PC11 Name___________________________

Chapter 3/4 Notes

Quadratic Functions

Date	Topic/Lesson	Assignment
	3.1 – Graphing Quadratic Functions of the Form $y = x^2$ and $y = x^2 + k$	After Notes – p.131: 1-2,5-6 (for 6, use basic count, NOT table of values)
	3.2 – Graphing Quadratic Functions of the Form $y = ax^2 + k$	After Notes – p.136: 1 (skip table of values) AND 3.2 Worksheet
	3.3 – Graphing Quadratic Functions of the Form $y = a(x – h)^2$	After Notes – p.141: 1, 3abc (skip table of values) AND 3.3 Worksheet
	3.4 – Graphing Quadratic Functions in Vertex Form $y = a(x – h)^2 + k$	After Notes – p.145: 2, 3bdf, 4ad AND 3.4 Worksheet
	3.5 – Finding the Equation of a Parabola	After Notes ex 2 – p.153: 1abcefg, After Notes ex 4 – p.154: 2abcegh
	4.1 – Completing the Square when a = 1	After Notes – p. 163: 4abcde AND p. 171: 3acd, 4e, 5a
	4.2 – Completing the Square when a ≠ 1	After Notes – p.163: 4fghij AND p. 171: 3efgh, 4ab, 5i
	4.3A – Intercepts and Projectile Applications of Quadratic Functions	After Notes ex 2 – p.164: 5abcfk (complete the square first if necessary) After Notes ex 3 – 4.3A Worksheet
	4.3B – Area Applications	After Notes – p.179: 2, 3, 11
	4.3C – Financial Applications	After Notes – p.179: 1, 6, 12
	Practice Test	Chapter 3/4 Practice Test
	Review	p.157: 1acd, 2, 3abc p.183: 1-3, 6, 7, 9 p. 179: 5
	Chapter Test	**Chapter 3/4 Test**

3.1 – Graphing Quadratic Functions of the Form $y = x^2$ and $y = x^2 + k$

A **quadratic function** is a function that has a second degree polynomial or "degree 2," which has an x^2 term, but nothing higher.

Examples:

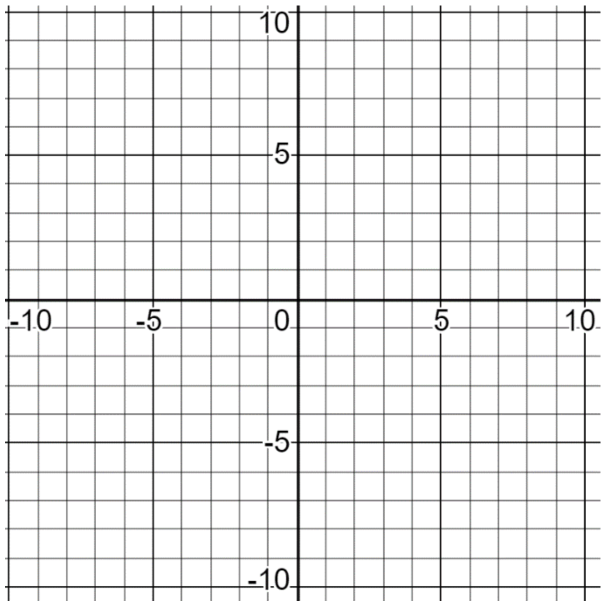
NON-examples:

$y = x^2$

Investigate the shape of the simplest quadratic function, $y = x^2$, by making a table of values and graphing:

$y = x^2$

x	*y*
-3	
-2	
-1	
0	
1	
2	
3	



Vertex:

A of S eqn:

y-int:

x-int(s):

Domain:

Range:

Opens:

Max/Min:

When you graph a quadratic function, the resulting shape is called a ____________________.

Key features of a parabola:

- The __________________ is the "middle" of the parabola, where it "turns around."
- Parabolas are _________________. The axis of symmetry is a vertical line that splits the parabola down the middle. It is described by an equation (x = a number).
- Some parabolas open _____. These parabolas have a ______________ height (at the vertex). If a parabola opens _________, then it has a ______________ height, located at the vertex.
- **Domain**: all quadratic functions have a domain of all real numbers. (x values go to the left and right forever)
  Range: since parabolas have either a minimum or a maximum, their range is restricted.
- **x-intercepts**: on any graph, you can find the x-intercepts by looking for any points where the graph crosses the x-axis.
  y-intercepts: on any graph, you can find the y-intercept by looking for the point where the graph crosses the y-axis.

What happens to y = x^2 when we add (or subtract) it by a constant? That is, $y = x^2 + k$?

y = x^2

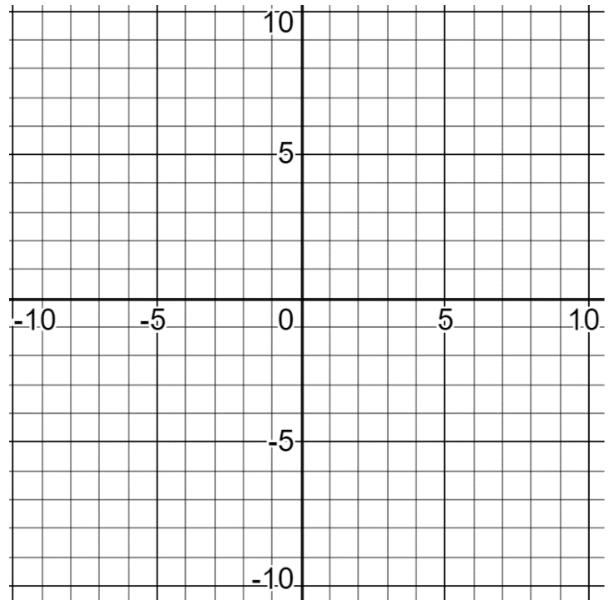
graphing shortcut

a) To quickly graph $y = x^2$, use the **basic count**:
Start at (0,0) and go **over 1,**

over 2,

over 3,

over 4,



***k* value**

$y = x^2 + k$

b) Graph $y = x^2 + 5$ using a table of values:

x	*y*
-3	
-2	
-1	
0	
1	
2	
3	

Notice:

Vertex: Domain:

A of S eqn: Range:

y-int: Opens:

x-int(s): Max/Min:

c) Graph $y = x^2 - 4$ by count method:

k value is:

Vertex is:

Then do **basic count**:

Vertex: Domain:

A of S eqn: Range:

y-int: Opens:

x-int(s): Max/Min:

y* = x^2 + *k

The *k* value:

3.2 – Graphing Quadratic Functions of the Form $y = ax^2 + k$

What happens to y = x^2 when we multiply it by a constant? That is, $y = ax^2$?

***a* value**

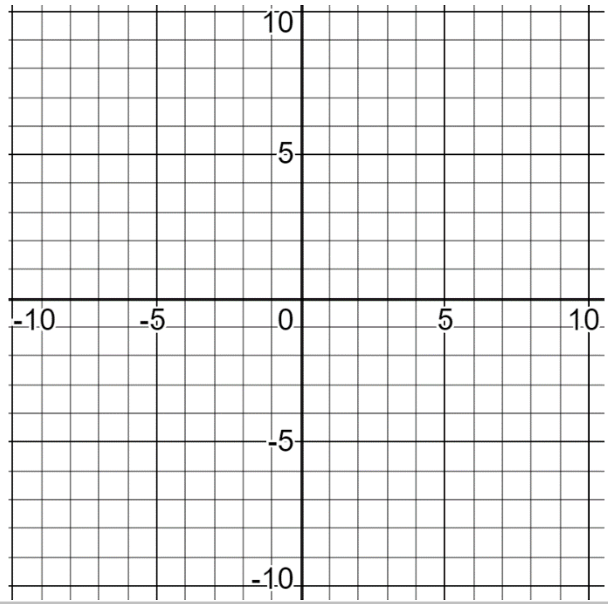
$y = ax^2$

1.a) Graph $y = x^2$ using the count.

b) Graph $y = 2x^2$ using a table of values

x	y
-3	
-2	
-1	
0	
1	
2	
3	

Notice:

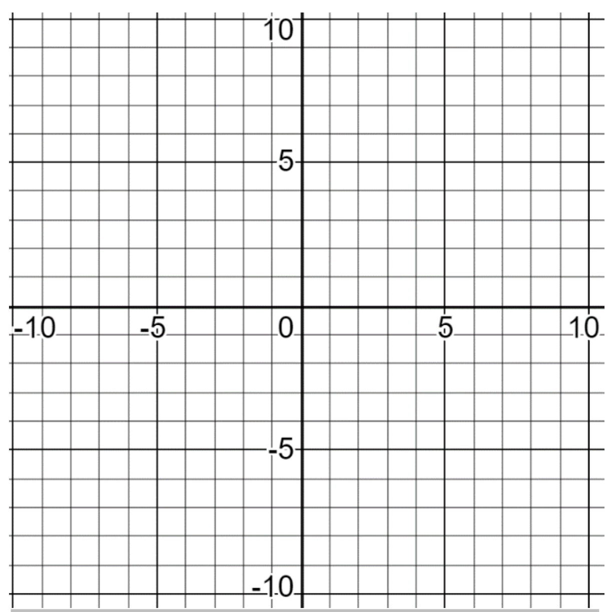


c) Graph $y = \frac{1}{2}x^2$ using the count method:

The *a* value:

2. Graph $y = -x^2$ using a table of values

x	y
-3	
-2	
-1	
0	
1	
2	
3	



Vertex: Domain:

A of S eqn: Range:

y-int: Opens:

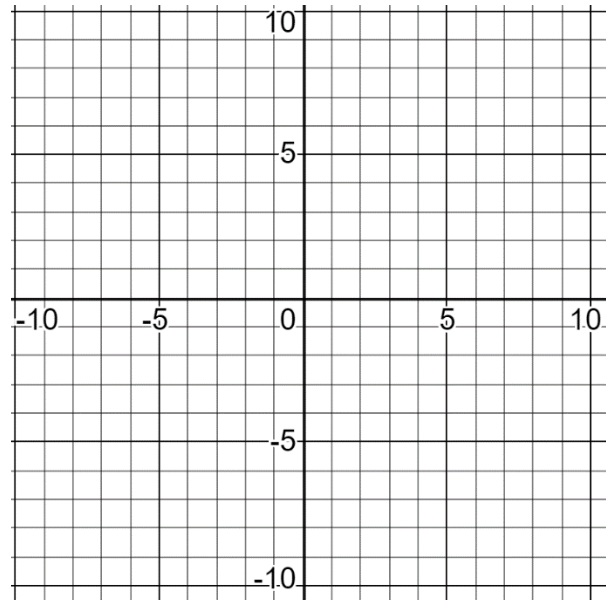
When the *a* value is negative:

$y = ax^2 + k$

Now that we know what the “a” and the “k” values both do, we can graph quadratic functions that have both a vertical stretch and/or reflection and a vertical translation:

3. Graph and find key info for

$$f(x) = -3x^2 + 3$$



Vertex: Domain:

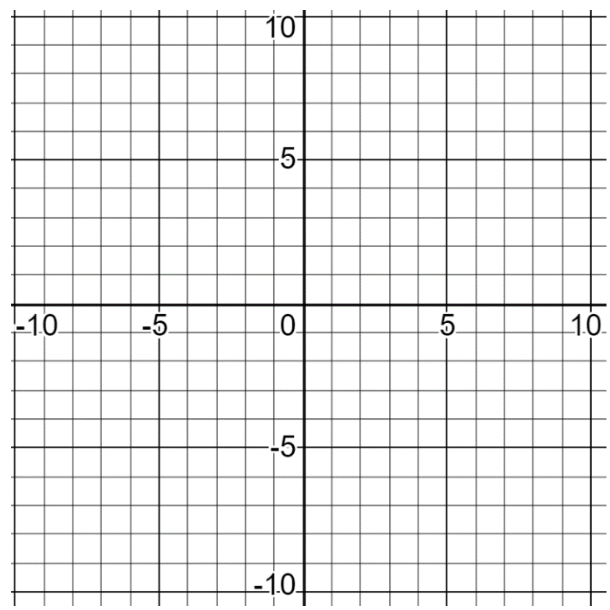
A of S eqn: Range:

y-int: Opens:

x-int(s): Max/Min:

4. Graph and find key info for

$$y + 2 = \frac{1}{2}x^2$$



Vertex: Domain:

A of S eqn: Range:

y-int: Opens:

x-int(s): Max/Min:

3.3 – Graphing Quadratic Functions of the Form $y = a(x - h)^2$

***h* value**

$y = (x - h)^2$

If "k" gives us a vertical shift, how can we move the graph horizontally?

1. a) Graph $y = x^2$ using the count

b) Graph $y = (x - 4)^2$ using a table of values

x	y
1	
2	
3	
4	
5	
6	
7	

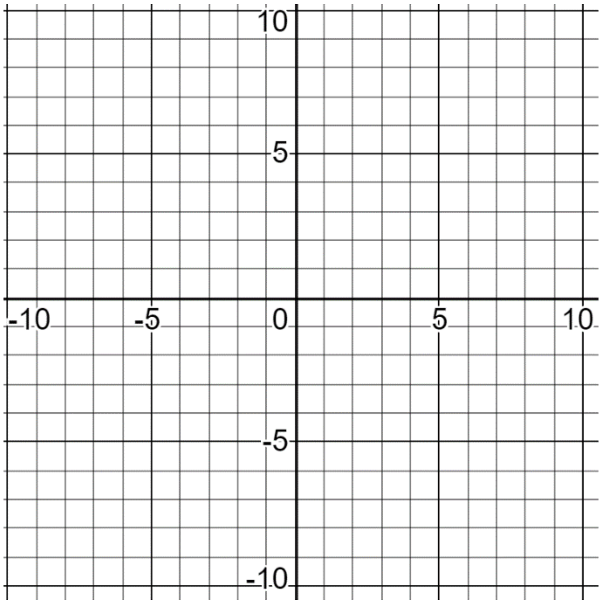
Notice:

Vertex: Domain:

A of S eqn: Range:

y-int: Opens:

x-int(s): Max/Min:



c) Graph $y = (x + 3)^2$ using the count method:

Vertex: Domain:

A of S eqn: Range:

y-int: Opens:

x-int(s): Max/Min:

h value <u>Mental Switch:</u>

Practice

y = a(x – h)²

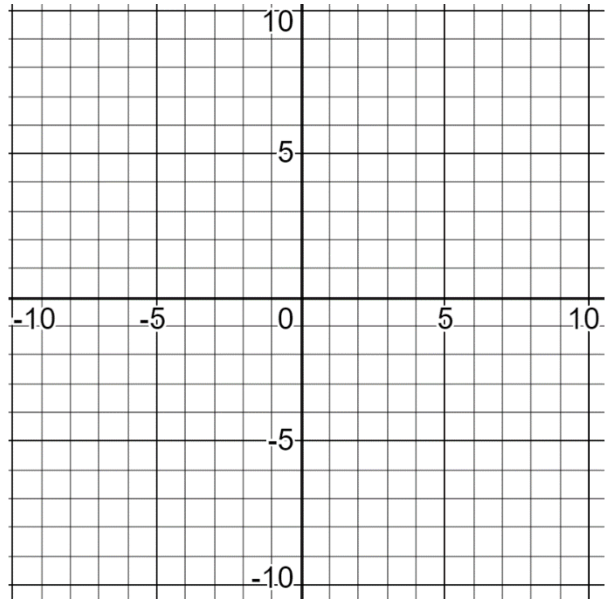
2. Graph $y = 2(x-2)^2$ using the count method

Vertex: Domain:

A of S eqn: Range:

y-int: Opens:

x-int(s): Max/Min:



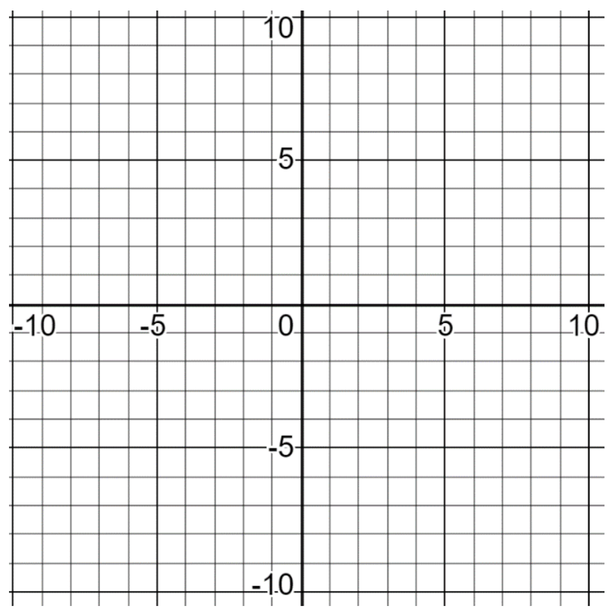
3. Graph $y = -\frac{1}{2}(x+4)^2$ using the count method

Vertex: Domain:

A of S eqn: Range:

y-int: Opens:

x-int(s): Max/Min:



3.4 – Investigating Quadratic Functions in Vertex Form: $y = a(x-h)^2 + k$

We can now graph any quadratic function written in "vertex" form: $y = a(x-h)^2 + k$

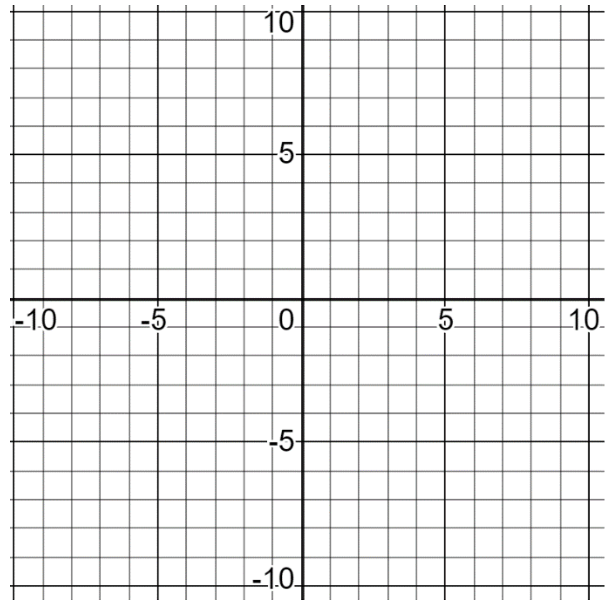
1. Graph $y = (x-2)^2 - 4$

Vertex: Domain:

A of S eqn: Range:

y-int: Opens:

x-int(s): Max/Min:



The Vertex

How does the vertex relate to the equation?

2. Graph and find key info for

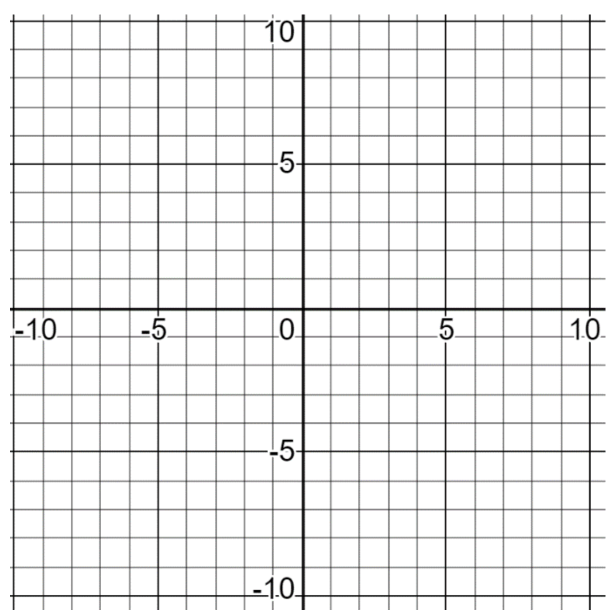
$f(x) = 2(x+3)^2 - 8$

Vertex: Domain:

A of S eqn: Range:

y-int: Opens:

x-int(s): Max/Min:



***Note:** depending on the book you are using, you might see this called "graphing form", "standard form" or "general form." We use "vertex form" because this equation helps us easily identify the vertex!*

Vertex Form: $y = a(x - h)^2 + k$

Notes:

a

h

k

3. Graph and find key info for

$y - 3 = -\frac{1}{3}(x - 3)^2$

Vertex:

A of S eqn:

y-int:

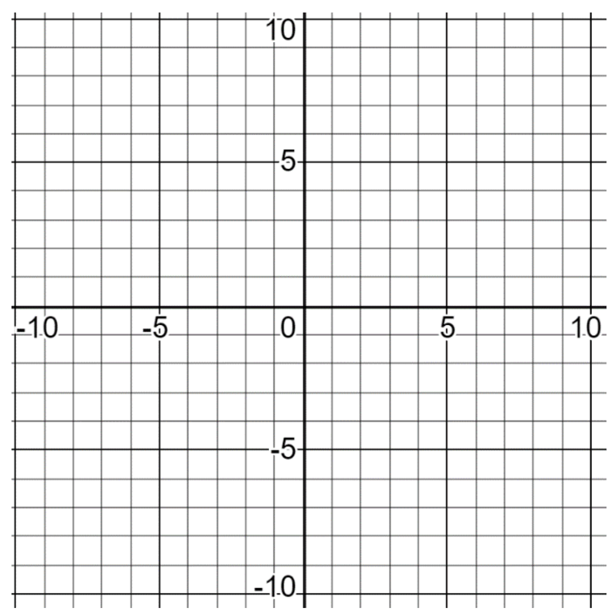
x-int(s):

Domain:

Range:

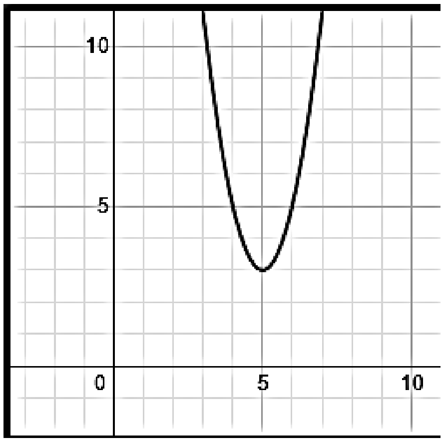
Opens:

Max/Min:



finding the equation of a parabola from a graph

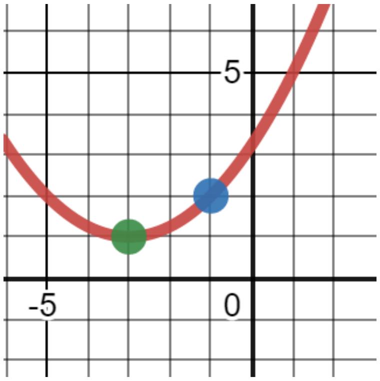
Example 1 – Determine the equation of the following parabola:



Steps:

1) Identify the vertex, which will give you "h" and "k". Remember the "h" sign switch!
2) Find another point on the graph that is easy to read, and count how far "over" it is from the vertex. *What would you expect the "up" to be in the basic count?*
3) Compare the graph's "up" count to the basic count. *What "a" value would you need to multiply by to turn the basic "up" into the graph's "up"?*
4) Write the formula $y = a(x-h)^2 + k$ with the "a", "h" and "k" values now filled in.

Example 2 – Determine the equation of the following parabola:



finding the equation of a parabola from the vertex and another point

Example 3 – A parabola with vertex (1, -2) passes through the point (4, 1). Find the equation.

Steps:

1) Write the formula $y = a(x - h)^2 + k$
2) Fill in what you know (usually the vertex)
3) Sub in a point and solve for what's missing (usually "a")
4) Rewrite the equation (now that you know a, h, k)

x- and y-intercepts

Example 4 – Find the equation of a quadratic function whose graph has vertex (–4, 8) and an *x*-intercept of – 6.

Hint: x-intercepts always have coordinates of…
and y-intercepts always have coordinates of…

4.1 – Completing the Square when *a = 1*

Vertex Form

vs

General Form

When a quadratic function is in VERTEX (or Graphing) FORM $y = a(x-h)^2 + k$
we can easily find its vertex and graph the parabola.

However, quadratic functions are often presented in the form $y = ax^2 + bx + c$
which is often called GENERAL FORM.

General form can be useful, but if we want to graph the parabola or find its vertex, we will need to convert from General Form to Vertex Form, using a process called
____.

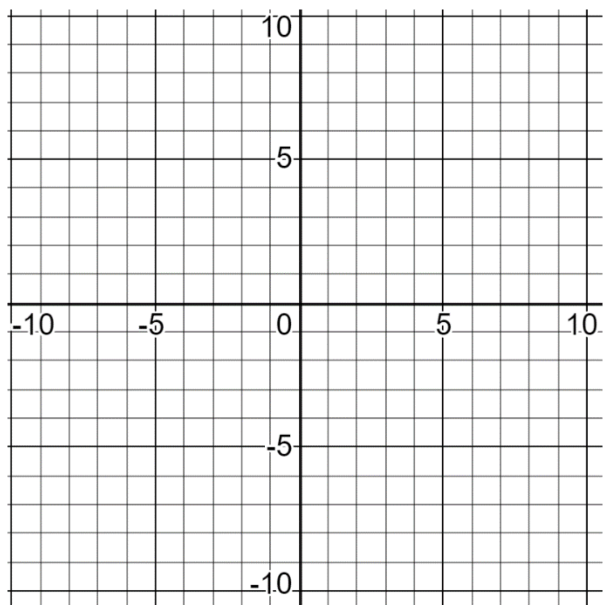
Completing the Square

Example 1 – Rewrite $y = 14 + 10x + x^2$ in vertex form by completing the square. Then sketch the graph.

STEPS:

1) Rearrange so squared term is first and *x* term is second.
2) Find the *a, b, c* values
3) Take half the b-value (you'll need this later), then square it.
4) Add and subtract the result to your quadratic function after the *x* term.
5) Make sure the new term you <u>added</u> is the third term.
6) Factor the trinomial and add the two last terms.

Shortcut for factoring the trinomial:



Example 2 – Change $y = x^2 - 4x - 1$ into Vertex Form, then state the vertex.

Example 3 – Change $y = x^2 + 5x + 2$ into Vertex Form, then state the vertex. (Use fractions, not decimals.)

4.2 – Completing the Square when *a ≠ 1*

When a $\neq 1$

When the *a* value is different from 1, there are a few more steps to Complete the Square and convert from General Form ($y = ax^2 + bx + c$) to Vertex Form ($y = a(x-h)^2 + k$).

Example 1 – Change $y = -2x^2 + 4x + 5$ into Vertex form and then state the vertex.

STEPS:

(Re-order first if needed)

1) Group the first two terms together.
2) Factor the *a* value out.
3) Find the new *b* value. Take half and square it.
4) Add and subtract the result IN THE BRACKETS.
5) Get the subtracted result out of the brackets by multiplying to the coefficient in front of the brackets.
6) Factor the trinomial and add the two outside terms.

Example 2 – Change $y = 3x^2 - 12x + 11$ into Vertex Form and state the vertex.

Example 3 – Change $y = 5x - 3x^2 + 1$ into Vertex Form using exact values and state the vertex.

4.3A – Intercepts and Projectile Applications of Quadratic Functions

Finding intercepts with algebra

y-intercepts

y-intercepts occur when the graph crosses the y-axis, which means the point is "over nothing" (x is zero). We can find the y-intercepts of any function by setting x=0 and evaluating for y.

Example 1 – Determine the y-intercept of the following parabolas:

a) $f(x) = (x+5)^2 - 10$

b) $g(x) = 3x^2 + 14x - 12$

x-intercepts

x-intercepts occur when the graph touches the x-axis, which means its height (y) is zero. We can find the x-intercepts of any function by setting y=0 and solving for x.

Before we try to solve, first consider how many x-intercepts are possible when graphing a parabola:

Where do the 2 solutions come from?

Example 2 – Determine the x-intercepts of the following parabolas.

a) $y = x^2 - 4$

b) $y = x^2 - 7$

Notice that we need the function in VERTEX FORM in order to solve.

c) $y = (x-2)^2 - 9$

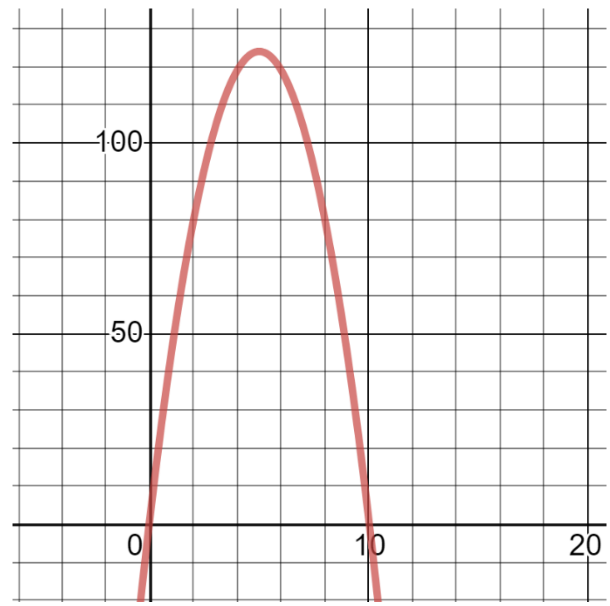
d) $y = 2(x+1)^2 - 3$

What do the intercepts represent in this situation?

Example 3 – The path of a rocket fired over a lake is described by the function

$h(t) = -4.9t^2 + 49t + 1.5$ where *h(t)* is the height of the rocket, in metres, and *t* is time in seconds, since the rocket was fired.

a) What is the maximum height reached by the rocket? How many seconds after it was fired did the rocket reach this height?
b) How high was the rocket above the lake when it was fired?
c) At what time does the rocket hit the ground?
d) What domain and range are appropriate in this situation?
e) How high was the rocket after 7s? Was it on its way up or down?



4.3B – Maximizing Area Applications of Quadratic Functions

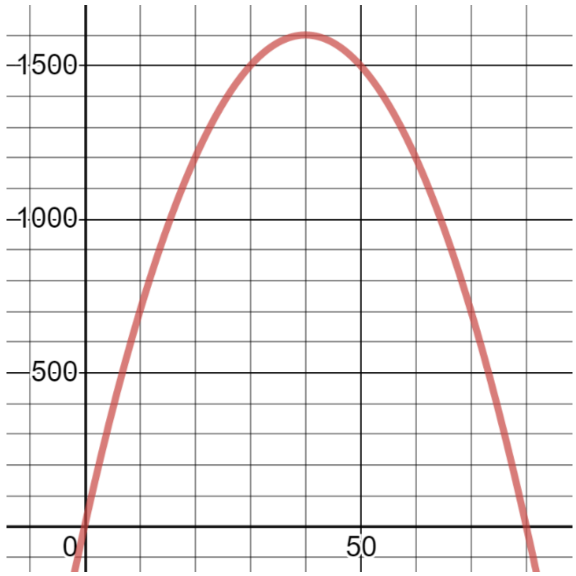
The Vertex helps us find the MAX or MIN in a real-life scenario.

When we model a situation and it generates a quadratic function, we can find the MAXIMUM (or MINIMUM) value for the function by finding the vertex.

Example 1 – At a concert, organizers are roping off a rectangular area for sound equipment. There is 160m of fencing available to create the perimeter. What dimensions will give the maximum area, and what is the maximum area?

Steps:

1) Write an equation for perimeter, and write an equation for area for a rectangle.
2) Use the two equations to create a quadratic function in general form.
3) Complete the square to change the quadratic function into standard form.
4) Identify the maximum area, and then the dimensions for the maximum area.



Example 2 – A rancher has 800m of fencing to enclose a rectangular cattle pen along a river bank. There is no fencing needed along the river bank. Find the dimensions that would enclose the largest area.

4.3C – Financial Applications of Quadratic Functions

The Vertex helps us find the MAX or MIN in a real-life scenario. With financial applications, we may want to maximize profit or minimize cost.

When we model a situation and it generates a quadratic function, we can find the MAXIMUM (or MINIMUM) value for the function by finding the vertex.

Example 1 – A sporting goods store sells basketball shorts for $8. At this price their weekly sales are approximately 100 items. Research says that for every $2 increase in price, the manager can expect the store to sell five fewer pairs of shorts. Determine the maximum revenue the manager can expect based on these estimates. What selling price will give that maximum revenue, and how many shorts will be sold?