PC11 Name___Key___

Chapter 7 Notes

Inequalities

Date	Topic/Lesson	Assignment
	7.3A - Inequalities in 1 Variable Part 1	After Notes – 7.3A Worksheet AND p.270: 2all
	7.3B - Inequalities in 1 Variable Part 2	After Notes – 7.3B Worksheet AND p.271: 3a-e
	7.5 - Applications of Quadratic Inequalities	After Notes – 7.5 Worksheet AND p281: 13, 14
	Practice Test	Chapter 7 Practice Test
	Review	p.283: 4, 5, 8, 10, 12
	Unit Test	**Chapter 7 Unit Test**

7.3A – Inequalities in One Variable, Part 1

Warmup

How do we read these inequalities (from left to right)? $5 > 2$ $-3 < -1$

"5 is greater than 2" "−3 is less than −1"

What does each symbol mean? $>$ $<$ $\geq$ $\leq$

$>$	$<$	$\geq$	$\leq$
Greater than Right of Above	less than Left of Lower than	Greater than or equal to	Less than or equal to

How do you say this aloud? $x \geq 4$

"x is greater than or equal to 4"

What are some possible answers?

X could be 4, 5, 6, 4.5, 600, etc....

What is the primary difference between an **equation** and an **inequality**?

An equation has one (or two) solutions, whereas an inequality can have a whole range of solutions

Solving Linear Inequalities

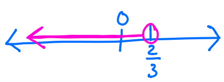
Example 1 – Solve the following inequality and graph on a number line: $3x – 7 < -5$

$3x - 7 < -5$
$\quad +7 \quad +7$

$\frac{3x}{3} < \frac{2}{3}$

$\boxed{x < \frac{2}{3}}$

open circle means don't include that number



all answers to the left of $\frac{2}{3}$

Example 2a – What are some possible answers to $-2x < 6$?

Test $x=0$? $-2(0) \overset{?}{<} 6$ $0 \overset{?}{<} 6$ ✓

$x=0$

$x=1$? $-2(1) \overset{?}{<} 6$ $-2 \overset{?}{<} 6$ ✓

Lots more......

Example 2b – Solve the following inequality and graph on a number line: $-2x < 6$

$\frac{-2x}{-2} = \frac{6}{-2}$

$x = -3$

if an equation

But... $\frac{-2x}{-2} < \frac{6}{-2}$

$x < -3$

test $x = -4$?

$-2(-4) \overset{?}{<} 6$

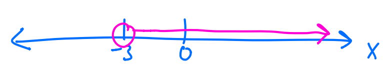
$8 \cancel{<} 6$

!! doesn't work

why? if multiplying/dividing by neg number must flip inequality sign.

$\frac{-2x}{-2} < \frac{6}{-2}$ flip

$\boxed{x > -3}$



How is solving an inequality like solving an equation? How is it different?

- Solve a linear inequality by isolating x (same as eq)
- make sure x is on the left of the inequality (different)
  ↳ just to make it easier graph on a number line
- If you mult/divide by a neg, flip the inequality sign (different)

Quadratic Inequalities

Solving quadratic inequalities requires a new technique, because we cannot just get “x by itself” on one side of the inequality.

There are two approaches we can take to solving quadratic inequalities:

graphing and test intervals

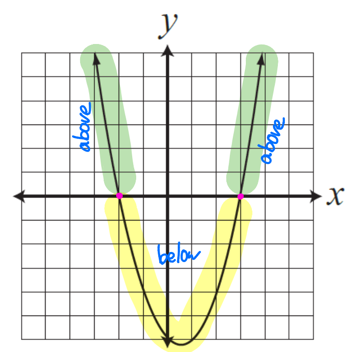
Solving Quadratic Inequalities by Graphing

Example 3 – Use the quadratic function $\overset{y}{\cancel{f(x)}} = x^2 - x - 6$ and its graph to answer the following:

a) Solve $x^2 - x - 6 = 0$

b) Solve $x^2 - x - 6 < 0$ and graph on a number line.

c) Solve $x^2 - x - 6 > 0$ and graph on a number line.

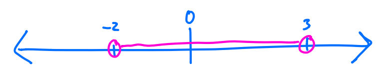


From the graph, you can see the parabola has

x-intercepts at −2 and 3.
On x-axis

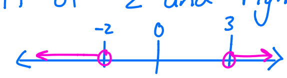
a) Therefore, $x^2 - x - 6 = 0$ when x = −2 and x = 3

The parabola is BELOW the x-axis when x…. between −2 and 3



b) Therefore, $x^2 - x - 6 < 0$ when $-2 < x < 3$ (domain)

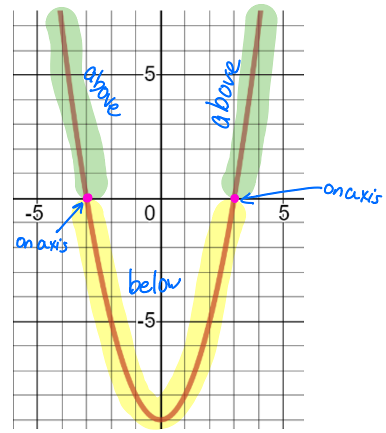
The parabola is ABOVE the x-axis when x… is left of −2 and right of 3



c) Therefore, $x^2 - x - 6 > 0$ when $x < -2$ $x > -3$

notice : 2 solutions (regions or intervals)

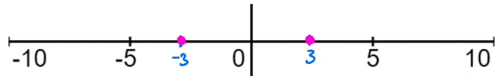
Example 4 – From the graph of the quadratic function $f(x) = x^2 - 9$, state the solution to the following and graph on a number line:



a) $x^2 - 9 = 0$

On x-axis

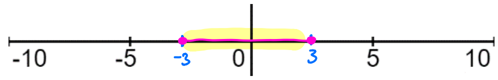
$x = \pm 3$



b) $x^2 - 9 \leq 0$

below or on x-axis

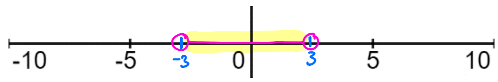
$-3 \leq x \leq 3$



c) $x^2 - 9 < 0$

below x-axis

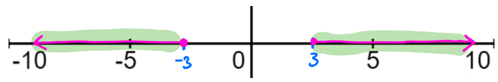
$-3 < x < 3$



d) $x^2 - 9 \geq 0$

above or on x-axis

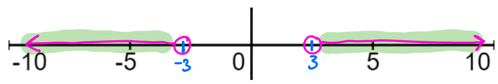
$x \leq -3, \; x \geq 3$



e) $x^2 - 9 > 0$

above x-axis

$x < -3, \; x > 3$

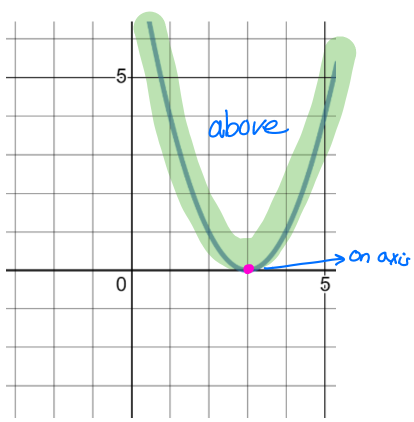


$x^2 - 9 = 0$

$(x+3)(x-3) = 0$

$x = \pm 3$

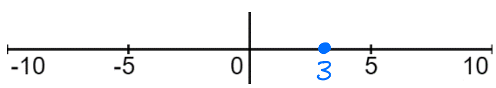
Example 5 – From the graph of the quadratic function $f(x) = x^2 - 6x + 9$, state the solution to the following and graph on a number line:



a) $x^2 - 6x + 9 = 0$

x on axis

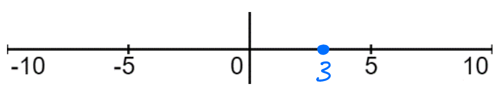
$x = 3$



b) $x^2 - 6x + 9 \leq 0$

below or on x-axis

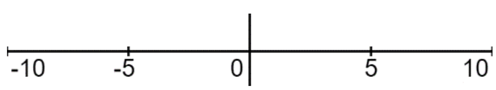
$x = 3$



c) $x^2 - 6x + 9 < 0$

below x-axis

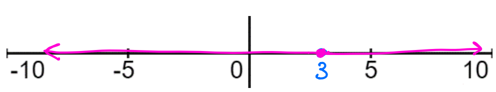
No Solution



d) $x^2 - 6x + 9 \geq 0$

above or on x-axis

all real numbers $x \in \mathbb{R}$

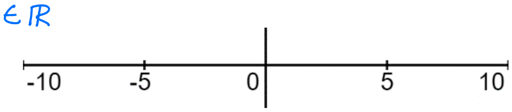


e) $x^2 - 6x + 9 > 0$

above x-axis

$\{x \in \mathbb{R} \mid x \neq 3\}$

all real numbers except $x = 3$



Can factor

$x^2 - 6 + 9 = 0$

$(x-3)(x-3) = 0$

$x = 3$

Example 1 – Solve $x^2 + 2x > 8$ **by graphing**, **and then using test intervals**. Graph the solution on a number line.

Graphing Steps

1. Get everything to the left side of the inequality so that zero is on the right.
2. Find the roots (*x*-intercepts) of the quadratic.
3. **Sketch** a graph and use the visual to solve the inequality.
   → if the quadratic is > 0, find the domain where the graph is **above** the x-axis
   → if the quadratic is < 0, find the domain where the graph is **below** the x-axis

$x^2 + 2x > 8$
$\quad\;\; -8 \quad -8$

① $x^2 + 2x - 8 > 0$

② x-ints

$x^2 + 2x - 8 = 0$

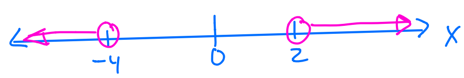
$(x+4)(x-2) = 0$

$x = -4 \quad x = 2$

Parabola opens up since "a" value pos

where is parabola above x-axis?

$x < -4 \qquad x > 2$



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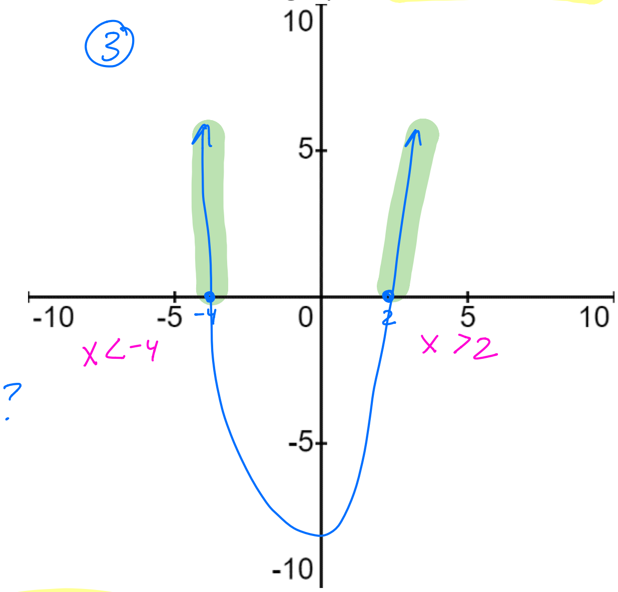


Test Intervals Steps

1. Set inequality to zero. Find the critical numbers (the zeros) of the quadratic.
2. Use the critical numbers to split the domain (x-values) into separate test intervals, and make an *x*-axis diagram of the resulting test intervals. Label each interval.
3. Test a value from each interval using the **original** inequality.

① $x^2 + 2x > 8$

$x^2 + 2x - 8 > 0$

$x^2 + 2x - 8 = 0$

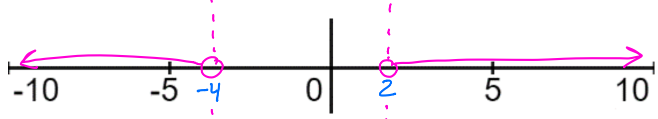
$(x+4)(x-2) = 0$

zeros/critical numbers

$x = -4 \qquad x = 2$

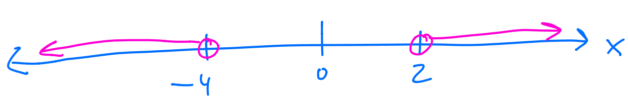
②

$x < -4 \qquad -4 < x < 2 \qquad x > 2$



③

Test $x = -5$	Test $x = 0$	Test $x = 3$
$x^2 + 2x \overset{?}{>} 8$	$x^2 + 2x > 8$	$x^2 + 2x > 8$
$(-5)^2 + 2(-5) > 8$	$0^2 + 2(0) > 8$	$(3)^2 + 2(3) > 8$
$25 - 10 > 8$	$0 > 8$	$9 + 6 > 8$
$15 > 8$	✗	$15 > 8$
✓		✓



$\boxed{\text{sol } x < -4 \qquad x > 2}$

Example 2 – Solve $2x^2 - 7x - 15 \leq 0$ using both methods and graph the solution on a number line.

*if the quadratic is ≥ 0, find the domain where the graph is **above or on** the x-axis

*if the quadratic is ≤ 0, find the domain where the graph is **below or on** the x-axis

Graphing:

$2x^2 - 7x - 15 = 0$

$x^2 - 7x - 30 = 0$

$\left(x - \frac{10}{2}\right)\left(x + \frac{3}{2}\right) = 0$

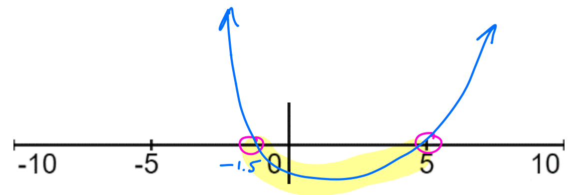
$(x-5)(2x+3) = 0$

$x = 5$

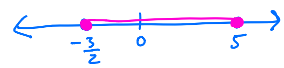
$x = -\frac{3}{2}$ or -1.5

$2x^2 - 7x - 15 \leq 0$

where is parabola on or below x-axis? $\Rightarrow$ between $-\frac{3}{2}$ and 5



sol: $-\frac{3}{2} \leq x \leq 5$

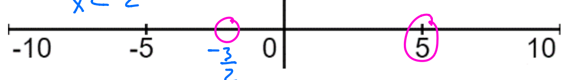


Test Intervals:

solve for x, same as above

$x = -\frac{3}{2}$ $\quad x = 5$

$x < -\frac{3}{2}$ $\quad -\frac{3}{2} < x < 5$ $\quad x > 5$



test $x = -2$

$2x^2 - 7x - 15 \overset{?}{\leq} 0$

$2(-2)^2 - 7(-2) - 15 \overset{?}{\leq} 0$

$2(4) + 14 - 15 \overset{?}{\leq} 0$

$7 \not\leq 0$

test $x = 0$

$2(0)^2 - 7(0) - 15 \overset{?}{\leq} 0$

$-15 \overset{?}{\leq} 0$ ✓

test $x = 6$

$2(6)^2 - 7(6) - 15 \overset{?}{\leq} 0$

$72 - 42 - 15 \overset{?}{\leq} 0$

$15 \not\leq 0$

sol: $-\frac{3}{2} \leq x \leq 5$

Example 3 – Solve $x^2 + 1 > 3x$. Then graph the solution on a number line.

$x^2 + 1 > 3x$

$x^2 - 3x + 1 > 0$ greater than 0

→ open up, where is it above x-axis

x-intercepts:

$x^2 - 3x + 1 = 0$

doesn't factor so quad formula →

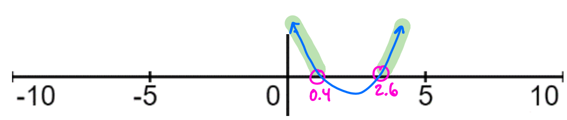
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$

$x = \frac{3 \pm \sqrt{9-4}}{2}$

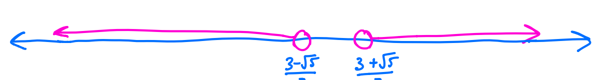
$x = \frac{3 \pm \sqrt{5}}{2}$ → $x = \frac{3+\sqrt{5}}{2} \cong 2.6$, $x = \frac{3-\sqrt{5}}{2} \cong 0.4$

use decimals for sketch

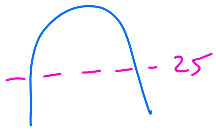


Use exact values for solution

$x < \frac{3-\sqrt{5}}{2}$, $x > \frac{3+\sqrt{5}}{2}$



7.5 – Applications of Quadratic Inequalities



Example 1 – The height in meters of a projectile shot from the top of a building is given by $h(t) = -16t^2 + 60t + 25$, where t represents the time in seconds the projectile is in the air. Find the time interval that the projectile is above 25m, to the nearest hundredth.

↳ means $h(t) > 25$

$$-16t^2 + 60t + 25 > 25$$

-25 $\quad$ -25

$$\boxed{-16t^2 + 60 > 0}$$

when is my parabola above x-axis?

"a" is neg so parabola opens down

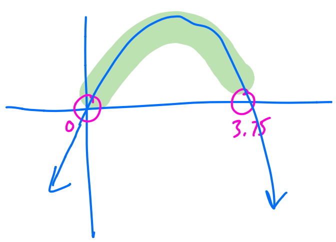
x-ints

$-16t^2 + 60t = 0$

$-4t(4t - 15) = 0$

$t = 0 \qquad t = \frac{15}{4} = 3.75$

Sketch



$0 < t < 3.75$

The projectile is above 25m between 0 seconds and 3.75 seconds

Example 2 – The sale price of a stereo is given by the function

price

$$S(x) = 200 - 0.1x, \quad 0 \le x \le 2000$$

where x is the number of stereos produced each day. It costs \$18 000 per day to operate the factory and \$15 for material to produce each stereo.

a) Find the equation for the daily revenue. Revenue = number sold x price

b) Find the equation for the daily cost of producing stereos.

c) Find the interval that produces a profit. Profit = Revenue – Cost

a) $R(x) = (x)(200 - 0.1x)$

$R(x) = 200x - 0.1x^2$

b) $C(x) = 18000 + 15x$

c) $P = R - C$

$= (200x - 0.1x^2) - (18000 + 15x)$

$= 200x - 0.1x^2 - 18\,000 + 15x$

$P = -0.1x^2 + 185x - 1800$

want positives >0 so making \$

$\boxed{-0.1x^2 + 185x - 1800 > 0}$

$-0.1x^2 + 185x - 1800 = 0$ solve!

$$x = \frac{-185 \pm \sqrt{(185)^2 - 4(-0.1)(-18000)}}{2(0.1)}$$

$$x = \frac{-185 \pm \sqrt{27025}}{-0.2}$$

$x = 103.04$

$x = 1746.96$

$0 < x < 103.04$ ✗ 103 ✓ 104 $103.04 < x < 1746.96$ ✓ 1746 ✗ 1747 $x > 1746.96$

103.04 1746.96 2000

test $x = 1$

$-0.1(1)^2 + 185(1) - 18\,000 \overset{?}{>} 0$

✗

$x = 110$

$-0.1(110)^2 + 185(110) - 18000 \overset{?}{>} 0$

$590 > 0$ ✓

test $x = 2000$

$-0.1(2000)^2 + 185(2000) - 18000 \overset{?}{>} 0$

✗

can't have a decimal of a stereo so round to closest whole number within interval!

The interval that gives us a profit is

$104 \le x \le 1746$