PC11 Name____________________________

Chapter 5 Notes

Quadratic Equations

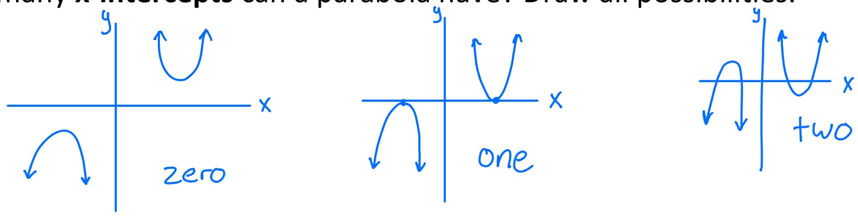
Date	Topic/Lesson	Assignment
*Answers for many radical questions are in a different format in the workbook. Please refer to your “Ch 5 Radical Answers” page for certain questions.		
	1.1 - Intro to Quadratic Equations and Factoring Review	After Notes ex 5 – p.10: 6, 7, 8ace After Notes ex 8 – p.13: 11
	1.2 - Factoring Review, continued	After Notes – p.19: 2a-h, 4a-d, 7left
	5.1 - Solving Quadratic Equations by Factoring	After Notes – p.188: 4, 5 (by factoring), 6a-k (skip b, d), 7a-d
	5.2A - Solving Quadratic Equations by Square Root Principle and by Completing the Square (a = 1)	After Notes ex 2 – p.195: 2left (see answer page for 2c,i), After Notes ex 4 – p.197: 6a-g
	5.2B – Solving Quadratic Equations by Completing the Square (a ≠ 1	After Notes – p.198: 7 (see answer page for 7a,f,h,i)
	5.3 - The Quadratic Formula	After Notes ex 4 – p.202: 2a-e After Notes ex 5 – p.203: 3a-f, 4a-g (see answer page for 3abcef and 4adef)
	5.5 - Applications of Quadratic Equations	After Notes ex 1 – p.216: 2 After Notes ex 2 – p.216: 3 After Notes ex 4 – p.217: 8-9 (for 9: V=LxWxH), 15-16
	Practice Test	Chapter 5 Practice Test
	Review	p.219: 1a-f, 3a-d, 5a-d, 6a-d,g,h, 8,12,13 (13: Profit = Revenue – Cost) (see answer page for 6abd)
	Unit Test	**Chapter 5 Unit Test**

1.1 – Introduction to Quadratic Equations and Factoring Review

A function of *degree 2* (meaning the highest exponent on the variable is 2) is called a **Quadratic Function**.

When a quadratic function is graphed, a **parabola** results.

How many **x-intercepts** can a parabola have? Draw all possibilities:



What is the *y* value at an *x*-intercept? the height is zero, so $y=0$

Therefore, to find the *x*-intercepts of a parabola, we can set ***y* = 0 (or *f(x)* = 0)** and solve the resulting **quadratic equation**.

When ***y*** is set to 0, we call the question a **quadratic equation** instead of a quadratic function.

Quadratic function: $f(x) = x^2 - x - 6$ vs Quadratic equation: $x^2 - x - 6 = 0$

f(x) or y in it — x's only, incl x^2

Terminology

The ***x-intercepts*** of the parabola are the **zeros** of the quadratic *function.* They are also called the **solutions** or **roots** of the quadratic *equation*.

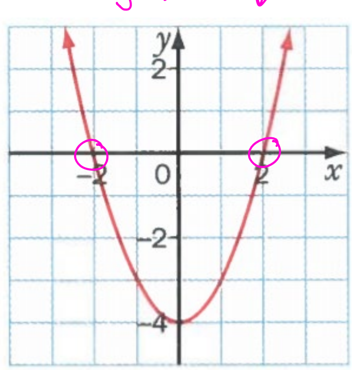
Finding x-intercepts by graphing

One method to find the **zeros** of a **quadratic function** is to graph it and visually determine the ***x-intercepts*** of the parabola.

Example 1 – Solve the **quadratic equation** $x^2 - 4 = 0$ by graphing the **quadratic function** $f(x) = x^2 - 4$ and determining the **x-intercepts** of the parabola:

X-intercepts at $(-2,0)$ and $(2,0)$
so solutions to equation $x^2-4=0$
are $x=-2$, $x=2$

$y=x^2-4$



What are the limitations of graphing?
Hard to answer when x-intercepts are at decimals / large values or if vertex is hard to plot, etc.

You can often find the **roots** of a **quadratic equation** by factoring when in general form $ax^2 + bx + c = 0$. Remember, the **roots** or **solutions** of the quadratic **equation** correspond to the **zeros** of the quadratic **function**, and the **x-intercepts** of the **parabola**. We will review factoring before we try solving equations using factoring.

Factoring Review

Multiplying two binomials

Example 1 – Expand and Simplify: $(x-1)(x-7)$ use FOIL

$x^2 - 7x - x + 7 = x^2 - 8x + 7$

Remember: expanding and factoring are opposite operations….they UNDO each other!

Steps for Factoring a Trinomial in the form $ax^2 + bx + c$, where a = 1

Step 1: If needed, re-order the terms in descending powers of the variable (biggest to smallest)

Step 2: With any factoring question, first check to see if you can factor out a GCF from ALL terms!

Step 3: Find two numbers that multiply to equal the **c** term and add to equal the **b** term (multiply to the outside, add to the middle)

Step 4: Factor into two binomials using the numbers from step 3, with the variable from the question placed first in each bracket

Factoring a trinomial in the form $x^2 + bx + c$

Example 2 – Factor the trinomial: $x^2 - 8x + 7$ …we should end up with $(x-1)(x-7)$!

$(x \quad)(x \quad)$

$(x-7)(x-1)$

need #s that (1, 7)

$-7 \times -1 = 7$

$-7 + -1 = -8$

Note: order doesn't matter

Example 3 – Factor: $z^2 - 2z - 8$

$(z \quad)(z \quad)$

$(z-4)(z+2)$

$-4 \times 2 = -8$

$-4 + 2 = -2$

(1,8) (2,4)

Example 4 – Factor: $-30 + 7m + m^2$

Reorder

$m^2 + 7m - 30$

$(m \quad)(m \quad)$

$(m+10)(m-3)$

$10 \times -3 = -30$

$10 + -3 = 7$

(1,30)(2,15)(3,10) (5,6)

$(1,12)(2,6)(3,4)$

Example 5 – Factor: $-5h^2 - 20h + 60$

$\frac{-5h^2}{-5} - \frac{20h}{-5} + \frac{60}{-5}$

$-5(h^2 + 4h - 12)$

$-5(h \quad)(h \quad)$

$-5(h+6)(h-2)$

$\underline{6} \times \underline{-2} = -12$

$\underline{6} + \underline{-2} = 4$

Always check to see if there is a GCF you can factor out first! IF there is a negative number in front of the x^2, factor out the negative as well.

Difference of Squares $\quad \boldsymbol{a^2 - b^2 = (a+b)(a-b)}$

Example 6 – Factor: $x^2 - 9$

$\sqrt{x^2} = x$

$\sqrt{9} = 3$

$(x+3)(x-3)$

Example 7 – Factor: $x^4 - 16$

note: can only factor if negative in between not positive

$\sqrt{x^4} = x^2$

$\sqrt{16} = 4$

$(x^2+4)(\underline{x^2-4})$

Another difference of squares, keep factoring if you can

$\sqrt{x^2} = x$

$\sqrt{4} = 2$

$(x^2+4)(x+2)(x-2)$

Example 8 – Factor: $50x^2 - 2y^2$

difference but 50 + 2 are not perfect squares ..... check for GCF!

$2(25x^2 - y^2)$

$\sqrt{25x^2} = 5x$

$\sqrt{y^2} = y$

$2(5x+y)(5x-y)$

Perfect square trinomials

$a^2 + 2ab + b^2 = (a+b)^2$

$a^2 - 2ab + b^2 = (a-b)^2$

eg: $36x^2 + 12x + 1$

$(6x+1)^2$

When $a \neq 1$in a trinomial of the form $\boldsymbol{ax^2 + bx + c}$, and it can't be factored out, then another process is needed.

One process you can use is called **Decomposition**, and uses factoring by grouping.

<u>Steps for factoring $\boldsymbol{ax^2 + bx + c}$, $\boldsymbol{a \neq 1}$ **by Decomposition**</u>

Step 1: If needed, re-order the terms in **descending** powers of the variable
Step 2: As with any factoring question, check to see if you can <u>factor out a GCF</u>
Step 3: Find two numbers that multiply to equal *ac* and add to equal *b* *(multiply to the product* $(\times)$ *of the "outsides" first* $\times$ *last, and add to the middle!)*
Step 4: Rewrite the expression but split or *decompose* the middle (*b*) term, using the two numbers from step 3
Step 5: Now the expression has FOUR terms, so we can **<u>factor by grouping</u>** the first two terms and the last two terms
Step 6: When factoring by grouping, the two resulting <u>binomials need to be identical</u>! These matching binomials are now the COMMON FACTOR, and can be factored out…and what is left become the components of the second bracket.

Another process you can use is called the "**ac" method**

<u>Steps for factoring $\boldsymbol{ax^2 + bx + c}$, $\boldsymbol{a \neq 1}$ **by "ac" method**</u>

Step 1: If needed, re-order the terms in **descending** powers of the variable
Step 2: Check to see if you can <u>factor out a GCF</u>. This step is **MANDATORY.**
Step 3: Find two numbers that multiply to equal *ac* and add to equal *b* *(multiply to the product* $(\times)$ *of the "outsides" first* $\times$ *last, and add to the middle!)*
Step 4: Write the expression as two factors using the numbers you found in step 3 and each with a denominator of "a"
Step 5: Now simplify each fraction in the brackets
Step 6: If a fraction remains, the denominator becomes the coefficient of the x term in that binomial.

<u>AC method</u>

Example 1 – Factor: $4x^2 + 11x + 6$

$x^2 + 11x + 24$

(1,24)
(2,12)
(3,8)
(4,6)

$\underline{8} \times \underline{3} = 24$

$\underline{8} + \underline{3} = 11$

$\left(x + \frac{8}{4}\right)\left(x + \frac{3}{4}\right)$

$(x+2)(4x+3)$

<u>Decomp method</u>

$4x^2 + 11x + 6$

$\underbrace{4x^2 + 8x} + \underbrace{3x + 6}$

$4x(x+2) + 3(x+2)$

$(x+2)(4x+3)$

Example 2 – Factor: $-7m - 10 + 6m^2$

Reorder $6m^2 - 7m - 10$

$m^2 - 7m - 60$

$\underline{-12} \times \underline{5} = -60$

$\underline{-12} + \underline{5} = -7$

$(m - \frac{12}{6})(m + \frac{5}{6})$

$(m-2)(6m+5)$

Decomp

$6m^2 - 7m - 10$

$6m^2 - 12m + 5m - 10$

$6m(m-2) + 5(m-2)$

$(m-2)(6m+5)$

Example 3 – Factor: $8p^2 - 18p - 5$

$p^2 - 18p - 40$

$\underline{-20} \times \underline{2} = -40$

$\underline{-20} + \underline{2} = -18$

$(p - \frac{20 \div 4}{8 \div 4})(p - \frac{2 \div 2}{8 \div 2})$

$(p - \frac{5}{2})(p - \frac{1}{4})$

$(2p-5)(4p-1)$

Decomp

$8p^2 - 18p - 5$

$8p^2 + 2p - 20p - 5$

$2p(4p+1) - 5(4p+1)$

$(4p+1)(2p-5)$

Example 4 – Factor: $24x^2 - 10xy - 4y^2$ → put y's at end of brackets

GCF $2(12x^2 - 5xy - 2y^2)$

$2(x^2 - 5xy - 24y^2)$

$\underline{-8} \times \underline{3} = -24$

$\underline{-8} + \underline{3} = -5$

$2(x - \frac{8}{12}y)(x + \frac{3}{12}y)$

$2(x - \frac{2}{3}y)(x + \frac{1}{4}y)$

$2(3x-2y)(4x+y)$

Decomp

$2(12x^2 - 5xy - 2y^2)$

$2(12x^2 - 8xy + 3xy - 2y^2)$

$2[4x(3x-2y) + y(3x-2y)]$

$2(3x-2y)(4x+y)$

Example 5 – Factor: $-16x^3 + 20x^2 + 24x$

GCF $-4x(4x^2 - 5x - 6)$

remember first # must be pos so take out neg from highest power → $-4x(x^2 - 5x - 24)$

$\underline{-8} \times \underline{3} = -24$

$\underline{-8} + \underline{3} = -5$

$-4x(x - \frac{8}{4})(x + \frac{3}{4})$

$-4x(x-2)(4x+3)$

Decomp

$-4x(4x^2 - 5x - 6)$

$-4x(4x^2 + 3x - 8x - 6)$

$-4x[x(4x+3) - 2(4x+3)]$

$-4x(4x+3)(x-2)$

5.1 – Solving Quadratic Equations by Factoring

Recall that you can often find the **roots** of a **quadratic equation** by factoring when in general form $ax^2 + bx + c = 0$. Remember, the **roots** or **solutions** of the quadratic **equation** correspond to the **zeros** of the quadratic **function**, and the **x-intercepts** of the **parabola**. There may be 0, 1, or 2 solutions to a quadratic equation.

Solving quadratic equations by factoring

Steps:

highest to lowest power

1) Get everything to one side of the equation so that only zero is on the other.
2) Identify *a*, *b*, and *c* values, and **factor** accordingly.
3) The roots are the *x*-values that will make the product of the binomials zero. If either of the binomials equal zero, then the product of the binomials will equal zero (this is called the **Zero Factor Property**). Therefore, identify the *x* values that make each binomial equal to zero.

if *a* = 1

Example 1 – Solve and check $x^2 + 3x = -2$

$+2 \qquad +2$

$1 \times 2 = 2$

$1 + 2 = 3$

$x^2 + 3x + 2 = 0$

$a=1 \quad b=3 \quad c=2$

opposite # will zero

$(x+1)(x+2) = 0$

$x+1 = 0$, $-1 \quad -1$, $x = -1$

$x+2 = 0$, $-2 \quad -2$, $x = -2$

Check:

$x^2 + 3x \mid -2$

$(-1)^2 + 3(-1)$

$1 - 3$

-2 ✓

$x^2 + 3x \mid -2$

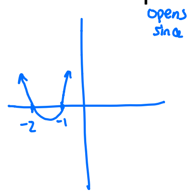
$(-2)^2 + 3(-2)$

$4 - 6$

-2 ✓

Sketch the Graph:

opens up since $a = +1$



Example 2 – a) Solve $x^2 - 8x - 40 = 8$

$-8 \qquad -8$

$-12 \times 4 = -48$

$-12 + 4 = -8$

$x^2 - 8x - 48 = 0$

$(x-12)(x+4) = 0$

$x = 12 \qquad x = -4$

b) Solve $\frac{2x^2}{2} + \frac{6x}{2} - \frac{108}{2} = \frac{0}{2}$

$a \neq 1$

since an equation do same action to left & right of =

simplified

$x^2 + 3x - 54 = 0$

$(x+9)(x-6) = 0$

$x = -9 \qquad x = 6$

c) Solve $\frac{1}{2}x^2 - x - 4 = 0$

$\times 2 \quad \times 2 \quad \times 2 \quad \times 2$

clear fractions

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4 \qquad x = -2$

if $a \neq 1$

When $a \neq 1$, factor by "decomposition" or "ac" method.

Example 3 – Solve $3x^2 - 5x + 2 = 0$

$x^2 - 5x + 6 = 0$

$\frac{-3}{} \ x \ \frac{-2}{} = 6$

$\frac{-3}{} + \frac{-2}{} = -5$

$(x - \frac{3}{3})(x - \frac{2}{3}) = 0$

$(x-1)(3x-2) = 0$

$x - 1 = 0$

$x = 1$

$3x - 2 = 0$

$+2 \quad +2$

$\frac{3x}{3} = \frac{2}{3}$

$x = \frac{2}{3}$

Trick for finding roots:

$(3x - 2) = 0$

$(x - \frac{2}{3}) = 0$

$x = +\frac{2}{3}$

bring the coefficient back and change the sign

If $c = 0$

Example 4 – Solve $3x^2 = -5x$

$+5x \quad +5x$

organized ✓

gcf X

$3x^2 + 5x = 0$

$(x)(3x+5) = 0$

$x = 0$

$x = \frac{-5}{3}$

*

difference of squares

Example 5 – Solve $b = 0?$

a) $x^2 - 25 = 0$

$\sqrt{x^2} = x$

$\sqrt{25} = 5$

$(x+5)(x-5)$

$x = -5 \quad x = +5$

or

$x = \pm 5$

b) $\frac{8p^2}{2} - \frac{18}{2} = \frac{0}{2}$

GCF = 2

$4p^2 - 9 = 0$

$\sqrt{4p^2} = 2p$

$\sqrt{9} = 3$

$(2p+3)(2p-3) = 0$

$p = \frac{-3}{2} \quad p = \frac{3}{2}$

$p = \pm\frac{3}{2}$

c) $49 - 4x^2 = 0$

reorder

GCF = −1

$\frac{-4x^2}{-1} + \frac{49}{-1} = \frac{0}{-1}$

$4x^2 - 49 = 0$

$\sqrt{4x^2} = 2x$

$\sqrt{49} = 7$

$(2x+7)(2x-7)$

$x = \pm\frac{7}{2}$

5.2A – Solving Quadratic Equations by Square Root Principle and Completing the Square

square root principle

When there is no *bx* term in a quadratic equation, first look to see if it is a *difference of squares* (is each term a perfect square, and is there a subtraction sign in between?). If it is not a difference of squares, it can be solved by the *square root principle.*

The general form of a quadratic equation that you could solve using the square root principle is $ax^2 + c = 0$. or $a(x-h)^2 + k = 0$

Example 1 – Solve $9y^2 - 22 = 0$
$ax^2 + c = 0$

1) Get everything that <u>isn't</u> squared to one side.
2) Square root both sides.
3) Consider both the positive and negative square roots.
4) Simplify any radical solutions as much as possible in exact form.

$9y^2 - 22 = 0$
$+22 \quad +22$

$\frac{9y^2}{9} = \frac{22}{9}$

$y^2 = \frac{22}{9}$

$\pm\sqrt{y^2} = \pm\sqrt{\frac{22}{9}}$

$y = \pm\sqrt{\frac{22}{9}} = \frac{\sqrt{22}}{\sqrt{9}}$ ← remove any perfect squares

$\boxed{y = \pm\frac{\sqrt{22}}{3}}$

no bx term ✓

Example 2 – Solve: a) $2x^2 - 11 = 87$
$+11 \quad +11$

$\frac{2x^2}{2} = \frac{98}{2}$

$\pm\sqrt{x^2} = \pm\sqrt{49}$

$\boxed{x = \pm 7}$

b) $\frac{50y^2}{50} = \frac{72}{50}$ ÷2 ÷2

$\pm\sqrt{y^2} = \pm\sqrt{\frac{36}{25}}$

$y = \pm\frac{\sqrt{36}}{\sqrt{25}}$

$\boxed{y = \pm\frac{6}{5}}$

c) $\pm\sqrt{(x+3)^2} = \pm\sqrt{16}$

$x + 3 = \pm 4$
$-3 \quad -3$

$x = -3 \pm 4$

$x = -3+4$, $\boxed{x = 1}$ $x = -3-4$, $\boxed{x = -7}$

d) $3x^2 - 8 = 0$
$+8 \quad +8$

$\frac{3x^2}{3} = \frac{8}{3}$

$\pm\sqrt{x^2} = \pm\sqrt{\frac{8}{3}}$

$\boxed{x = \pm\sqrt{\frac{8}{3}}}$ ← workbook answers look different since they further reduce radicals

or $x = \pm\frac{2\sqrt{6}}{3}$

e) $\pm\sqrt{(x-1)^2} = \pm\sqrt{12}$

$x - 1 = \pm\sqrt{12}$
$+1 \quad +1$

$\boxed{x = 1 \pm \sqrt{12}}$

or

$x = 1 \pm 2\sqrt{3}$

completing the square when a = 1

Sometimes factoring quadratic equations is not possible, as you cannot find the two numbers that multiply to *c* (or *ac*) and add to *b*. When this is the case, you can still solve the quadratic equation by a method called *completing the square.*

Example 3 – Solve $x^2 - 24 = -10x$ by completing the square. This example can be easily solved by factoring, but we will use it to introduce how to complete the square.

1) Get *c* to one side of the equation.
2) If the *a* value is 1, find the *b* value, half it, and square it.
3) Add the number from step 2 to BOTH sides of the equation.
4) Factor the trinomial on the left (we created a perfect square trinomial, so it will lead to 2 brackets that are exactly the same).
5) Solve using the square root property.

① $x^2 - 24 = -10x$

$\quad +10x \qquad +10x$

$x^2 + 10x - 24 = 0$

$\quad +24 \qquad +24$

② $x^2 + 10x = 24$ $\qquad b = 10 \qquad 10 \div 2 = 5 \qquad 5^2 = 25$

③ $x^2 + 10x + 25 = 24 + 25$

④ $(x + 5)^2 = 49$

⑤ $\pm\sqrt{(x+5)^2} = \pm\sqrt{49}$

$x + 5 = \pm 7$

$\quad -5 \qquad -5$

$x = -5 + 7$ or $-5 - 7$

$x = 2, -12$

Example 4 – Solve by completing the square. Express the solutions in exact form.

a) $w^2 - 4w - 11 = 0$

$\quad +11 \qquad +11 \qquad b = \frac{-4}{2} = -2$

$w^2 - 4w + 4 = 11 + 4 \qquad (-2)^2 = 4$

$w^2 - 4w + 4 = 15$

$\pm\sqrt{(w-2)^2} = \pm\sqrt{15}$

$w - 2 = \pm\sqrt{15}$

$\quad +2 \qquad +2$

$w = 2 \pm \sqrt{15}$

b) $x^2 + 5x + 7 = 0$

$\qquad -7 \qquad -7$

$x^2 + 5x = -7$

$\hookrightarrow \frac{5}{2} \to \left(\frac{5}{2}\right)^2 \to \frac{25}{4}$

$x^2 + 5x + \frac{25}{4} = -7 + \frac{25}{4}$

$\left(x + \frac{5}{2}\right)^2 = \frac{-28}{4} + \frac{25}{4}$

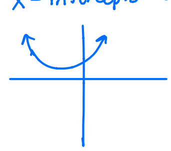
$\left(x + \frac{5}{2}\right)^2 = \frac{-3}{4}$

try to $\pm\sqrt{\ }$ both sides

↑ can't $\sqrt{\ }$ a negative

No Solution

(no x-intercepts on graph)



c) $m^2 - 5m + 3 = 0$

$\qquad -3 \qquad -3$

$m^2 - 5m = -3$

$\hookrightarrow -\frac{5}{2} \qquad \left(-\frac{5}{2}\right)^2 = \frac{25}{4}$

$m^2 - 5m + \frac{25}{4} = \frac{-3}{1}\frac{\times 4}{\times 4} + \frac{25}{4} = \frac{-12}{4} + \frac{25}{4}$

$\pm\sqrt{\left(m - \frac{5}{2}\right)^2} = \pm\sqrt{\frac{13}{4}}$

$m - \frac{5}{2} = \frac{\pm\sqrt{13}}{2} + \frac{5}{2}$

$\quad +\frac{5}{2}$

$m = \frac{5 \pm \sqrt{13}}{2}$

5.2B – Solving Quadratic Equations by Completing the Square $(a \neq 1)$

completing the square when $a \neq 1$

If $a \neq 1$, there are a few more considerations when *completing the square*.

Example 1 – Solve $2x^2 - 5x - 1 = 0$ by completing the square

1) Get c to one side of the equation.
2) Factor the a value out of the left side. ← or combine these steps and just ÷ all terms by "a"
3) Divide both sides by the a value to leave $x^2 + bx$ on the left side of the equation. THEN find the b value, half it, and square it.
4) Add the number from step 3 to BOTH sides of the equation.
5) Factor the resulting trinomial on the left.
6) Solve using the square root principle and answer in exact form.

$2x^2 - 5x - 1 = 0$

① $2x^2 - 5x = 1$

② $2(x^2 - \frac{5}{2}x) = 1$

③ $\frac{2}{2}(x^2 - \frac{5}{2}x) = \frac{1}{2}$

$-\frac{5}{2} \div 2 = -\frac{5}{2} \times \frac{1}{2} = -\frac{5}{4}$

$(-\frac{5}{4})^2 = \frac{25}{16}$

④ $x^2 - \frac{5}{2}x + \frac{25}{16} = \frac{1}{2}{}^{\times 8}_{\times 8} + \frac{25}{16}$

factor ↓ $= \frac{8}{16} + \frac{25}{16}$

⑤ $(x - \frac{5}{4})^2 = \frac{33}{16}$

⑥ $\pm\sqrt{(x - \frac{5}{4})^2} = \pm\sqrt{\frac{33}{16}}$

$x - \frac{5}{4} = \frac{\pm\sqrt{33}}{4}$

$+5/4$ $+5/4$

$x = \frac{5 \pm \sqrt{33}}{4}$

Example 2 – Solve by completing the square: $3x^2 - 2 = -4x$

Reorder: $\frac{3x^2}{3} + \frac{4x}{3} = \frac{2}{3}$

$x^2 + \frac{4}{3}x = \frac{2}{3}$

$\frac{4}{3} \div 2 = \frac{4}{3} \times \frac{1}{2} = \frac{4}{6}{}^{\div 2}_{\div 2} = \frac{2}{3}$

$(\frac{2}{3})^2 = \frac{4}{9}$

$x^2 + \frac{4}{3}x + \frac{4}{9} = \frac{2}{3}{}^{\times 3}_{\times 3} + \frac{4}{9} \rightarrow \frac{6}{9} + \frac{4}{9} = \frac{10}{9}$

$\pm\sqrt{(x + \frac{2}{3})^2} = \pm\sqrt{\frac{10}{9}}$

$x + \frac{2}{3} - \frac{2}{3} = \frac{\pm\sqrt{10}}{3} - \frac{2}{3}$

$x = \frac{-2 \pm \sqrt{10}}{3}$

An easy way to half a fraction is to double the denom-inator.

Example 3 – Solve by completing the square. Answer to the nearest hundredth.
2 decimals

$-2x^2 - 3x + 7 = 0$

$\frac{-2x^2}{-2} - \frac{3x}{-2} = \frac{-7}{-2}$

$x^2 + \frac{3}{2}x = \frac{7}{2}$

$\frac{3}{2} \div 2 = \frac{3}{2} \times \frac{1}{2} = \frac{3}{4}^2 = \frac{9}{16}$

$x^2 + \frac{3}{2}x + \frac{9}{16} = \frac{7^{\times 8}}{2_{\times 8}} + \frac{9}{16} \rightarrow \frac{56}{16} + \frac{9}{16}$

$\pm\sqrt{\left(x + \frac{3}{4}\right)^2} = \pm\sqrt{\frac{65}{16}}$

$x + \frac{3}{4} - \frac{3}{4} = \frac{\pm\sqrt{65}}{4} - \frac{3}{4}$

$x = \frac{-3 \pm \sqrt{65}}{4}$

$x = (-3 + \sqrt{65}) \div 4 \cong 1.27$
$x = (-3 - \sqrt{64}) \div 4 \cong -2.77$

$x \cong 1.27, -2.77$

Example 4 – Solve by completing the square: $3x^2 + 6x - 1 = 0$

$\frac{3x^2}{3} + \frac{6x}{3} = \frac{1}{3}$

$x^2 + 2x = \frac{1}{3}$

$2 \div 2 = 1 \quad 1^2 = 1$

$x^2 + 2x + 1 = \frac{1}{3} + \frac{1^{\times 3}}{1_{\times 3}} \rightarrow \frac{1}{3} + \frac{3}{3} = \frac{4}{3}$

$\pm\sqrt{(x+1)^2} = \pm\sqrt{\frac{4}{3}}$

$x + 1 = \pm\frac{2}{\sqrt{3}}$

$x = -1 \pm \frac{2}{\sqrt{3}}$

This format is ok we will learn how to simplify radicals in a future chapter

or

$x = \frac{-3 \pm 2\sqrt{3}}{3}$

5.3 – The Quadratic Formula

Quadratic equations can be solved by graphing, factoring (5.1), the square root property (5.2) and/or completing the square (5.2). Each of these methods have advantages and limitations.

quadratic formula

Any quadratic equation can be solved using something called the *quadratic formula*. If the quadratic equation is in general form ($ax^2 + bx + c = 0$), the quadratic formula is:

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

The derivation of the quadratic formula can be found in your workbook on p.85.

$ax^2 + bx + c = 0$

Example 1 – Solve $2x^2 - 5x + 2 = 0$ using the quadratic formula

$a = 2$
$b = -5$
$c = 2$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(2)}}{2(2)}$$

$$x = \frac{5 \pm \sqrt{25 - 16}}{4}$$

$$x = \frac{5 \pm \sqrt{9}}{4} \rightarrow \frac{5 \pm 3}{4}$$

$\frac{5+3}{4} = \frac{8}{4} = 2$

$\frac{5-3}{4} = \frac{2}{4} = \frac{1}{2}$

$x = 2, \frac{1}{2}$

Example 2 – Solve $3x^2 - x = -1$ using the quadratic formula

$3x^2 - x + 1 = 0$

$a = 3$
$b = -1$
$c = 1$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(1)}}{2(3)}$$

$$x = \frac{1 \pm \sqrt{1 - 12}}{6}$$

$$x = \frac{1 \pm \sqrt{-11}}{6}$$

can't square root a negative

$X =$ No Solution

discriminant

There could be 0, 1, or 2 resulting roots, depending on the **discriminant**, the expression under the square root ($b^2 - 4ac$). Solutions can be written in simplest radical form, or decimal form.

How would the discriminant determine the *nature of the roots* (the number of roots)?

If $b^2 - 4ac > 0$, there will be two different roots
positive (2 x-intercepts)

If $b^2 - 4ac = 0$, there is one (repeated) root
$\pm\sqrt{0}$ (1 x-intercept)
± 0 same thing

If $b^2 - 4ac < 0$, no solution, no (real) roots
negative (0 x-intercepts)

* you cannot square a negative number

how many solutions?

$a=3 \quad b=2 \quad c=-4$

Example 3 – Determine the nature of the roots, and then solve $3x^2 + 2x - 4 = 0$ using the quadratic formula

$b^2 - 4ac$

$(2)^2 - 4(3)(-4)$

$4 + 48$

$D = 52$

pos so 2 solutions

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ → D

$x = \frac{-2 \pm \sqrt{52}}{2(3)}$

$x = \frac{-2 \pm \sqrt{52}}{6}$

clear the fraction

Example 4 – Determine the nature of the roots for $\frac{1}{4}x^2 - 3x + 9 = 0$

$\times 4 \quad \times 4 \quad \times 4 \quad \times 4$

$x^2 - 12x + 36 = 0$

$a = 1 \quad b = -12 \quad c = 36$

$b^2 - 4ac$

$= (-12)^2 - 4(1)(36)$

$= 144 - 144$

$= 0$

$\therefore$ 1 solution

Example 5 – Solve using the quadratic formula. Leave answers in exact form.

a) $x(x-2) = 1$

$-1 \quad -1$

$x^2 - 2x - 1 = 0$

$a = 1$
$b = -2$
$c = -1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-1)}}{2(1)}$

$x = \frac{2 \pm \sqrt{4+4}}{2}$

$x = \frac{2 \pm \sqrt{8}}{2}$

b) $\frac{x^2}{2} - \frac{5x}{6} = \frac{-3}{2}$

$3(6) \quad (6) \quad 3(6)$

$3x^2 - 5x = -9$

$3x^2 - 5x + 9 = 0$

$a = 3$
$b = -5$
$c = 9$

$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(9)}}{2(3)}$

$x = \frac{5 \pm \sqrt{25 - 108}}{6}$

$x = \frac{5 \pm \sqrt{-83}}{6}$

No solutions

Example 1 – The sum of a number and twice its reciprocal is $\frac{9}{2}$. Find the number.

$+ \quad x \quad (2) \quad \frac{1}{x} \quad =$

clear fractions

$(2x)\,x + (2x)\,2\left(\frac{1}{x}\right) = \frac{9}{2}(2x)$

$2x^2 + 4 = 9x$

$-9x \qquad -9x$

$2x^2 - 9x + 4 = 0$

→ check to see if it factors?

$\underline{-8} \times \underline{-1} = 8$

$\underline{-8} + \underline{-1} = -9$

$x^2 - 9x + 8 = 0$

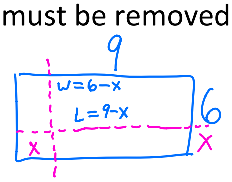
$\left(x - \frac{8}{2}\right)\left(x - \frac{1}{2}\right) = 0$

$(x-4)(2x-1) = 0$

$x = 4, \frac{1}{2}$

The numbers could be 4 or $\frac{1}{2}$

Example 2 – The length and width of a rectangular sheet of plywood is 6ft by 9ft. How much must be removed equally from the length and width to reduce the area to half its original size?



old area $= 6 \times 9 = 54$

new area $= \frac{1}{2}(54) = 27$

new Area ⇒ $(9-x)(6-x) = 27$

$54 - 9x - 6x + x^2 = 27$

$-27 \qquad -27$

$x^2 - 15x + 27 = 0$

$a=1 \quad b=-15 \quad c=27$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{15 \pm \sqrt{(-15)^2 - 4(1)(27)}}{2(1)}$$

$$x = \frac{15 \pm \sqrt{117}}{2}$$

$x = \frac{15 + \sqrt{117}}{2}$ $\qquad$ $x = \frac{15 - \sqrt{117}}{2}$

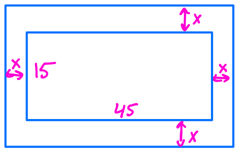
~~$x = 12.908$~~ To big $\qquad$ $x = 2.092$

Can't cut over 12 ft since board is only 9ft x 6ft

2.092 ft must be removed from the length and width.

New dimensions: 6.91 x 3.91

Example 3 – A 15cm by 45cm painting has a frame surrounding it. If the frame is the same width all around, and the total area of the frame is 325cm^2, how wide is the frame?



Small $A = 15 \times 45 = 675$

frame $A = 325$

Total $A = 675 + 325 = 1000$

Area $= L \times W$

$(45 + 2x)(15 + 2x) = 1000$

$675 + 90x + 30x + 4x^2 = 1000$

$-1000 \qquad -1000$

$4x^2 + 120x - 325 = 0$

$a = 4 \quad b = 120 \quad c = -325$

$x = \dfrac{-120 \pm \sqrt{120^2 - 4(4)(-325)}}{2(4)}$

$x = \dfrac{-120 \pm \sqrt{19600}}{8}$

$x = \dfrac{-120 \pm 140}{8}$

$x = \dfrac{-120 + 140}{8}$

$x = 2.5$

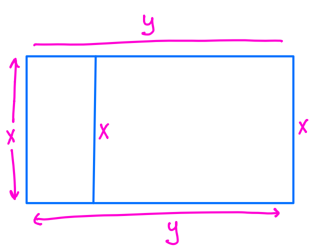
$x = \dfrac{-120 - 140}{8}$

$x \neq -32.5$

reject since neg

The width of the frame is 2.5 cm

Example 4 – A farmer uses 90m of fencing to enclose two adjacent rectangular pens. Find the dimensions that enclose a total area of 300m^2.



Fencing $= 3x + 2y$

$3x + 2y = 90\text{m}$

$-3x \qquad -3x$

isolate y

$\dfrac{2y}{2} = \dfrac{-3x}{2} + \dfrac{90}{2}$

$y = \dfrac{-3}{2}x + 45$

sub into equation

Area $= L \times W$

$xy = 300\text{m}^2$

$x\left(-\frac{3}{2}x + 45\right) = 300$

$(2)\ -\frac{3}{2}x^2 + (2)\ 45x = (2)\ 300$ → clear fractions

$-3x^2 + 90x = 600$

$-600 \qquad -600$

$\dfrac{-3x^2}{-3} + \dfrac{90x}{-3} \dfrac{-600}{-3} = \dfrac{0}{-3}$

$x^2 - 30x + 200 = 0$

$(x - 10)(x - 20)$

$x = 10, 20$

also find y

$y = \frac{-3}{2}x + 45$

$y = \frac{-3}{2}(10) + 45$ or $\frac{-3}{2}(20) + 45$

$y = 30$

$y = 15$

The dimensions are either 10m x 30m or 20m x 15m.