrt{32x^8y^{11}}$

d) $\sqrt[3]{40a^4b^8c^{15}}$ e) $\sqrt[3]{54a^5b^{10}}$ f) $\sqrt[3]{\frac{x^{13}}{64}}$

g) $\sqrt[4]{162x^3y^{11}z^5}$ h) $\sqrt[4]{m^7}$

Changing Mixed Radicals to Entire Radicals

Usually we work with radicals in simplest form (mixed radicals, with the smallest possible radicand). But we can change mixed radicals to entire radicals. This allows us to compare radicals.

Example 3 – Without using a calculator, determine which number is larger. (Change to entire radicals and compare)

$4\sqrt{3}$ or $3\sqrt{5}$

Example 4 – Change to Entire (assume variables are positive)

a) $5\sqrt{2}$ b) $7\sqrt{3}$ c) $x^3\sqrt{x}$ d) $-2x\sqrt{6x}$

e) $2\sqrt[3]{7}$ f) $3a^2b\,\sqrt[3]{b^2c}$ g) $\frac{3x^2y}{5}\sqrt[3]{2xy^2}$

1.5 – Adding and Subtracting Radical Expressions

Like Radicals

'Like Radicals' work very similarly to 'Like Terms'.

Simplify: $3x + 2x$

Simplify: $3\sqrt{2} + 2\sqrt{2}$

Like radicals have ____

Steps for adding & subtracting like radicals:

Example 1 – Simplify

a) $7\sqrt{3} - 2\sqrt{3}$

b) $-5\sqrt[3]{10} - 6\sqrt[3]{10}$

c) $4\sqrt{2} - 5\sqrt[3]{2}$

d) $2\sqrt{75} + 3\sqrt{3}$

e) $-\sqrt{27} + 3\sqrt{5} - \sqrt{80} - 2\sqrt{12}$

f) $\sqrt{9b} - 3\sqrt{16b}, \;\; b \geq 0$

In example f, why does *b* have to be greater than or equal to zero?

Example 2 – Simplify (assume variables are positive)

a) $\sqrt{27xy} + \sqrt{8xy}$

b) $4\sqrt[3]{16} + 3\sqrt[3]{54}$

c) $3x\sqrt{63y} - 5\sqrt{28x^2y}$

d) $\frac{5}{2}\sqrt[3]{16x^4y^5} - xy\sqrt[3]{54xy^2}$

1.6A – Multiplying & Dividing Radical Expressions

multiplying radicals

Example 1 – Multiply $2\sqrt{5}\,(3\sqrt{5})$ Verify your answer:

To multiply radicals:

In general:

Example 2 – Simplify: a) $5\sqrt{3}\,(\sqrt{6})$ b) $2\sqrt{6}\,(4\sqrt{8})$

c) $-3\sqrt{2x}\,(4\sqrt{3x})\;\; x \geq 0$

d) $-2\sqrt[3]{11}(4\sqrt[3]{2} - 3\sqrt[3]{3})$

e) $(4\sqrt{2} + 3)(\sqrt{7} - 5\sqrt{14})$

f) $(\sqrt[3]{x} + \sqrt[3]{y})(\sqrt[3]{x^2} - \sqrt[3]{xy} + \sqrt[3]{y^2})$

dividing radicals

Example 3 – Divide $\frac{6\sqrt{12}}{3\sqrt{6}}$

Verify your answer:

To divide radicals:

In general:

Example 4 – Simplify: a) $\frac{-24\sqrt[3]{14}}{8\sqrt[3]{2}}$ b) $\frac{2\sqrt{51}}{\sqrt{3}}$ c) $\frac{\sqrt{18x^3}}{\sqrt{3x}}, x > 0$

1.6B – Rationalizing the Denominator

rationalizing the denominator

A final answer cannot have a radical in the denominator. Therefore, you may have to '**rationalize the denominator**' – a process that will eliminate the radical from the denominator without changing the value of the expression.

If the denominator is a radical *monomial*, multiply the numerator and denominator by that radical.

Example 1 – Rationalize:

a) $\frac{3}{\sqrt{5}}$

b) $\sqrt{\frac{2}{7}}$

c) $6\sqrt{\frac{3}{4x}}, x > 0$

d) $\frac{6}{7\sqrt{5}}$

e) $\frac{2\sqrt{5}}{3\sqrt[3]{6}}$

f) $\sqrt[3]{\frac{2}{y}}$

g) $\frac{2}{\sqrt{x+1}}$

If the denominator is a radical *binomial*, multiply the numerator & denominator by its ***conjugate.***

Example 2 – Rationalize:

a) $\frac{3}{\sqrt{x}-2}$

b) $\frac{2+\sqrt{2}}{3\sqrt{5}-4}$

c) $\frac{\sqrt{a}+\sqrt{2b}}{\sqrt{a}-\sqrt{2b}}$, $a, b \geq 0$

1.7 – Radical Equations

Radical Equations are mathematical equations that include a radical, such as

$$2\sqrt{6x} - 1 = 11$$

If the index of the radical is even, there are restrictions on the variable: since it is not possible to find the square root of a negative number, the radicand cannot be negative.

Example 1 – Find the restriction on the variable:

a) $2\sqrt{6x} - 1 = 11$

b) $\sqrt{x+2} = 49$

c) $7\sqrt{-2x+3} = 35$

d) $\sqrt{3x+4} = \sqrt{2x-4}$

Steps to solving radical equations:

1. Find the restrictions on the variable in the radicand (if the index is even). Remember, the radicand must be set to ≥ 0 and then solved (if you multiply or divide by a negative number to both sides, FLIP the inequality).
2. Get the radical all by itself on one side of the equation.
3. If the index is 2, square both sides (if index is 3, cube both sides, etc.) and then solve for the variable.
4. See if the solution is affected by the restriction.
5. Check the answer using the original equation to see if solutions are valid or extraneous.

Example 2 – Solve: a) $2\sqrt{6x} - 1 = 11$

b) $-8 + \sqrt{\frac{3y}{5}} = 2$

Example 3 – Solve: a) $4+\sqrt{2x-3}=1$ b) $-2\sqrt{x-5}=16$

Example 4 – Solve:

a) $\sqrt{10x-7}=3\sqrt{x}$ b) $2\sqrt{x}=\sqrt{7x+6}$

Example 5 – Solve:

a) $\sqrt{x+1}=x-1$ b) $m-\sqrt{2m+3}=6$