PC11 Name______KEY_______

Chapter 3/4 Notes

Quadratic Functions

Date	Topic/Lesson	Assignment
	3.1 – Graphing Quadratic Functions of the Form $y = x^2$ and $y = x^2 + k$	After Notes – p.131: 1-2,5-6 (for 6, use basic count, NOT table of values)
	3.2 – Graphing Quadratic Functions of the Form $y = ax^2 + k$	After Notes – p.136: 1 (skip table of values) AND 3.2 Worksheet
	3.3 – Graphing Quadratic Functions of the Form $y = a(x – h)^2$	After Notes – p.141: 1, 3abc (skip table of values) AND 3.3 Worksheet
	3.4 – Graphing Quadratic Functions in Vertex Form $y = a(x – h)^2 + k$	After Notes – p.145: 2, 3bdf, 4ad AND 3.4 Worksheet
	3.5 – Finding the Equation of a Parabola	After Notes ex 2 – p.153: 1abcefg, After Notes ex 4 – p.154: 2abcegh
	4.1 – Completing the Square when $a = 1$	After Notes – p. 163: 4abcde AND p. 171: 3acd, 4e, 5a
	4.2 – Completing the Square when $a \neq 1$	After Notes – p.163: 4fghij AND p. 171: 3efgh, 4ab, 5i
	4.3A – Intercepts and Projectile Applications of Quadratic Functions	After Notes ex 2 – p.164: 5abcfk (complete the square first if necessary) After Notes ex 3 – 4.3A Worksheet
	4.3B – Area Applications	After Notes – p.179: 2, 3, 11
	4.3C – Financial Applications	After Notes – p.179: 1, 6, 12
	Practice Test	Chapter 3/4 Practice Test
	Review	p.157: 1acd, 2, 3abc p.183: 1-3, 6, 7, 9 p. 179: 5
	Chapter Test	**Chapter 3/4 Test**

3.1 – Graphing Quadratic Functions of the Form $y = x^2$ and $y = x^2 + k$

A **quadratic function** is a function that has a second degree polynomial or "degree 2," which has an x^2 term, but nothing higher.

Examples: $y = 2x^2 + 3x + 7$
$y = -(x+1)^2 - 3$
$y = \frac{x^2}{3}$
$y = 2 - x^2 + \frac{1}{4}x$

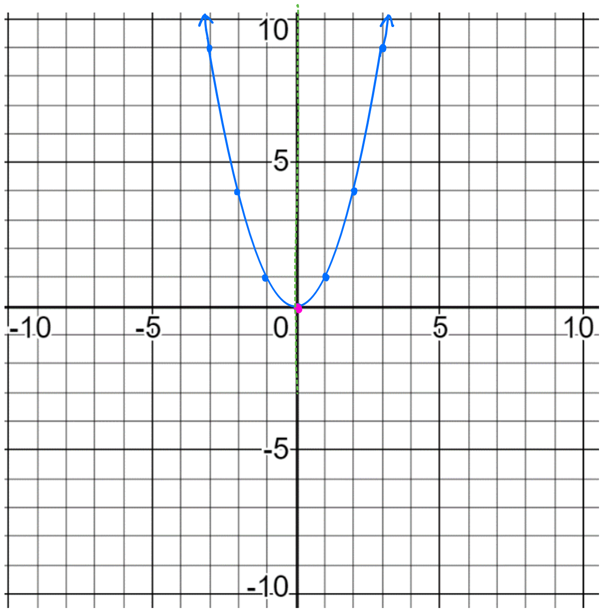
NON-examples: $y = \frac{1}{x^2} + x^2 + 1$
$y = \sqrt{x} + x$
$y = \frac{1}{2}x - 5$

$y = x^2$

Investigate the shape of the simplest quadratic function, $y = x^2$, by making a table of values and graphing:

$y = x^2$

x	$y = (x)^2$
-3	$9 = (-3)^2$
-2	$4 = (-2)^2$
-1	$1 = (-1)^2$
0	$0 = 0^2$
1	$1 = 1^2$
2	$4 = 2^2$
3	$9 = 3^2$



Vertex: $(0,0)$

A of S eqn: $x = 0$

y-int: $(0,0)$

x-int(s): $(0,0)$

Domain: $x \in \mathbb{R}$
x = all real numbers

Range: $y \geq 0$

Opens: up

Max/[✓ Min]: of $y = 0$

When you graph a quadratic function, the resulting shape is called a parabola.

Key features of a parabola:

- The vertex is the "middle" of the parabola, where it "turns around."
- Parabolas are symmetrical. The axis of symmetry is a vertical line that splits the parabola down the middle. It is described by an equation (x = a number).
- Some parabolas open up. These parabolas have a minimum ∪ height (at the vertex). If a parabola opens down, then it has a maximum ∩ height, located at the vertex.
- **Domain**: all quadratic functions have a domain of all real numbers. (x values go to the left and right forever) $\{x \in \mathbb{R}\}$ x is an element of the real numbers or x = all real numbers or $x = \infty$
  Range: since parabolas have either a minimum or a maximum, their range is restricted.
  ∪ $y \geq \#$ or ∩ $y \leq \#$
- **x-intercepts**: on any graph, you can find the x-intercepts by looking for any points where the graph crosses the x-axis. height = 0, so $y = 0$
  y-intercepts: on any graph, you can find the y-intercept by looking for the point where the graph crosses the y-axis. left/right nothing so $x = 0$

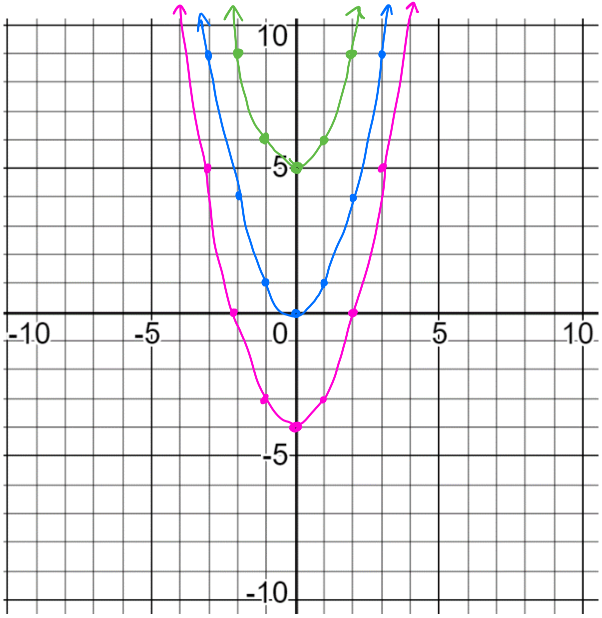
$y = x^2$

graphing shortcut

***k* value**

$y = x^2 + k$

What happens to $y = x^2$ when we add (or subtract) it by a constant? That is, $y = x^2 + k$?



a) To quickly graph $y = x^2$, use the **basic count**:

Start at (0,0) and go **over 1,** up 1

always start back at the vertex

over 2, up 4

over 3, up 9

over 4, up 16

b) Graph $y = x^2 + 5$ using a table of values:

vertex

x	y	$= x^2 + 5$
-3	14	$= (-3)^2 + 5 = 9 + 5 = 14$
-2	9	$= (-2)^2 + 5 = 4 + 5 = 9$
-1	6	$= (-1)^2 + 5 = 1 + 5 = 6$
$\boxed{0}$	$\boxed{5}$	$= 0^2 + 5 = 5$
1	6	$= 1^2 + 5 = 6$
2	9	$= 2^2 + 5 = 9$
3	14	$= 3^2 + 5 = 14$

repeats (symmetry)

Notice:

- $y = x^2 + 5$ is same shape as $y = x^2$ (same count)
- It has moved up 5 units
- The "up 5" came from the "+ 5" after x^2. We call this value "k"

Vertex: (0, 5)

Domain: $x \in \mathbb{R}$ or all real #s

A of S eqn: $x = 0$

Range: $y \geq 5$

y-int: (0, 5)

Opens: up

x-int(s): none

Max/[✓ Min]: $y = 5$

c) Graph $y = x^2 - 4$ by count method:

k value is: -4

Vertex is: (0, -4)

Then do **basic count**: over 1 up 1

always start back at vertex

over 2 up 4

over 3 up 9

over 4 up 16

Vertex: (0, -4)

Domain: $x \in \mathbb{R}$

A of S eqn: $x = 0$

Range: $y \geq -4$

y-int: (0, -4)

Opens: up

x-int(s): (-2, 0)(2, 0)

Max/[✓ Min]: $y = -4$

y = x² + k

The *k* value: translates (shifts) the parabola up (if "k" is pos) or down (if "k" is neg)

This is called a "vertical translation"

3.2 – Graphing Quadratic Functions of the Form $y = ax^2 + k$

What happens to $y = x^2$ when we multiply it by a constant? That is, $y = ax^2$?

***a* value**

$y = ax^2$

1.a) Graph $y = x^2$ using the count.

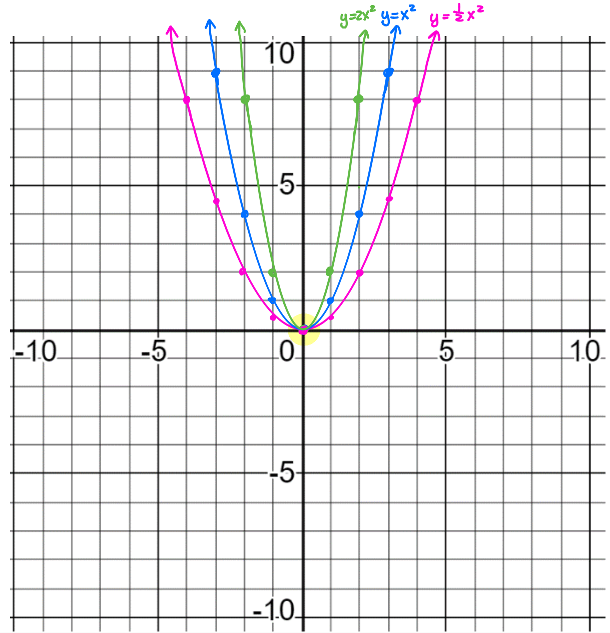
b) Graph $y = 2x^2$ using a table of values

x	y	$= 2(x)^2$
-3	18	$= 2(-3)^2 = 2(9) = 18$
-2	8	$= 2(-2)^2 = 2(4) = 8$
-1	2	$= 2(-1)^2 = 2(1) = 2$
0	0	$= 2(0)^2 = 2(0) = 0$
1	2	$= 2(1)^2 = 2$
2	8	$= 2(2)^2 = 8$
3	18	$= 2(3)^2 = 18$

symmetry

Notice:

- the parabola $y = 2x^2$ is taller/skinnier than $y = x^2$
- the "a" value of 2 causes it to rise twice as fast



called a "vertical stretch"

The *a* value: alters the "up" count

expansion → If $a > 1$, parabola taller/skinnier

compression → if $a < 1$, parabola shorter/wider

c) Graph $y = \frac{1}{2}x^2$ using the count method:

$V(0,0)$ $a = \frac{1}{2}$ (multiply "up" count by $\frac{1}{2}$)

Over 1 up $1 \times \frac{1}{2} \rightarrow$ up $\frac{1}{2}$

Over 2 up $4 \times \frac{1}{2} \rightarrow$ up 2

over 3 up $9 \times \frac{1}{2} \rightarrow$ up 4.5

over 4 up $16 \times \frac{1}{2} \rightarrow$ up 8

2. Graph $y = -x^2$ using a table of values

x	y	$= -(x)^2$
-3	-9	$= -(-3)^2 = -9$
-2	-4	$= -(-2)^2 = -4$
-1	-1	$= -(-1)^2 = -1$
0	0	$= -(0)^2 = 0$
1	-1	$= -(1)^2 = -1$
2	-4	$= -(2)^2 = -4$
3	-9	$= -(3)^2 = -9$

Same count but negative

Now it's a "down" count

Vertex: $(0,0)$

Domain: $x \in \mathbb{R}$

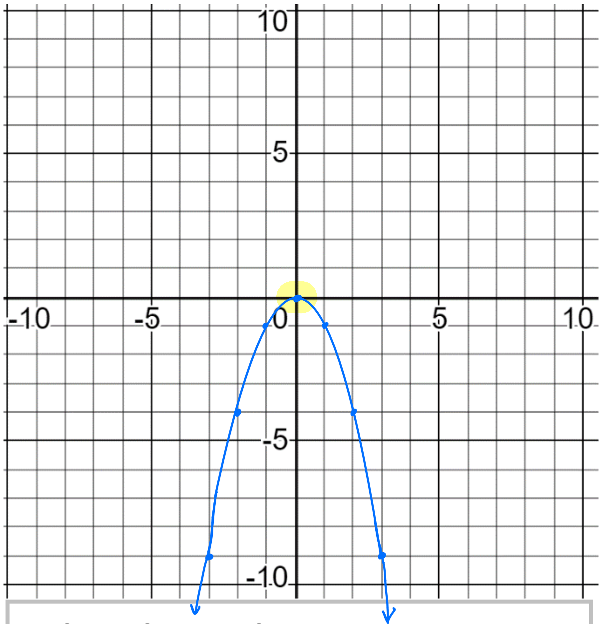
A of S eqn: $x = 0$

Range: $y \leq 0$

y-int: $(0,0)$

Opens: Down

called a "vertical reflection"



When the *a* value is negative:

- The parabola opens down
- all "up" counts become "down" counts

$y = ax^2 + k$

Now that we know what the "a" and the "k" values both do, we can graph quadratic functions that have both a vertical stretch and/or reflection and a vertical translation:

3. Graph and find key info for

$$\overset{y\ =\ a\,x^2\ +\ K}{f(x) = -3x^2 + 3}$$

$a = -3 \quad K = 3 \quad V(0,3)$

(basic "up" count x a)

Over 1 up 1 x (-3) = down 3

Over 2 up 4 x(-3) = down 12

Vertex: (0,3)

Domain: $X \in \mathbb{R}$

A of S eqn: $x=0$

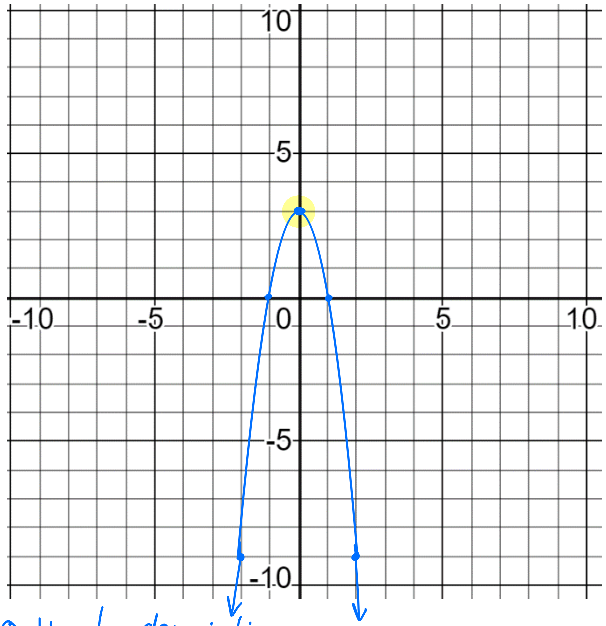
Range: $y \leq 3$

y-int: (0,3)

Opens: Down

x-int(s): (-1,0)(1,0)

[✓ Max]/Min: $y=3$



Optional description

- vertical expansion by 3
- vertical reflection
- vertical translation up 3

4. Graph and find key info for

$$\underset{-2}{y + 2} = \underset{-2}{\tfrac{1}{2}x^2}$$

must rearrange into $y=ax^2+K$

$y = \frac{1}{2}x^2 - 2$

$a = \frac{1}{2} \quad K = -2 \rightarrow V(0,-2)$

Over 1 up1 × ½ = up 0.5

Over 2 up 4 × ½ = up 2

over 3 up 9 × ½ = up 4.5

Over 4 up 16 × ½ = up 8

Vertex: (0, -2)

Domain: $X \in \mathbb{R}$

A of S eqn: $x=0$

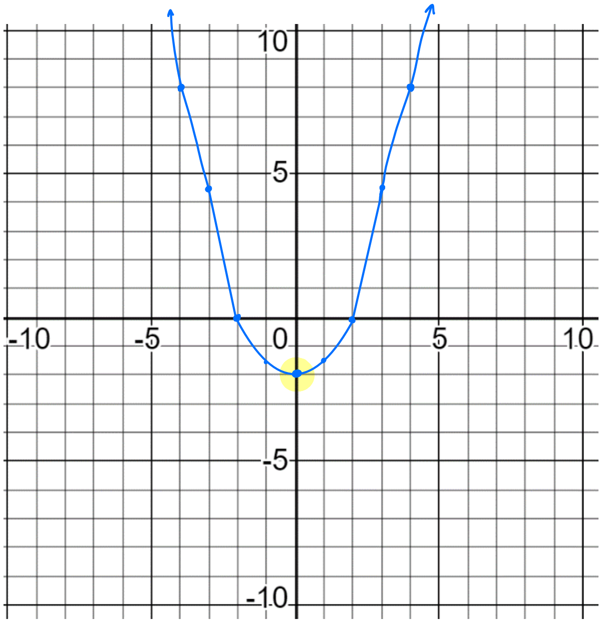
Range: $y \geq -2$

y-int: (0,-2)

Opens: Up

x-int(s): (-2,0)(2,0)

Max/[✓ Min]: $y=-2$



Optional description

- vertical compression by ½
- vertical translation down 2

3.3 – Graphing Quadratic Functions of the Form $y = a(x-h)^2$

***h* value**

$y = (x-h)^2$

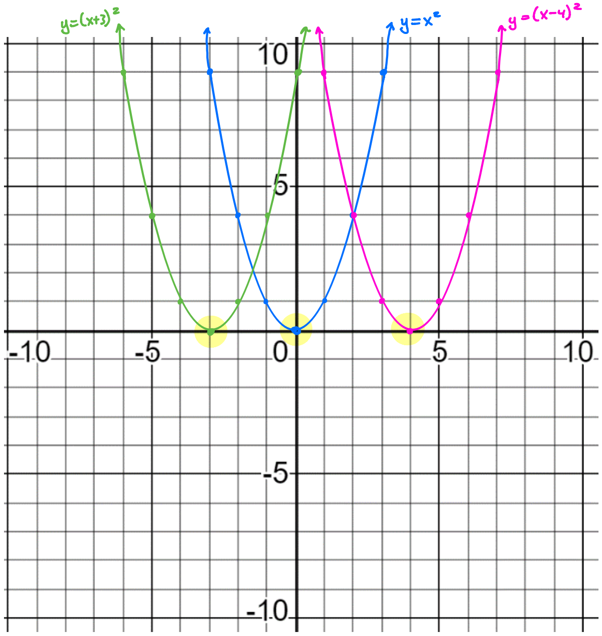
If "k" gives us a vertical shift, how can we move the graph horizontally?

1. a) Graph $y = x^2$ using the count

b) Graph $y = (x-4)^2$ using a table of values

vertex →

x	y	$= (x-4)^2$
1	9	$= (1-4)^2 = (-3)^2 = 9$
2	4	$= (2-4)^2 = (-2)^2 = 4$
3	1	$= (3-4)^2 = (-1)^2 = 1$
$\boxed{4}$	$\boxed{0}$	$= (4-4)^2 = 0^2 = 0$
5	1	$= (5-4)^2 = 1^2 = 1$
6	4	$= (6-4)^2 = 2^2 = 4$
7	9	$= (7-4)^2 = 3^2 = 9$



Notice:

- to generate the "basic" y-values the x-values are all 4 more (+4) than our "basic" x-values
- $y=(x-4)^2$ has same shape as $y=x^2$, but its translated 4 units right. This is due to the "h" value of 4.

Vertex: (4,0)

Domain: $x \in \mathbb{R}$

A of S eqn: $x=4$

Range: $y \geq 0$

need to imagine basic count over 4 up 16 →

y-int: (0,16)

Opens: up

x-int(s): (4,0)

Max/[✓ Min]: $y=0$

c) Graph $y = (x+3)^2$ using the count method:

↳ switch sign h = −3

Vertex is moved 3 left so it's at (-3,0)

Basic count (a=1)

Vertex: (-3,0)

Domain: $x \in \mathbb{R}$

A of S eqn: $x=-3$

Range: $y \geq 0$

y-int: (0,9)

Opens: up

x-int(s): (-3,0)

Max/[✓ Min]: $y=0$

h value <u>Mental Switch:</u>

- Switch the sign of the constant in the brackets to find "h".

Practice

y = a(x – h)²

read from graph

$y = a(x-h)^2$

2. Graph $y = 2(x-2)^2$ using the count method

$a = 2$

↳ mental switch

$h = 2 \therefore v(2,0)$

basic up count ×2

over 1 up 1 ×2 → up 2

Over 2 up 4 ×2 → up 8

Vertex: (2,0)

Domain: $x \in \mathbb{R}$

A of S eqn: $x = 2$

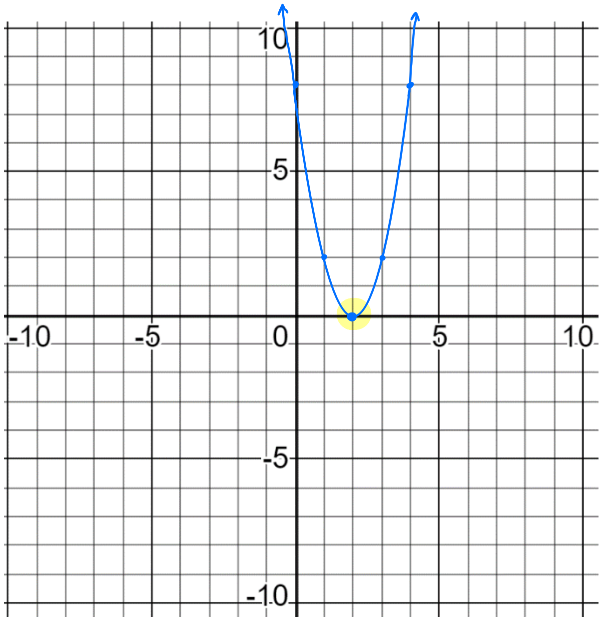
Range: $y \geq 0$

y-int: (0, 8)

Opens: up

x-int(s): (2,0)

Max/[✓ Min]: $y = 0$



$y = a(x-h)^2$

3. Graph $y = -\frac{1}{2}(x+4)^2$ using the count method

$a = -\frac{1}{2}$

$h = -4$

$\therefore V = (-4,0)$

basic count $\times(-\frac{1}{2})$

over 1 up 1 $\times(-\frac{1}{2})$ → down 0.5

Over 2 up 4 $\times(-\frac{1}{2})$ → down 2

over 3 up 9 $\times(-\frac{1}{2})$ → down 4.5

Over 4 up 16 $\times(-\frac{1}{2})$ → down 8

Vertex: (-4,0)

Domain: $x \in \mathbb{R}$

A of S eqn: $x = -4$

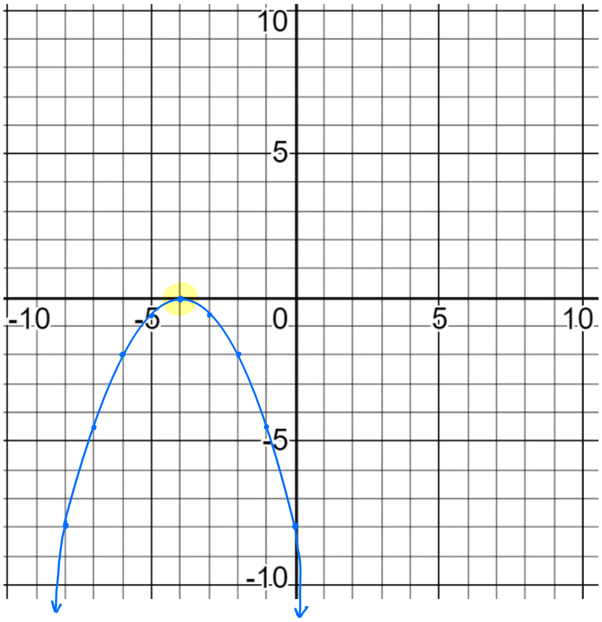
Range: $y \leq 0$

y-int: (0, -8)

Opens: down

x-int(s): (-4,0)

[✓ Max]/Min: $y = 0$



3.4 – Investigating Quadratic Functions in Vertex Form: $y = a(x - h)^2 + k$

We can now graph any quadratic function written in “vertex” form: $y = a(x - h)^2 + k$

1. Graph $y = 1(x-2)^2 - 4$

$a=1$ $h=2$ $k=-4$

vertex is 2 right and down 4

(2, −4)

basic count

over 1 up 1

over 2 up 4

over 3 up 9

(h, k)

Vertex: (2, 4)

Domain: $x \in \mathbb{R}$

A of S eqn: $x = 2$

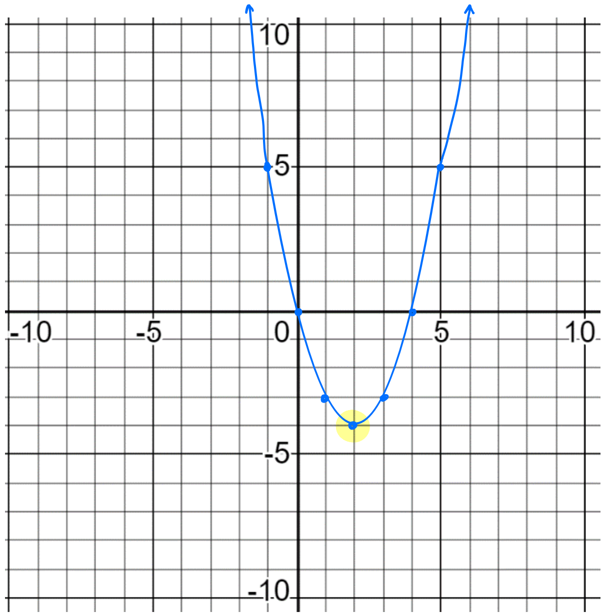
Range: $y \geq -4$

y-int: (0, 0)

Opens: up

x-int(s): (0,0) (4,0)

Max/[✓ Min]: $y = -4$



The Vertex

How does the vertex relate to the equation?

The vertex is always at (h, k) for

$y = a(x-h)^2 + k$ (remember sign switch for “h”)

2. Graph and find key info for

$y = a(x-h)^2 + k$

$f(x) = 2(x+3)^2 - 8$

$a=2$ $h=-3$ $k=-8$ ∴ v(−3, −8)

over 1 $1 \times 2 \rightarrow$ up 2

over 2 $4 \times 2 \rightarrow$ up 8

over 3 $9 \times 2 \rightarrow$ up 18

Vertex: (−3, −8)

Domain: $x \in \mathbb{R}$

A of S eqn: $x = -3$

Range: $y \geq -8$

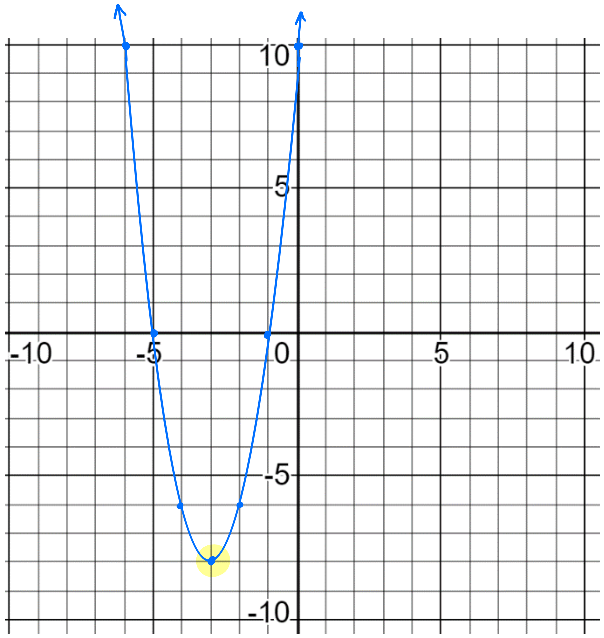
y-int: (0, 10)

Opens: up

x-int(s):

(−5,0)(−1,0)

Max/[✓ Min]: $y = -8$



Note: *depending on the book you are using, you might see this called "graphing form", "standard form" or "general form." We use "vertex form" because this equation helps us easily identify the vertex!*

Vertex Form: $\boldsymbol{y = a(x - h)^2 + k}$

multiply basic "up" count by a

vertex (h, k)
a of s x = h
max y = k

Notes:

a
- alters up/ down count
- if neg, parabola opens down (vertical reflection)
- if a > 1, vertical expansion
- if a < 1, vertical compression

h
- represents horizontal translation
- remember sign switch

k
- represents vertical translation (no sign switch)

3. Graph and find key info for

rewrite → $y = a(x-h)^2 + k$

$y - 3 = -\frac{1}{3}(x-3)^2 \rightarrow y = -\frac{1}{3}(x-3) + 3$

+3 +3

$a = -\frac{1}{3}$ $h = 3$ $k = 3$

$\therefore v(3,3)$

over 1 up 1 x $-\frac{1}{3}$ → down $0.\overline{3}$

over 2 up 4 x $-\frac{1}{3}$ → down $1.\overline{3}$

over 3 up 9 x $-\frac{1}{3}$ → down 3

Vertex: (3, 3)

Domain: $x \in \mathbb{R}$

A of S eqn: x = 3

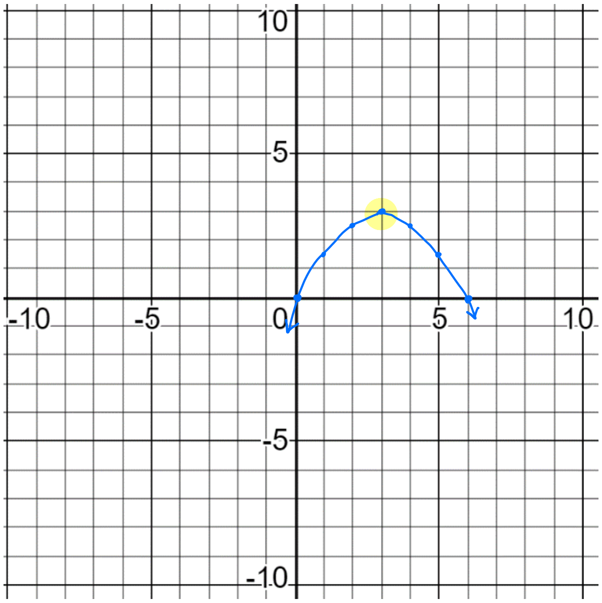
Range: $y \leq 3$

y-int: (0, 0)

Opens: down

x-int(s): (0, 0) (6, 0)

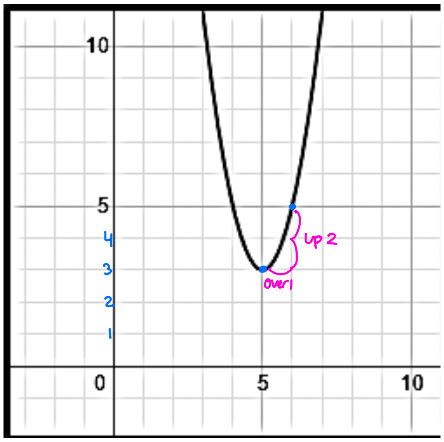
[✓ Max]/Min: y = 3



3.5 – Finding the Equation of a Parabola

finding the equation of a parabola from a graph

Example 1 – Determine the equation of the following parabola:



$V\ (\underset{h}{5}, \underset{k}{3})$

Over 1 up 2 (instead of "basic" over 1 up 1 ....)
must be double tall $a = 2$

$a = 2 \quad h = 5 \quad k = 3$

$y = a(x-h)^2 + k$

$y = 2(x-5)^2 + 3$

Steps:

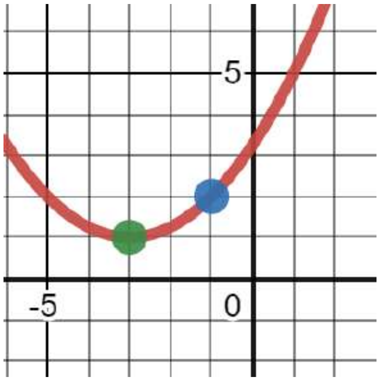
1) Identify the vertex, which will give you "h" and "k". Remember the "h" sign switch!

2) Find another point on the graph that is easy to read, and count how far "over" it is from the vertex. *What would you expect the "up" to be in the basic count?*

3) Compare the graph's "up" count to the basic count. *What "a" value would you need to multiply by to turn the basic "up" into the graph's "up"?*

4) Write the formula $y = a(x-h)^2 + k$ with the "a", "h" and "k" values now filled in.

Example 2 – Determine the equation of the following parabola:



$V\ (-3, 1)$ Point $(-1, 2)$

1) $V\ (\underset{h}{-3}, \underset{k}{1})$

2) easy to see $(-1,2)$, which is "over 2 up 1" from vertex
(think: basic count over 2 up 4 $\times$??? → up 1)

3) need to multiply 4 by $\frac{1}{4}$ to get 1

4) $a = \frac{1}{4} \quad h = -3 \quad k = 1$

sign switch

$y = \frac{1}{4}(x+3)^2 + 1$

finding the equation of a parabola from the vertex and another point

Example 3 – A parabola with vertex $(\overset{h}{1}, \overset{k}{-2})$ passes through the point $(\overset{x}{4}, \overset{y}{1})$. Find the equation.

↳ think : sub in!

$y = a(x-h)^2 + k$

$y = a(x-1)^2 - 2$

$1 = a(4-1)^2 - 2$ remember

$1 = a(3)^2 - 2$ BEDMAS

$1 = 9a - 2$

$+2 \qquad +2$

$\frac{3}{9} = \frac{9a}{9}$

$a = \frac{3 \div 3}{9 \div 3} = \frac{1}{3}$

$y = \frac{1}{3}(x-1)^2 - 2$

Steps:

1) Write the formula $y = a(x-h)^2 + k$
2) Fill in what you know (usually the vertex)
3) Sub in a point and solve for what's missing (usually "a")
4) Rewrite the equation (now that you know a, h, k)

x- and y-intercepts

Example 4 – Find the equation of a quadratic function whose graph has vertex $(\overset{h}{-4}, \overset{k}{8})$ and an *x*-intercept of – 6. → a point

Hint: x-intercepts always have coordinates of… (?, 0) y is always 0

and y-intercepts always have coordinates of… (0, ?) x is always 0

$\therefore$ x-int of -6 → $(\overset{}{\underset{x}{-6}}, \underset{y}{0})$

1) $y = a(x-h)^2 + k$

2) $y = a(x-(-4))^2 + 8$ note: $(x-(-4))^2 \rightarrow (x+4)^2$ there is the sign switch

$y = a(x+4)^2 + 8$

3) $0 = a(-6+4)^2 + 8$

$0 = a(-2)^2 + 8$

$0 = 4a + 8$

$-8 \qquad -8$

$\frac{-8}{4} = \frac{4a}{4}$

$-2 = a$

4) $y = -2(x+4)^2 + 8$

4.1 – Completing the Square when *a = 1*

Vertex Form

vs

General Form

When a quadratic function is in VERTEX (or Graphing) FORM $y = a(x-h)^2 + k$
we can easily find its vertex and graph the parabola.

However, quadratic functions are often presented in the form $y = ax^2 + bx + c$
which is often called GENERAL FORM.

General form can be useful, but if we want to graph the parabola or find its vertex, we will need to convert from General Form to Vertex Form, using a process called Completing the Square.

- allows us to graph, find vertex, find x-intercepts

Completing the Square

Example 1 – Rewrite $y = 14 + 10x + x^2$ in vertex form by completing the square. Then sketch the graph.

STEPS:

1) Rearrange so squared term is first and *x* term is second.
2) Find the *a, b, c* values
3) Take half the b-value (you'll need this later), then square it.
4) Add and subtract the result to your quadratic function after the *x* term.
5) Make sure the new term you <u>added</u> is the third term. ⊕ first then ⊖
6) Factor the trinomial and add the two last terms.

Shortcut for factoring the trinomial:

The "saved" value goes in your $(x + \_)^2$ bracket

$$\overset{ax^2}{} \quad \overset{+bx}{} \quad \overset{+c}{}$$

① $y = x^2 + 10x + 14$

② $a = 1 \quad b = 10 \quad c = 14$

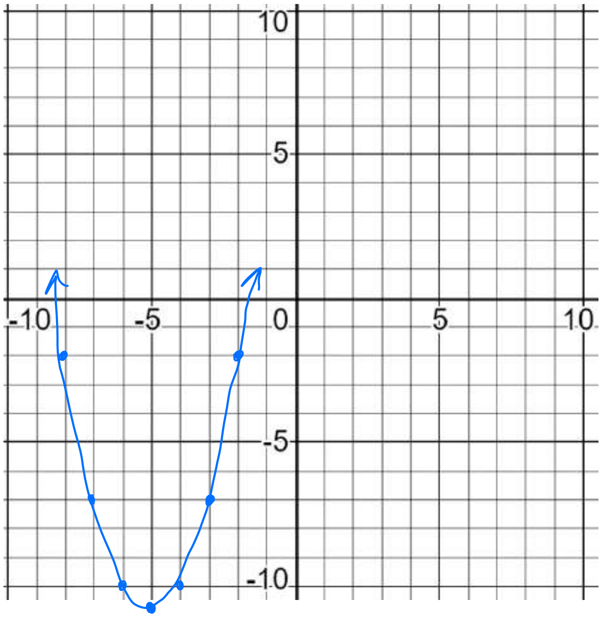
③ $b = 10 \quad \frac{b}{2} = \frac{10}{2} = \underset{\text{save for later}}{\underline{\underline{5}}} \quad (5)^2 = \underset{\text{use}}{\underline{25}}$

④ $y = x^2 + 10x + 25 \underline{- 25 + 14}$ ⑤ ✓

⑥ $y = (x + 5)^2 - 11$

$a = 1 \quad h = -5 \quad k = -11$

$V = (-5, -11) \quad a = 1 \rightarrow$ basic count



Example 2 – Change $y = x^2 - 4x - 1$ into Vertex Form, then state the vertex.

$$y = x^2 - 4x - 1$$

$$a=1 \quad b=-4 \quad c=-1$$

$$b \rightarrow \frac{-4}{2} = \underset{\text{save}}{\underbrace{-2}} \qquad (-2)^2 = \underset{\text{use}}{\underbrace{4}}$$

$$y = \underbrace{x^2 - 4x + 4} - 4 - 1$$

$$\boxed{\begin{array}{c} y = (x-2)^2 - 5 \\ V(2, -5) \end{array}}$$

Example 3 – Change $y = x^2 + 5x + 2$ into Vertex Form, then state the vertex. (Use fractions, not decimals.)

$$y = x^2 + 5x + 2$$

$$a=1 \quad b=5 \quad c=2$$

$$b \rightarrow \underset{\text{save}}{\underbrace{\frac{5}{2}}} \qquad \left(\frac{5}{2}\right)^2 = \underset{\text{use}}{\underbrace{\frac{25}{4}}}$$

$$y = \underbrace{x^2 + 5x + \frac{25}{4}} - \frac{25}{4} + 2 \quad \rightarrow \quad \frac{-25}{4} + \frac{2^{\times 4}}{1_{\times 4}} \rightarrow \frac{-25}{4} + \frac{8}{4} = \frac{-17}{4}$$

$$\boxed{\begin{array}{c} y = \left(x + \frac{5}{2}\right)^2 - \frac{17}{4} \\ V\left(\frac{-5}{2}, \frac{-17}{4}\right) \end{array}}$$

4.2 – Completing the Square when *a* ≠ 1

When a ≠ 1

When the *a* value is different from 1, there are a few more steps to Complete the Square and convert from General Form ($y = ax^2 + bx + c$) to Vertex Form ($y = a(x - h)^2 + k$).

Example 1 – Change $y = -2x^2 + 4x + 5$ into Vertex form and then state the vertex.

STEPS:

(Re-order first if needed)

1) Group the first two terms together.
2) Factor the *a* value out.
3) Find the new *b* value. Take half and square it.
4) Add and subtract the result IN THE BRACKETS.
5) Get the <u>subtracted</u> result <u>out</u> of the brackets by multiplying to the coefficient in front of the brackets.
6) Factor the trinomial and add the two outside terms.

① $y=\underline{-2x^2+4x}\ +5$

② $y=-2(x^2\ \underset{\substack{\uparrow\\ \text{new } b}}{\underline{-2}}x)+5$

③ $\frac{b}{2}=\frac{-2}{2}=\underset{\text{save}}{-1}\qquad(-1)^2=\underset{\text{use}}{1}$

④ $y=\overset{\times}{-2}(x^2-2x+1\overset{+2}{-1})+5$

⑤ $y=-2(x^2-2x+1)\ \underline{+2\ +5}$

⑥ $y=-2(x-1)^2+7$

$V=(1,7)$

Example 2 – Change $y = \underline{3x^2 - 12x} + 11$ into Vertex Form and state the vertex.

$\frac{b}{2}=\frac{-4}{2}=\underset{\text{save}}{-2}\qquad(-2)^2=\underset{\text{use}}{4}$

$y=\underline{3}(x^2-4x+4\ \overset{\times 3\ \rightarrow\ -12}{-4})+11$

$$\boxed{\begin{gathered}y=3(x-2)^2-1\\ v(2,-1)\end{gathered}}$$

Example 3 – Change $y = 5x - 3x^2 + 1$ into Vertex Form using exact values and state the vertex.

Reorder!

$$y = \underbrace{-3x^2 + 5x} + 1 \longrightarrow \frac{-25}{{}_{12}\cancel{36}} \cdot \frac{\overset{-1}{\cancel{-3}}}{1} = \frac{25}{12}$$

$$y = -3\left(x^2 - \frac{5}{3}x + \frac{25}{36} \overset{\times -3}{-\frac{25}{36}}\right) + 1$$

$$\frac{b}{2} = \frac{-5}{3} \times \frac{1}{2} = \underset{\text{save}}{\frac{-5}{6}} \qquad \left(-\frac{5}{6}\right)^2 = \underset{\text{use}}{\frac{25}{36}}$$

$$y = -3\left(x - \frac{5}{6}\right)^2 + \underbrace{\frac{25}{12} + 1} \longrightarrow \frac{25}{12} + \frac{1}{1} = \frac{25}{12} + \frac{12}{12} = \frac{37}{12}$$

$$\boxed{\begin{gathered} y = -3\left(x - \frac{5}{6}\right)^2 + \frac{37}{12} \\ V = \frac{5}{6}, \frac{37}{12} \end{gathered}}$$

4.3A – Intercepts and Projectile Applications of Quadratic Functions

Finding intercepts with algebra

y-intercepts

y-intercepts occur when the graph crosses the y-axis, which means the point is “over nothing” (x is zero). We can find the y-intercepts of any function by setting x=0 and evaluating for y.

Example 1 – Determine the y-intercept of the following parabolas:

a) $\overset{y}{\cancel{f(x)}} = (x+5)^2 - 10$

$y = (0+5)^2 - 10$

$y = 25 - 10$

$y = 15 \rightarrow (0,15)$

b) $\overset{y}{\cancel{g(x)}} = 3x^2 + 14x - 12$

$y = 3(0)^2 + 14(0) - 12$

$y = 0 + 0 - 12$

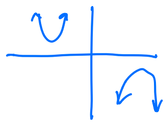
$y = -12 \rightarrow (0,-12)$

x-intercepts

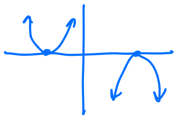
x-intercepts occur when the graph touches the x-axis, which means its height (y) is zero. We can find the x-intercepts of any function by setting y=0 and solving for x.

Before we try to solve, first consider how many x-intercepts are possible when graphing a parabola:

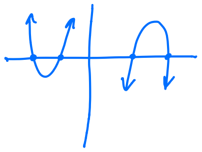
no x-intercepts



one x-intercept



two x-intercepts



Where do the 2 solutions come from?

Example 2 – Determine the x-intercepts of the following parabolas.

set $y=0$

a) $y = x^2 - 4$

$0 = x^2 - 4$

$+4 \quad +4$

$\pm\sqrt{4} = \pm\sqrt{x^2}$

$\pm 2 = x$

$(-2,0)\ (2,0)$

New!

undo $(\ )^2$

use $\pm\sqrt{\ }$

Check:

$x^2 = 4$

$2^2 = 4$

$(-2)^2 = 4$

b) $y = x^2 - 7$

$0 = x^2 - 7$

$+7 \quad +7$

$\pm\sqrt{7} = \pm\sqrt{x^2}$

$\pm\sqrt{7} = x$

$(-\sqrt{7},0)(\sqrt{7},0)$ exact values!

Notice that we need the function in VERTEX FORM in order to solve.

c) $y = (x-2)^2 - 9$

$0 = (x-2)^2 - 9$

$+9 \quad +9$

$\pm\sqrt{9} = \pm\sqrt{(x-2)^2}$

$\pm 3 = x - 2$

$+2 \quad +2$

$2 \pm 3 = x$

$2-3=-1 \qquad 2+3=5$

$(-1,0)\ (5,0)$

d) $y = 2(x+1)^2 - 3$

$0 = 2(x+1)^2 - 3$

$+3 \quad +3$

$\frac{3}{2} = \frac{2(x+1)^2}{2}$

$\pm\sqrt{\frac{3}{2}} = \pm\sqrt{(x+1)^2}$

$\pm\sqrt{\frac{3}{2}} = x + 1$

$-1 \quad -1$

$-1 \pm \sqrt{\frac{3}{2}} = x$

$\left(-1+\sqrt{\frac{3}{2}},0\right)\left(-1-\sqrt{\frac{3}{2}},0\right)$

What do the intercepts represent in this situation?

Example 3 – The path of a rocket fired over a lake is described by the function

$h(t) = -4.9t^2 + 49t + 1.5$ where *h(t)* is the height of the rocket, in metres, and *t* is time in seconds, since the rocket was fired.

a) What is the maximum height reached by the rocket? How many seconds after it was fired did the rocket reach this height? → vertex
b) How high was the rocket above the lake when it was fired? $t=0$
c) At what time does the rocket hit the ground? $h=0$
d) What domain and range are appropriate in this situation?
e) How high was the rocket after 7s? Was it on its way up or down?

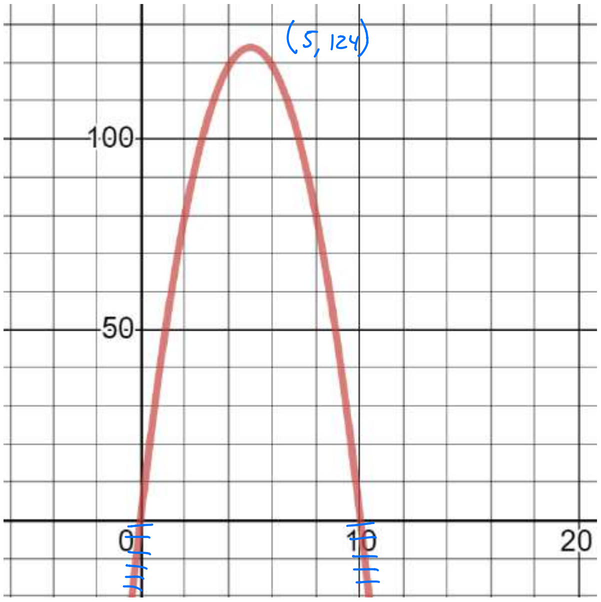
$$h(t) = -4.9\left(t^2 - 10t + 25 \overset{\times -4.9 \to 122.5}{-25}\right) + 1.5 \quad +$$

$$\frac{-10}{2} = \underset{\text{save}}{-5} \qquad (-5)^2 = \underset{\text{use}}{25}$$

$$h(t) = -4.9(t-5)^2 + 124$$

$$V = (\underset{t}{5}, \underset{h}{124})$$

a) The rocket reaches a max height of 124m 5 seconds after it was fired.



b) $h(0) = -4.9(0-5)^2 + 124$

$h(0) = -4.9(25) + 124$

$h(0) = 1.5\text{ m}$

The rocket was 1.5m above the ground when it was fired.

c) $\underset{-124}{0} = -4.9(x-5)^2 + \underset{-124}{124}$

$\frac{-124}{-4.9} = \frac{-4.9}{-4.9}(x-5)^2$

$\pm\sqrt{25.306} = \pm\sqrt{(x-5)^2}$

$\pm 5.03 = t - 5$

$t = 5 + 5.03 = 10.03\text{sec}$

$t = 5 - 5.03 = -0.03\text{ secs}$ X

↳ doesn't make sense

about 10 seconds after the rocket was fired it hits the ground.

d) domain → from when rocket is fired, until it hits the ground

D: $0 \leq t \leq 10.03$

range → height from ground to max height

R: $0 \leq h \leq 124$

e) $h = ?$ $t = 7$

$h(7) = -4.9(7-5)^2 + 124$

$h(7) = 104.4\text{m}$ → after vertex at 5 sec
→ so on its way down

The rocket was 104.4m after 7secs and on its way down.

4.3B – Maximizing Area Applications of Quadratic Functions

The Vertex helps us find the MAX or MIN in a real-life scenario.

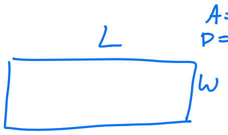
When we model a situation and it generates a quadratic function, we can find the MAXIMUM (or MINIMUM) value for the function by finding the vertex.

Example 1 – At a concert, organizers are roping off a rectangular area for sound equipment. There is 160m of fencing available to create the perimeter. What dimensions will give the maximum area, and what is the maximum area?

Steps:
1) Write an equation for perimeter, and write an equation for area for a rectangle.
2) Use the two equations to create a quadratic function in general form.
3) Complete the square to change the quadratic function into standard form.
4) Identify the maximum area, and then the dimensions for the maximum area.

let $x =$ length and $y =$ width

$A = L \times W$
$P = L + W + L + W$



① $160 = 2x + 2y$
$A = xy$

② $160 = 2x + 2y$

$\frac{160 - 2x}{2} = \frac{2y}{2}$

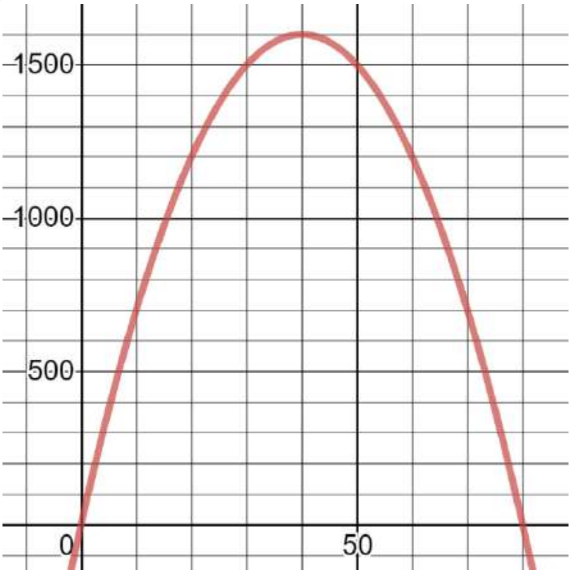
$80 - x = y$

$A = xy$

$A = x(80 - x)$

$A = 80x - x^2$

$A = -x^2 + 80x$



③ $A = -x^2 + 80x$

$x \cdot -1 \rightarrow 1600$

$A = -(x^2 - 80x + 1600 - 1600)$

$-\frac{80}{2} = -40$

$(-40)^2 = 1600$

$A = -(x - 40)^2 + 1600$

$V(\underset{x}{40}, \underset{A}{1600}) \rightarrow \text{max}$

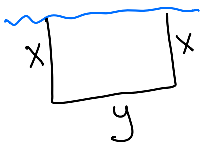
④ max area is $1600\,m^2$ when length is $40m$

(y): width is: $2x + 2y = 160$
$2(40) + 2y = 160$
$80 + 2y = 160$
$2y = 80$
$y = 40\,m$

or

$A = xy$
$\frac{1600}{40} = \frac{40y}{40}$
$40 = y$

The dimensions are 40m by 40m



$P = 2x + y$

$800 = 2x + y$

Example 2 – A rancher has 800m of fencing to enclose a rectangular cattle pen along a river bank. There is no fencing needed along the river bank. Find the dimensions that would enclose the largest area.

$$800 = 2x + y \rightarrow y = \underbrace{800 - 2x}$$

$$A = x\underline{\underline{y}}$$

$$A = x(800 - 2x)$$

$$A = 800x - 2x^2$$

$$A = -2x^2 + 800x$$

$$A = -2(x^2 - 400x + 40\,000 - \overset{\times -2 \rightarrow 80\,000}{40\,000})$$

$$-\frac{400}{2} = -200 \qquad (-200)^2 = 40\,000$$

$$A = -2(x - 200)^2 + 80\,000$$

$$V = (\underset{x}{200}, \underset{\max A}{80\,000})$$

$$y = 800 - 2x$$

$$y = 800 - 2(200)$$

$$y = 400\text{m}$$

The dimensions that give the max area of 80 000m² are 200m by 400m.

4.3C – Financial Applications of Quadratic Functions

The Vertex helps us find the MAX or MIN in a real-life scenario. With financial applications, we may want to maximize profit or minimize cost.

When we model a situation and it generates a quadratic function, we can find the MAXIMUM (or MINIMUM) value for the function by finding the vertex.

Revenue = number × price

Example 1 – A sporting goods store sells basketball shorts for $8. At this price their weekly sales are approximately 100 items. Research says that for every $2 increase in price, the manager can expect the store to sell five fewer pairs of shorts. Determine the maximum revenue the manager can expect based on these estimates. What selling price will give that maximum revenue, and how many shorts will be sold? let x = # of $2 increases in price

Current	new
number = 100	number = 100 − 5x
price = 8	price = 8 + 2x

$R = n \times p$

max → $R = (100-5x)(8+2x)$

$R = 800 + \underline{\underline{200x}} - \underline{\underline{40x}} - 10x^2$

$R = -10x^2 + 160x + 800$

$R = -10(x^2 - 16x + 64 \overset{\times -10 \to 640}{-64}) + 800$ $\frac{-16}{2} = -8$ $(-8)^2 = 64$

$R = -10(x-8)^2 + 1440$

$v(\underset{x}{8}, \underset{\max R}{1440})$

x = 8 means 8 price jumps

number = 100 − 5x
= 100 − 5(8)
= 100 − 40
= 60 shorts

price = 8 + 2x
= 8 + 2(8)
= 8 + 16
= $24

To get a max revenue of $1440, you should charge $24 and expect to sell 60 pairs of shorts.