

Chapter 2 Notes

Rationals

(Rational Expressions and Equations)

Date	Topic/Lesson	Assignment
	2.1 - Properties of Rational Expressions	After notes ex 1 p. 74: 1, 3 right After notes ex 2 p. 73 #4a-f, 5a-m (left)
	2.2 - Multiplying and Dividing Rational Expressions	After Notes ex 2 p. 79 #2a-m(left) After notes ex 3 p 81 3acegjkkm
	2.3 - Adding and Subtracting Rational Expressions	After Notes ex 1 p. 87 3a-k (left) After notes ex 2 p. 88 #4b-n (right)
	2.4 - Mixed Operations in Rational Expressions	After notes ex 1 p. 94 # 1 left After notes ex 3 p. 95 #2a-i (left)
	2.5 - Rational Equations	After notes ex 2 p. 101 #1, 4aceg After notes ex 3 p. 103 #5acegimnqs
	2.7 - Applications of Rational Expressions	After notes ex 3 p. 121 #2abc After notes ex 5 p. 122 3abe
	Practice Test	Chapter 6 Practice Test
	Review	P. 123 #1a-d, 2, 3ad, 4left, 5abc, 6abcegh, 11-14
	Chapter Test	Chapter 2 Test

2.1 – Properties of Rational Expressions

how zero
affects
division

A rational expression is ...

Evaluate $\frac{0}{3}$

When zero is divided by any non-zero real number, ...

Evaluate $\frac{7}{0}$

Division by zero is undefined because...

restricted
values

For the expression $\frac{3}{x-2}$, what value of x is restricted?

What is a **restricted value**?

Write a rule that explains how to determine restricted values:

Example 1 – Determine the restrictions for each rational expression:

a) $\frac{4a}{3b}$

b) $\frac{x-1}{(x+2)(x-3)}$

c) $\frac{2y^2}{y^2-4}$

**simplifying
rational
expressions**

When simplifying rational expressions:

1)

2)

Example 2 – Simplify the rational expressions. Keep a running list of restrictions.

a) $\frac{3x-3}{6x-6}$

b) $\frac{x-2}{x^2-4}$

c) $\frac{3x-6}{2x^2+x-10}$

d) $\frac{2y^2+y-10}{y^2+3y-10}$

e) $\frac{6-2m}{m^2-9}$

f) $\frac{x^2y+xy^2}{xy+y^2}$

See the bottom of page 71 for *Common Errors

2.2 – Multiplying & Dividing Rational Expressions

multiplication
& division
review

Warmup – Simplify

a) $\left(\frac{3}{-4}\right)\left(\frac{1}{2}\right)$

b) $\left(\frac{5}{8}\right)\left(\frac{-4}{15}\right)$

c) $\frac{2}{3} \div \frac{3}{4}$

d) $\frac{\frac{2}{5}}{\frac{-1}{10}}$

Explain how to multiply fractions:

Explain how to divide fractions:

Example 1 – Simplify and keep a running list of restrictions.

a) $\left(\frac{x+3}{2}\right)\left(\frac{x+1}{4}\right)$

b) $\left(\frac{4x^2}{3xy}\right)\left(\frac{y^2}{8x}\right)$

c) $\left(\frac{d}{2\pi r}\right)\left(\frac{2\pi rh}{d-2}\right)$

Example 2 – Simplify and keep a running list of restrictions.

a) $\frac{y^2-9}{r^3-r} \times \frac{r^2-r}{y+3}$

b) $\left(\frac{x^2-x-12}{x^2-9}\right)\left(\frac{x^2-4x+3}{x^2-4x}\right)$

Example 3 – Simplify and keep a running list of restrictions.

a) $\frac{m^2-6m-7}{m^2-49} \div \frac{m^2+8m+7}{m^2+7m}$

b) $\frac{3x+12}{3x^2-5x-12} \div \frac{12}{3x+4} \times \frac{2x-6}{x+4}$

2.3 – Adding & Subtracting Rational Expressions

adding &
subtracting
review

Warmup – Simplify each expression

a) $\frac{5}{6} - \frac{3}{8}$ b) $-\frac{2}{3} + \frac{4}{5}$ c) $\frac{7x+1}{x} + \frac{5x-2}{x}$ d) $\frac{7}{6x^2} - \frac{3}{8x^3}$

Write the steps to adding/subtracting fractions:

Example 1 – Simplify and identify all restrictions.

a) $\frac{10y-1}{4y-3} - \frac{8-2y}{4y-3}$ b) $\frac{2x}{xy} + \frac{4}{x^2} - 3$ c) $\frac{3}{3x+6} + \frac{1}{x+2}$

- Steps: 1) Factor as much as possible.
2) List restrictions. Do any relevant reducing.
3) Get common denominators.
4) Add or subtract numerators.
5) Do any further factoring and/or reducing.

Example 2 – Simplify and identify all restrictions.

a) $\frac{4}{x^2-1} + \frac{3}{1-x}$

b) $\frac{x-2}{x^2+x-6} - \frac{x^2+6x+5}{x^2+4x+3}$

c) $\frac{1}{x^2-1} - \frac{2}{x^2+x}$

d) $\frac{3x+9}{x^2+7x+10} + \frac{14}{x^2+3x-10}$

2.4 – Mixed Operations in Rational Expressions

When simplifying rational expressions with mixed operations, ORDER OF OPERATIONS is to be followed (**BEDMAS**).

Example 1 – Simplify & identify all restrictions.

$$\text{a) } \frac{x+5}{x+6} + \frac{1}{x+4} \div \frac{x+6}{x^2-x-20}$$

$$\text{b) } \left(\frac{x-3}{x^2-9} + \frac{x+3}{x^2+6x+9} \right) \left(\frac{x+3}{x+1} \right)$$

Complex Fractions – Rational Expressions that contain fractions in the numerators and/or denominators.

Example 2 – Simplify and identify all restrictions.

$$\frac{2 - \frac{4}{y}}{y - \frac{4}{y}}$$

Steps:

- 1) Get a common denominator for the numerator and then the denominator of the complex fraction.
- 2) Write each as one fraction.
- 3) Rewrite the division in a side-by-side manner and simplify.

Example 3 – Simplify and identify all restrictions.

a)
$$\frac{\frac{2}{5x} - \frac{3}{x^2}}{\frac{7}{2x} + \frac{3}{4x^2}}$$

b)
$$\frac{\frac{1}{x-1} + \frac{2}{x+2}}{\frac{2}{x+2} - \frac{1}{x-3}}$$

2.5 – Rational Equations

A rational equation is an equation containing at least one rational expression. Remember, when working with an equation, whatever you do to one side, you do to the other side.

Steps to solving rational equations:

- 1) Factor each denominator if possible.
- 2) Identify any restrictions (and do this throughout).
- 3) Multiply both sides of the equation by what would be the lowest common denominator in order to eliminate the fractions.
- 4) Solve the equation.
- 5) Check your solutions.

Example 1 – Solve

$$\text{a) } \frac{x}{2} + \frac{7}{3} = \frac{5}{6}$$

$$\text{b) } \frac{5}{3x} - \frac{1}{9} = \frac{4}{x}$$

Example 2 – Solve

$$\text{a) } \frac{2x}{x-4} = \frac{8}{x-4} + 1$$

$$\text{b) } \frac{9}{y-3} - \frac{4}{y-6} = \frac{18}{y^2-9y+18}$$

****When a solution is the same as a restricted value, it is called an EXTRANEIOUS solution.***

Example 3 – Solve

$$\text{a) } \frac{x}{x-5} - \frac{3}{x+1} = \frac{30}{x^2-4x-5}$$

$$\text{b) } \frac{3x}{x+2} - \frac{5}{x-3} = \frac{-25}{x^2-x-6}$$

2.7 – Applications of Rational Equations

There is no fool-proof way to solve a word problem. You should try to read the problem carefully, create a 'Let' statement for your variable, build your equation (sometimes using a table or diagram for assistance), and solve the equation. Then do a check.

Example 1 – Stella takes 4 hours to paint a room. It takes Jose 3 hours to paint the same area. How long will the paint job take if they work together?

	Time to Paint (hours)	Fraction of Work Done in 1 hour	Fraction of Work Done in x hours
Stella			
Jose			
Together			

Example 2 – Jenny takes 5 hours to install laminate flooring in the kitchen by herself. Mike can do the job alone in 6 hours. How long would it take them if they did it together?

Example 3 – Evan works twice as fast as JJ. If it takes them 13 minutes & 20 seconds together to shovel snow from the driveway, how long would it take JJ by himself?

Example 4 – A speedboat can travel 108km downstream in the same time it can travel 78km upstream. If the current of the river is 10km/h, what is the speed of the boat in still water?

$$s = \frac{d}{t} \quad d = st \quad t = \frac{d}{s}$$

	d (km)	s (km/h)	t (h)	Equation
downstream				
upstream				

Example 5 – Rob and Alissa ride a skateboard a distance of 4km. It takes Rob one more minute (hint: $\frac{1}{60}$ of an hour) than it takes Alissa, and Alissa travels 1km/h faster than Rob. At what speed is each traveling?