PC11 Name______________________________

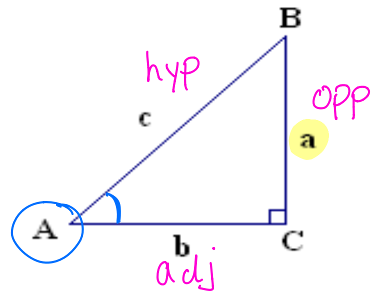
Chapter 8 Notes
Trigonometry *Key*

Date	Topic/Lesson	Assignment
	8.0 - Trig Review	After Notes ex 1 – 8.0 Trig. Review Worksheet #1-2 After Notes ex 3 – 8.0 W/S #3-5 After Notes ex 4 – 8.0 W/S #6
	8.1 - Angles and their Measure	After Notes ex 2 – p.291: 1, 2, 3a-h, 5left After Notes ex 5 – p.293: 7, 9a-h, 10, 11
	8.2 - The Three Trig Functions	After Notes ex 2 – p.300: 1, 2left, 3a-f After Notes ex 4 – p.303: 5a-d, 6a-d
	8.3A - Special Angles Part 1	Warm-up before Notes – p.312: 1(a-f), 2(a-f), After Quadrantal angles – p.313: 3(a-f) After Notes – p.313: 3(g-p)
	8.3B - Special Angles Part 2	After Notes ex 2 – p.313: 4 After Notes ex 3 – p.314: 5a-h
	8.5 - The Sine Law	After Notes – p.330: 6abcfhi
	8.6 - The Cosine Law	After Notes ex 1 – p.336: 2 After Notes ex 3 – p.338: 7acde, 8abe
	Test Prep	Chapter 8 Practice Test
	Review	p.341: 1left, 2a-d, 3, 4left, 6abc, 7bdfhj, 9a-e
	Chapter Test	**Chapter 8 Test**

8.0 – Trigonometry Review

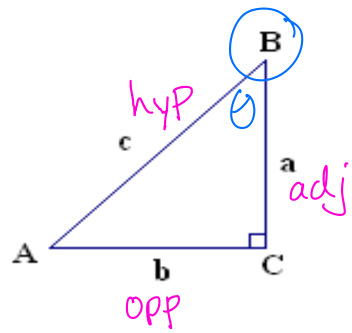
In trigonometry problems, all vertices (corners or angles) of the triangle are labeled with capital letters. The right angle is usually labeled *C*. Sides are usually labeled with lower case letters. The side opposite to <*A* will be labeled *a* and so on.

Whenever we do trigonometry problems on a right triangle, we focus on a target angle. The target angle can be any of the two angles that are **not the right angle.**



Once we have a target angle, we can name each side of the triangle. Let's suppose *A* is the target angle. Then side *a* is called the ___opposite___ side because it is on the other side of the triangle. Side *c* is always called the ___hypotenuse___ because it is the longest side, and side *b* is called the ___adjacent___ side as it is beside <*A*. If the target angle does not have an angle measurement on it, we represent it with the Greek letter theta, θ.

Suppose *B* is the target angle in the triangle on the right. Label all appropriate parts.



The three trigonometric ratios for right triangles are:

SINE	COSINE	TANGENT
$\sin\theta = \frac{opposite}{hypotenuse}$	$\cos\theta = \frac{adjacent}{hypotenuse}$	$\tan\theta = \frac{opposite}{adjacent}$
S O H	C A H	T O A

What is the point of the trigonometric ratios?

They relate the sides of a right triangle to its angles and they are used to calculate unknown angles or sides.

check calc in DEG

Example 1 – Solve each to the nearest hundredth. ↳ 2 decimal places

a) $\cos 42°$

$= 0.74$

b) $7\cdot\tan 67° = \frac{x}{7}\cdot 7$

$x = 7\cdot\tan 67$

$x = 16.49$

c) $\sin\theta = \frac{5}{9}$ need angle

$\theta = \sin^{-1}\left(\frac{5}{9}\right)$

$\theta = 33.75°$

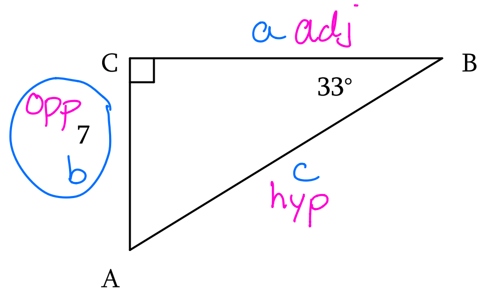
d) $\cos 35° = \frac{8}{x}$

$x = \frac{8}{(\cos 35°)}$

$x = 9.77$

In order to solve a right triangle, you must find the measurement of all three sides and all three angles.

Example 2 - Solve ΔABC to the nearest tenth.



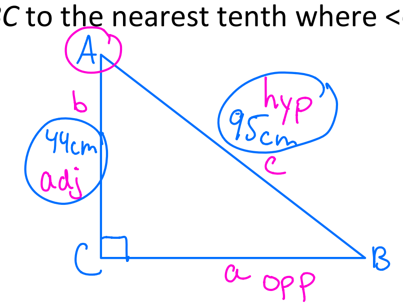
$A = 57°$ $a = 10.8$
$B = 33°$ $b = 7$
$C = 90°$ $c = 12.9$

$\boxed{\angle A} = 180° - 90° - 33°$
$\angle A = 57°$

$\boxed{a}$ $\tan\theta = \frac{O}{A}$
$\tan 33° = \frac{7}{a}$
$a = \frac{7}{\tan 33°}$
$a = 10.8$

$\boxed{c}$ $\sin\theta = \frac{O}{H}$
$\sin 33 = \frac{7}{b}$
$b = \frac{7}{\sin 33°}$
$b = 12.9$

Example 3 – Sketch & solve ΔABC to the nearest tenth where $<C=90°$, c =95cm & b =44cm



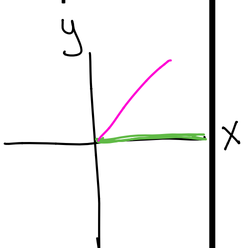
$A = 62.4°$ $a = 84.2cm$
$B = 27.6°$ $b = 44cm$
$C = 90°$ $c = 95cm$

$\boxed{a}$ $a^2 + b^2 = c^2$
$a^2 + 44^2 = 95^2$
$a^2 + 1936 = 9025$
$-1936 \quad -1936$
$a^2 = \sqrt{7089}$
$a = 84.2cm$

$\boxed{\angle A} = \cos\theta = \frac{A}{H}$
$\theta = \cos^{-1}\left(\frac{44}{95}\right)$
$\theta = 62.4°$

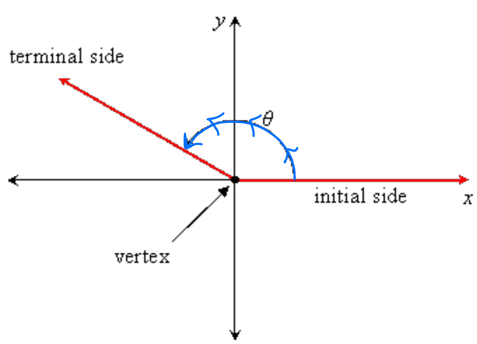
$\boxed{\angle B} = 180° - 90° - 62.4°$
$\angle B = 27.6°$

angles in standard position

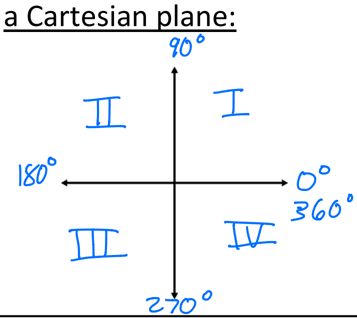


An angle that is drawn in **standard position** must have its vertex at the origin of the Cartesian plane, and its initial arm must coincide with the positive x-axis.

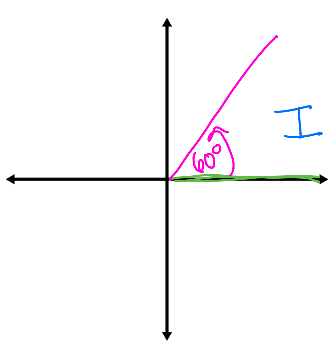
To draw angles in standard position, we use an **initial arm** (always the positive x-axis) and a **terminal arm** (the final position after a rotation). The angle is labeled **θ** (*theta*). The **vertex** of the angle must be at the origin (0, 0) of a Cartesian plane. Positive angles are measured in a counterclockwise direction. Here is an example:

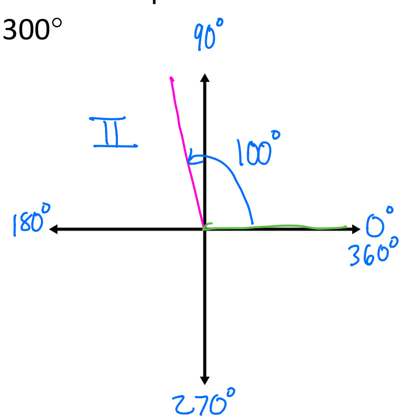


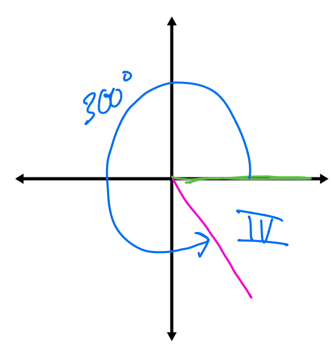
Label the four quadrants of a Cartesian plane:



Example 4 – Draw each angle in standard position and identify the quadrant in which it lies: a) 60° b) 100° c) 300°



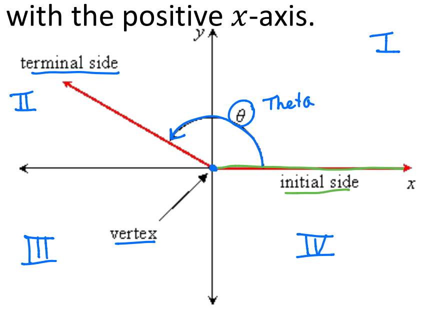




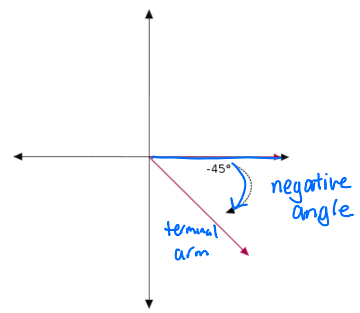
8.1 – Angles and Their Measure

angles in standard position

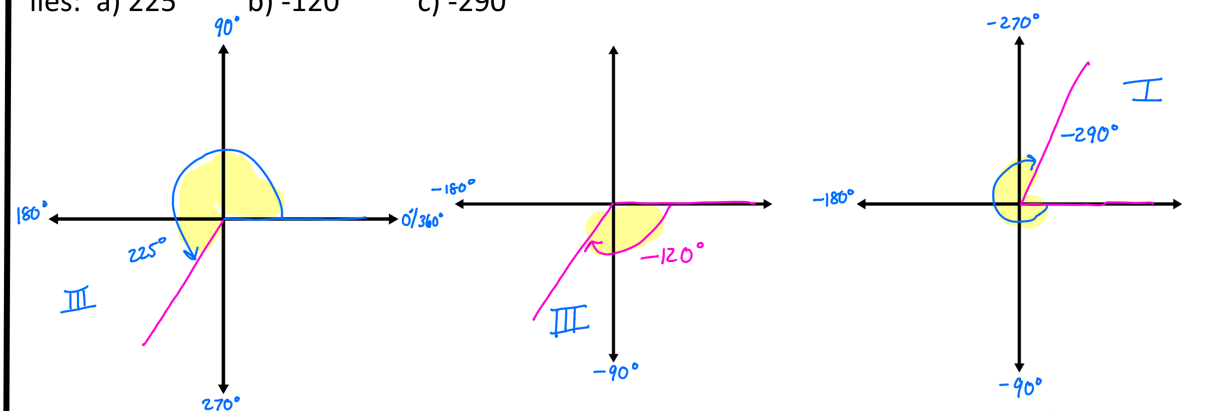
An angle that is drawn in **standard position** must have its vertex (corner) at the origin (0, 0) of the Cartesian plane, and its initial arm must coincide with the positive x-axis.



Clockwise angles have a negative measure:



Example 1– Draw each angle in standard position and identify the quadrant in which it lies: a) 225° b) -120° c) -290°

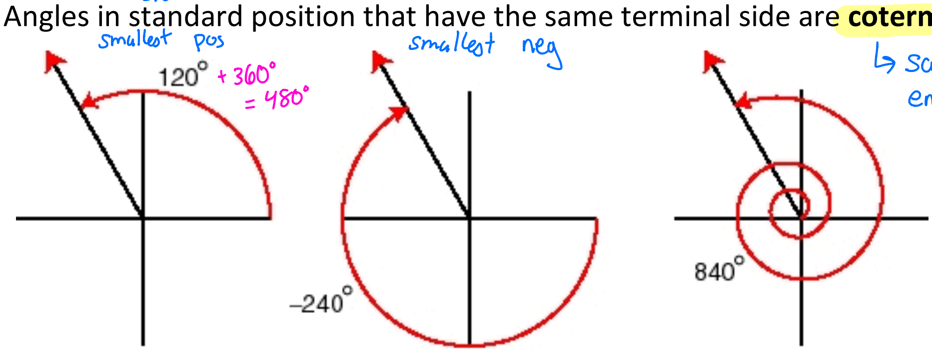


coterminal angles

Add 360° end up in same place

Angles in standard position that have the same terminal side are **coterminal**.

↳ same terminal arm end at same location



Example 2 – Find three coterminal angles (at least one negative) for:

a) 60°

$60° + 360° = 420°$

$60° + 360° + 360° = 780°$

$60° - 360° = -300°$

All coterminal

b) 225°

$225° + 360° = 585°$

$225° + 360° (2) = 945°$

$225° - 360° = -135°$

↳ or $225° + 360° (-1)$

What is a general formula to find **coterminal** angles?

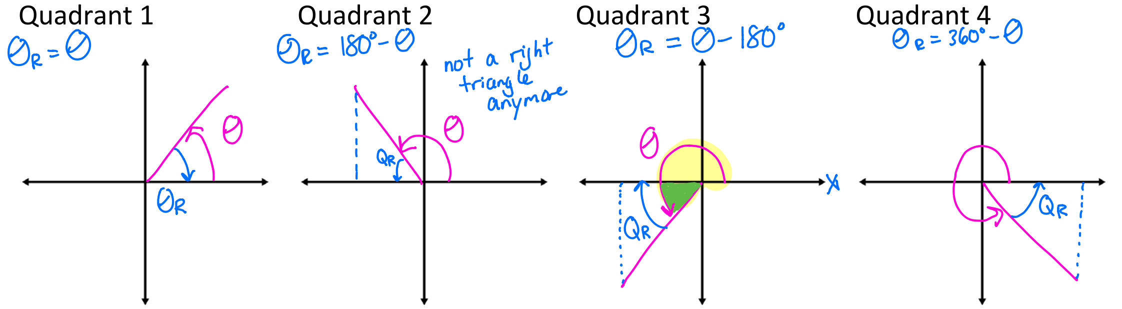
$\theta + 360° n$

↑ n stands for any counting number pos or neg (an integer)

reference angles

(helper angles)

For each **angle in standard position**, there is a corresponding acute angle called the **reference angle**, which is the acute angle between the terminal arm and the (nearest) *x*-axis. Thus, any reference angle is between 0° and 90°



must be nearest x-axis

Always use 90° or 360° never 90° or 270°

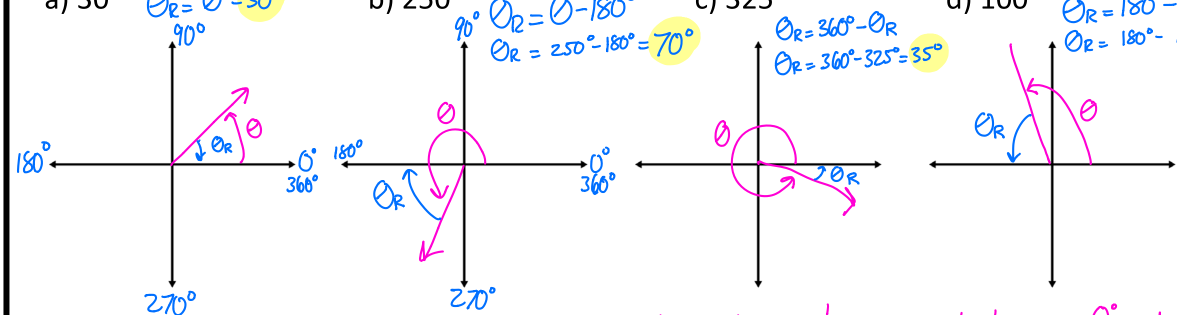
Example 3 – Draw each angle in standard position, and find the reference angle.

a) 30° $\theta_R = \theta = 30°$

b) 250° $\theta_R = \theta - 180°$, $\theta_R = 250° - 180° = 70°$

c) 325° $\theta_R = 360° - \theta_R$, $\theta_R = 360° - 325° = 35°$

d) 100° $\theta_R = 180° - \theta$, $\theta_R = 180° - 100° = 80°$



Example 4 – Find the reference angle for: get coterminal angle between 0° and 360°

a) 1450°

$1450 \div 360 = 4.0277\ldots$ ($=4$) pos # always round down

$1450 - 360(4)$

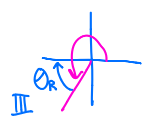
$1450 - 1440 = 10°$ (in Q I)

$\theta_R = \theta$ so $\theta_R = 10°$

b) -870°

$-870 \div 360 = 2.4\,66\ldots$ ($=3$) neg # always round up

$-870° + 360(3) = 210°$

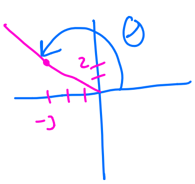


$\theta_R = \theta - 180°$

$\theta_R = 210° - 180° = 30°$

work book Q7

(-3, 2) sketch

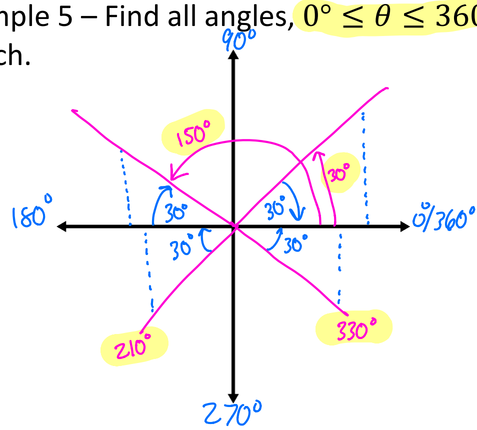


Example 5 – Find all angles, $0° \le \theta \le 360°$, that have reference angles of 30°. Do a sketch.

- 30°
- 180 − 30° = 150°

270 − 30° = 210°

360 − 30° = 330°



Find sin θ for each with your calculator:

sin 30° = 0.5 sin 210° = −0.5

sin 150° = 0.5 sin 330° = −0.5

Find cos θ for each with your calculator:

cos 30° = 0.866 cos 210 = −0.866

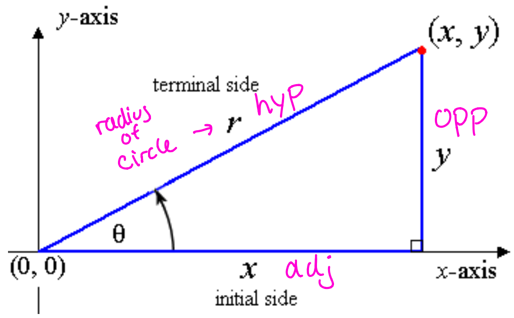
cos 150° = −0.866 cos 330 = 0.866

What do you notice? Why are some results positive and some negative?

- The values repeat.
- Coordinates can be negative depending on the quadrants

8.2 – The Three Trigonometric Functions

Suppose θ is an angle in standard position. Suppose the point at the end of the terminal arm is labeled $P(x, y)$, at a distance r from the origin.

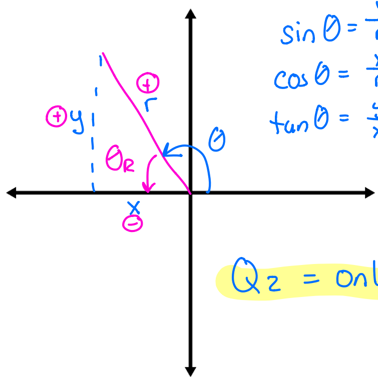


You can use a reference angle to determine the three trigonometric ratios in terms of x, y, and r. $\sin\theta = \frac{O}{H}$ $\cos\theta = \frac{A}{H}$ $\tan\theta = \frac{O}{A}$

$$\sin\theta = \frac{y}{r} \quad \cos\theta = \frac{x}{r} \quad \tan\theta = \frac{y}{x}$$

Trigonometry ratios in the four quadrants: (signs)

Quadrant 2 $\quad 90° < \theta < 180°$



$\sin\theta = \frac{y}{r} = \frac{+}{+} = +$

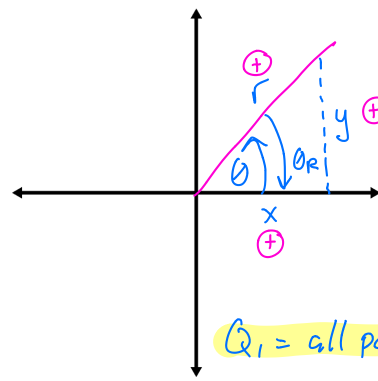
$\cos\theta = \frac{x}{r} = \frac{-}{+} = -$

$\tan\theta = \frac{y}{x} = \frac{+}{-} = -$

Q2 = only sin +

Quadrant 1 $\quad 0° < \theta < 90°$

radius (r) always pos



$\sin\theta = \frac{y}{r} = \frac{+}{+} = +$

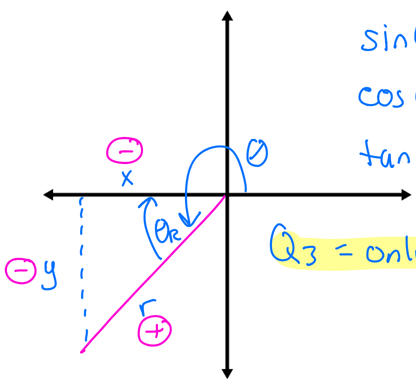
$\cos\theta = \frac{x}{r} = \frac{+}{+} = +$

$\tan\theta = \frac{y}{x} = \frac{+}{+} = +$

trig we've seen so far

Q1 = all pos

Quadrant 3 $\quad 180° < \theta < 270°$



$\sin\theta = \frac{y}{r} = \frac{-}{+} = -$

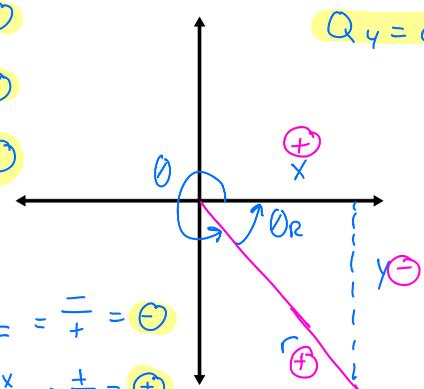
$\cos\theta = \frac{x}{r} = \frac{-}{+} = -$

$\tan\theta = \frac{y}{x} = \frac{-}{-} = +$

Q3 = only tan +

Quadrant 4 $\quad 270° < \theta < 360°$

Q4 = only cos +



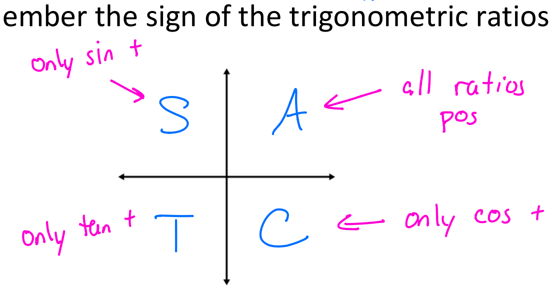
$\sin\theta = \frac{y}{r} = \frac{-}{+} = -$

$\cos\theta = \frac{x}{r} = \frac{+}{+} = +$

$\tan\theta = \frac{y}{x} = \frac{-}{+} = -$

CAST

Here is a way to remember the sign of the trigonometric ratios in each quadrant:

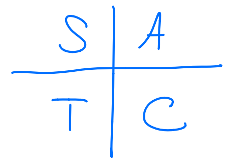


Example 1 – Identify the quadrant(s) for the angles satisfying the following conditions: $<$ = less than, $>$ = greater than

a) $\sin\theta < 0, \cos\theta > 0$ — $\ominus$, $\oplus$ — Q IV

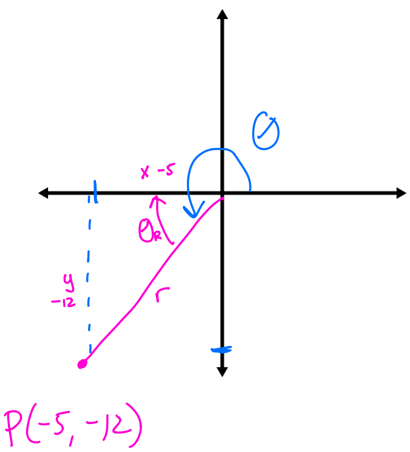
b) $\tan\theta < 0, \cos\theta < 0$ — $\ominus$, $\ominus$

Q II



Example 2 – The point *P*(-5, -12) lies on the terminal arm of an angle, θ, in standard position. Determine the exact trigonometric ratios for sinθ, cosθ, and tanθ.

$\frac{y}{r}$ $\quad \frac{x}{r}$ $\quad \frac{y}{x}$



☐ use pythagoras

$a^2 + b^2 = c^2$

$x^2 + y^2 = r^2$

$(-5)^2 + (-12)^2 = r^2$

$25 + 144 = r^2$

$\sqrt{169} = \sqrt{r^2}$

$r = 13$

$\sin\theta = \frac{y}{r} = \frac{-12}{13}$

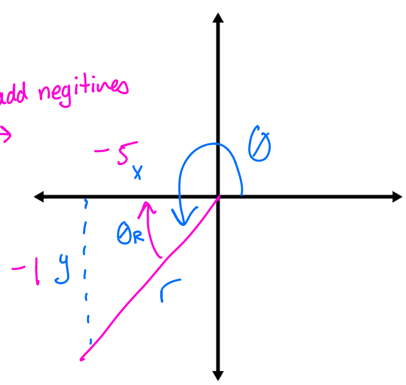
$\cos\theta = \frac{x}{r} = \frac{-5}{13}$

$\tan\theta = \frac{y}{x} = \frac{-12}{-5} = \frac{12}{5}$

Example 3 – Suppose θ is an angle in standard position with terminal arm in quadrant III, and $tan\theta = \frac{1}{5}$. Determine the exact values of sinθ and cosθ.

$\tan\theta = \frac{y}{x} = \frac{1}{5}$

add negitives →



☐ use pythagoras

$x^2 + y^2 = r^2$

$(-5)^2 + (-1)^2 = r^2$

$25 + 1 = r^2$

$\sqrt{26} = \sqrt{r^2}$

$r = \sqrt{26}$

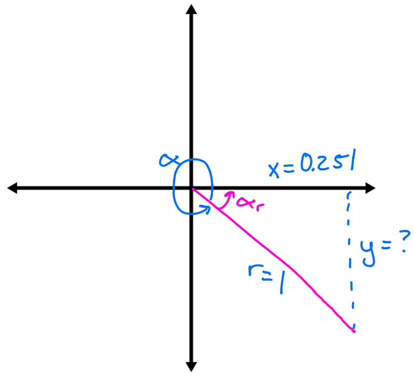
Keep root since want exact values

$\sin\theta = \frac{y}{r} = \frac{-1}{\sqrt{26}}$

$\cos\theta = \frac{x}{r} = \frac{-5}{\sqrt{26}}$

alpha

Example 4 – Find $\sin\alpha$ if $\cos\alpha = 0.251$ with α in Quadrant IV.



$\cos\alpha = \frac{0.251}{1} = \frac{x}{r}$

$\boxed{y}$ use pythagorus

$x^2 + y^2 = r^2$

$-x^2 \qquad -x^2$

$\sqrt{y^2} = \sqrt{r^2 - x^2}$

$y = \sqrt{(r^2 - x^2)}$

$y = \sqrt{(1^2 - 0.251^2)}$

$y = 0.967987$

* since down in pic add negative
* round to 3 decimals

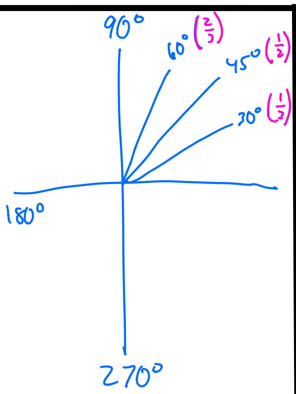
$y = -0.968$

$\sin\alpha = \frac{y}{r} = \frac{-0.968}{1}$

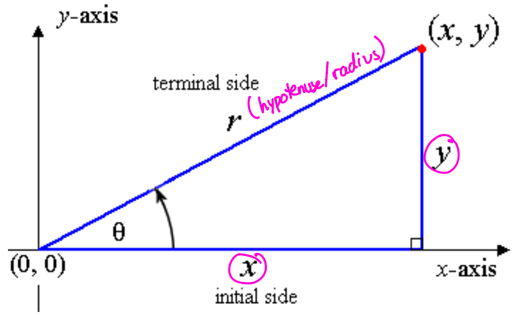
$\sin\alpha = -0.968$

$\tan\alpha = \frac{y}{x} = \frac{-0.968}{0.251}$

$\tan\alpha = -3.857$



Suppose θ is an angle in standard position. Suppose the point at the end of the terminal arm is labeled $P(x, y)$, at a distance $\boldsymbol{r}$ from the origin.



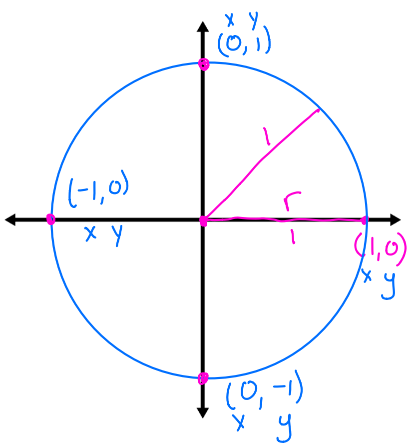
You can use a reference angle to determine the three trigonometric ratios in terms of x, y, and r.

$\frac{\Delta y}{\Delta x}$ = slope!

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x}$$

A **quadrantal angle** is an angle in standard position whose terminal arm lies on one of the axes. It's easiest to suppose the terminal arm, $\boldsymbol{r}$, has a length of 1.

Example – Find the values of sinθ, cosθ, and tanθ for each quadrantal angle on the Cartesian plane.



	0°/360°	90°	180°	270°
sinθ $\frac{y}{r}$	$\frac{0}{1} = 0$	$\frac{1}{1} = 1$	$\frac{0}{1} = 0$	$\frac{-1}{1} = -1$
cosθ $\frac{x}{r}$	$\frac{1}{1} = 1$	$\frac{0}{1} = 0$	$\frac{-1}{1} = -1$	$\frac{0}{1} = 0$
tanθ $\frac{y}{x}$ slope	$\frac{0}{1} = 0$	$\frac{1}{0} =$ undefined	$\frac{0}{-1} = 0$	$\frac{-1}{0} =$ undefined

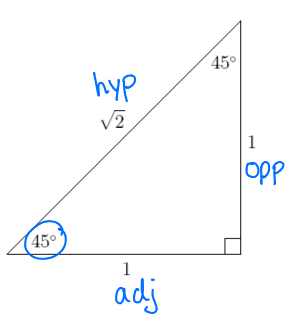
↳ remember vertical line = slope undefined

There are two right triangles in trigonometry that are especially significant because of their frequent occurrence.

sides have ratio of 1-1-√2

dashes, not negatives

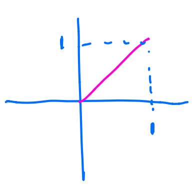
A **45°-45°-90° triangle** with legs of each 1 unit has a hypotenuse of $\sqrt{2}$.



$$\sin\theta = \frac{opposite}{hypotenuse} \qquad \cos\theta = \frac{adjacent}{hypotenuse} \qquad \tan\theta = \frac{opposite}{adjacent}$$

S O H C A H T O A

$$\sin 45° = \frac{1}{\sqrt{2}} \qquad \cos 45° = \frac{1}{\sqrt{2}} \qquad \tan 45° = \frac{1}{1} = 1$$



$m = \frac{1}{1} = 1$

tan ≈ slope!

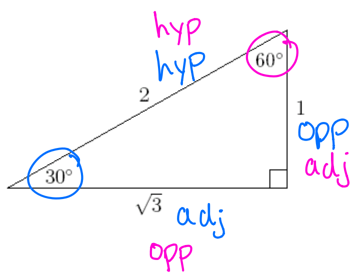
ratilizing denominator

$\frac{1}{\sqrt{2}} \to \frac{\sqrt{2}}{2}$ ✓

$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ ✓

$1-2-\sqrt{3}$

A **30°-60°-90° triangle** has legs of 1 unit and $\sqrt{3}$ units, with hypotenuse 2 units.

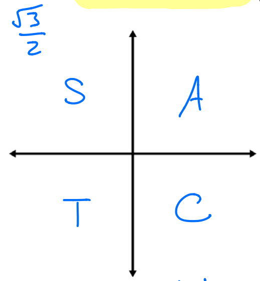


$sin30° = \frac{1}{2}$ $\quad cos30° = \frac{\sqrt{3}}{2}$ $\quad tan30° = \frac{1}{\sqrt{3}}$

$sin60° = \frac{\sqrt{3}}{2}$ $\quad cos60° = \frac{1}{2}$ $\quad tan60° = \frac{\sqrt{3}}{1}$

The trigonometric ratios are given as **exact values** (in fraction/radical form as opposed to an approximated decimal). ie $\frac{\sqrt{3}}{2}$

What is the CAST rule again?

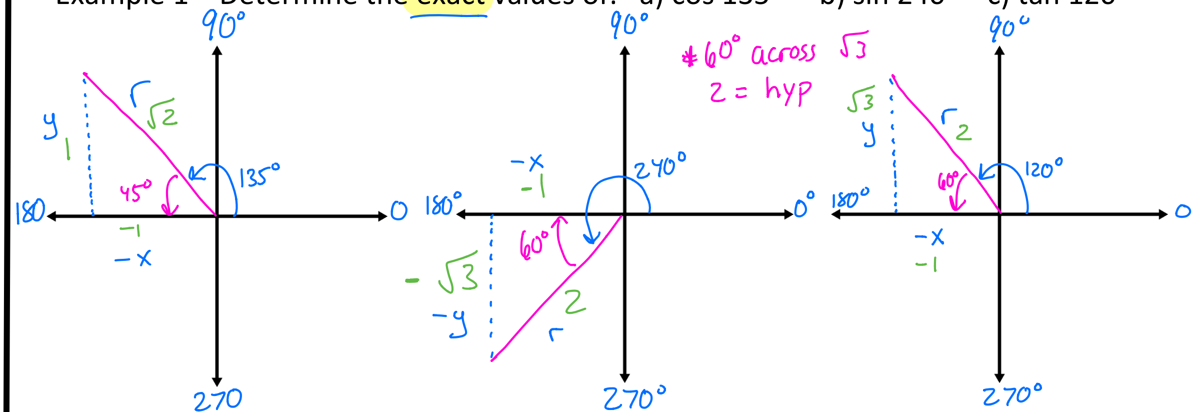


Steps

1) sketch
2) find θ_R
3) check for θ_R special angle
4) Label special triangle remember CAST
5) use SOH CAH TOA to answer

Example 1 – Determine the exact values of: a) cos 135° b) sin 240° c) tan 120°

→ clue, special triangle



$\theta_R = 180 - 135$
$\theta_R = 45°$

$\cos 135° = \frac{x}{r} = \frac{-1}{\sqrt{2}}$

$\theta_R = 240° - 180° = 60°$

$\sin 240° = \frac{y}{r} = \frac{-\sqrt{3}}{2}$

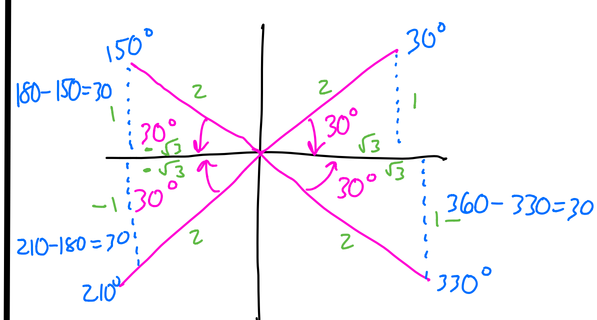
S | A
T | C

double check

$\theta_R = 180° - 120°$
$\theta_R = 60°$

$\tan 120° = \frac{\sqrt{3}}{-1} = -\sqrt{3}$

Example 2 – Evaluate sin 30°, sin 150°, sin 210°, and sin 330°.



$\sin\theta = \frac{y}{r}$

$\sin 30° = \frac{1}{2}$

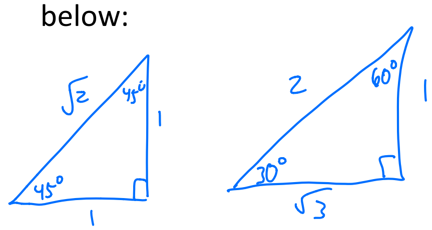
$\sin 150° = \frac{1}{2}$

$\sin 210° = -\frac{1}{2}$

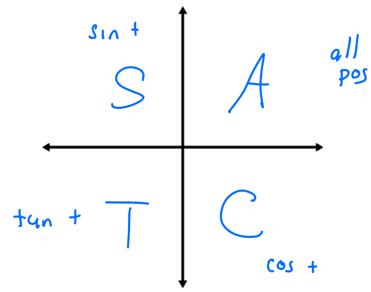
$\sin 330° = -\frac{1}{2}$

8.3B – Special Angles Part 2

Warmup 1 – Draw the 45°-45°-90° triangle and the 30°-60°-90° triangle below:

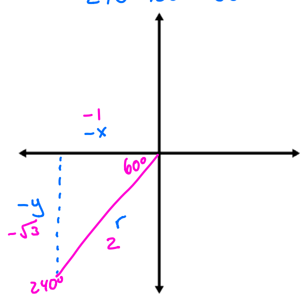


Warmup 2 – Quickly draw and explain the 'CAST' rule:



Example 1 – Find the exact value of cos 240°

240 − 180 = 60



$\cos\theta = \frac{x}{r}$

$\cos 240° = \frac{-1}{2}$

solving for angles

Steps for solving for angles given their sine, cosine, or tangent ratio:

1. Use the sign (+ or -) to determine the quadrant the solution is in. S | A / T | C
2. Solve for the reference angle. special △
3. Draw a diagram and use the reference angles to find the angle in standard position.

Example 2 – Solve for θ.

find all answers in a circle

a) $\sin\theta = -\frac{1}{\sqrt{2}}, 0° \le \theta < 360°$ b) $\cos\theta = \frac{1}{2}, 0° \le \theta < 360°$ c) $\sin\theta = -\frac{\sqrt{3}}{2}, 0° \le \theta \le 360°$

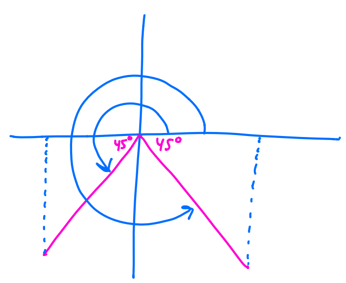
$Q_1: \theta = \theta_R$
$Q_2: \theta = 180° - \theta_R$
$Q_3: \theta = 180° + \theta_R$ * only diff one
$Q_4: \theta = 360° - \theta_R$

a)

① S | A / T | C sin neg in QIII/QIV

② $\frac{-1}{\sqrt{2}}$ sin so $\frac{y}{r}$ 45°-45°-90° △

③ sketch



Q3 angle = 180° + 45° = 225°

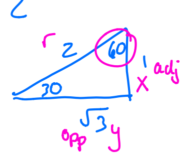
Q4 angle = 360° − 45° = 315°

$\theta = 225°, 315°$

b)

① S | A / T | C cos pos in I & IV

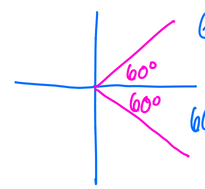
② $\frac{1}{2}$ so 30°-60°-90° △



cos $\frac{adj}{hyp} = \frac{x}{r}$

$\theta_R = 60°$

③



$\theta = 60°$

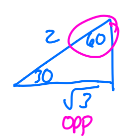
60 = 360 − 60

$\theta = 60°, 300°$

c)

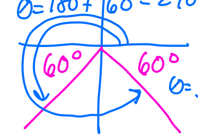
① S | A / T | C sin neg in QIII/QIV

② $\frac{-\sqrt{3}}{2}$ sin $\frac{opp}{hyp} = \frac{y}{r}$



$\theta_R = 60°$

③



$\theta = 180 + 60 = 240°$

$\theta = 360 - 60 = 300°$

$\theta = 240°, 300°$

Example 3 – Determine the measure of θ, to the nearest degree, given

not exact value, not special triangle

a) $\sin\theta = -0.8090$, where $0° \le \theta < 360°$ b) $\tan\theta = -0.7565$, where $0° \le \theta < 360°$

① $\begin{array}{c|c} S & A \\ \hline T & C \end{array}$ sin neg in Q III, Q IV

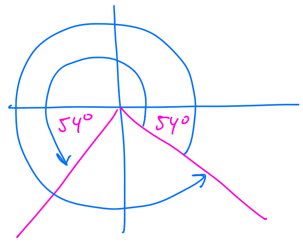
② can't use special triangle, so ignore +/- sign

$\sin\theta_R = 0.8090$

$\theta_R = \sin^{-1}(0.8090)$

$\theta_R = 54°$

③



Q III $= 180° + 54° = 234°$

Q IV $= 360° - 54° = 306°$

$\theta = 234°, 306°$

---

b) $\tan\theta = -0.7565$

① $\begin{array}{c|c} S & A \\ \hline T & C \end{array}$ tan neg in Q II, Q IV

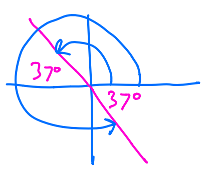
② ignore +/- sign

$\tan\theta_R = 0.7565$

$\theta_R = \tan^{-1}(0.7565)$

$\theta_R = 37°$

③



Q II $= 180° - 37° = 143°$

Q IV $= 360° - 37° = 323°$

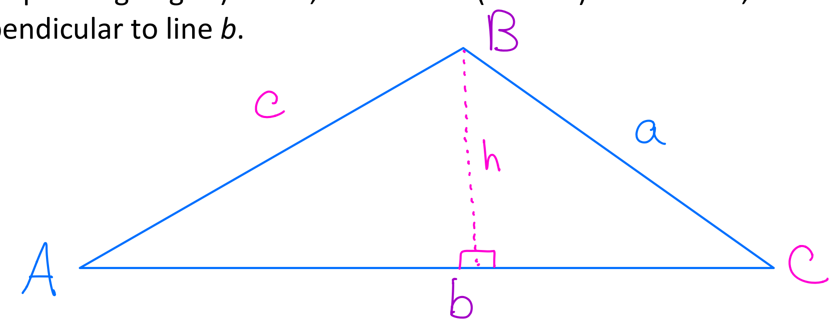
$\theta = 143°, 323°$

8.5 – The Sine Law

So far, you have learned how to use trigonometry when working with right triangles. Now, you will learn how to use trigonometry for **oblique triangles** (non-right triangles).

developing the sine law

Draw an oblique triangle *ABC* and label the sides *a*, *b*, & *c* (opposite the respective corresponding angles). Then, draw a line (call it *h*) from *B* to *b*, so that it is perpendicular to line *b*.



$\sin = \frac{opp}{hyp}$

Write a ratio for sin*A*, and then for sin*C*. Then, solve each for *h*.

$c \times \sin A = \frac{h}{c} \times c$

$c \times \sin A = h$

$a \times \sin C = \frac{h}{a} \times a$

$a \times \sin C = h$

Since each ratio is equal to *h*, they must also equal one another.

$$h = h$$

$$c \sin A = a \sin C$$

$$\frac{\sin A}{a} = \frac{\sin C}{c} \quad \text{or} \quad \frac{a}{\sin A} = \frac{c}{\sin C}$$

By using similar steps, you can also show the same for *b* and sin*B*.

sine law

For any triangle, the sine law states that the sides of a triangle are proportional to the sines of the opposite angles:

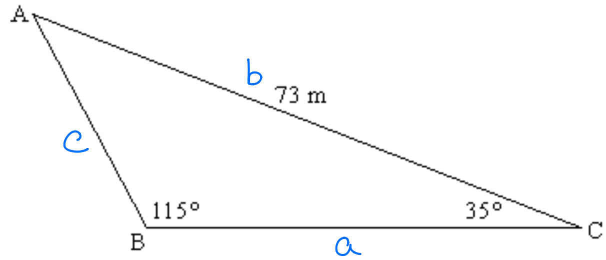
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad OR \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Tip: Round only at the end. Keep 3 or more decimals as you go along

solving a triangle

When **solving a triangle**, you must find all of the unknown angles and sides.

Example 1 – Solve the triangle (answer to the nearest tenth). 1 decimal



smallest	$A = 30°$	$a = 40.3\text{m}$
largest	$B = 115°$	$b = 73\text{m}$
medium	$C = 35°$	$c = 46.2\text{m}$

$180° - 115° - 35° = 30°$

$\boxed{a}$ looking for on top, must have all other measures

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\sin 30 \times \frac{a}{\sin 30} = \frac{73}{\sin 115} \times \sin 30$$

$$a = \frac{73 \times \sin 30}{\sin 115}$$ all in calc

$$a = 40.273$$

$\boxed{c}$

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

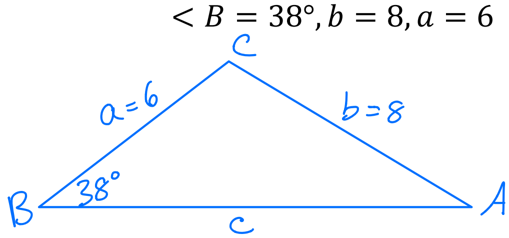
$$\sin 35 \times \frac{c}{\sin 35} = \frac{73}{\sin 115} \times \sin 35$$

$$c = \frac{73 \times \sin 35}{\sin 115}$$

$$c = 46.1996$$

Example 2 – Sketch and solve the triangle (round to the nearest whole number). no decimals

$$< B = 38°, b = 8, a = 6$$



smallest	$A = 27°$	$a = 6$
mid	$B = 38°$	$b = 8$
biggest	$C = 115°$	$c = 12$

Note: cannot use pythagoras since not right triangle

$\boxed{\angle A}$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$6 \times \frac{\sin A}{6} = \frac{\sin 38}{8} \times 6$$

$$\sin A = \frac{6 \times \sin 38}{8}$$

$\sin^{-1}$ $\sin^{-1}$

$$A = \sin^{-1}\left(\frac{6 \times \sin 38}{8}\right)$$

$$A = 27.4998$$

$\boxed{\angle C}$ $180° - 38° - 27.4998° = 114.50016$

$\boxed{c}$

$$\frac{c}{\sin C} = \frac{b}{\sin B}$$

$$\sin 114 \times \frac{c}{\sin 114.50016} = \frac{8}{\sin 38} \times \sin 114$$

$$c = \frac{8 \times \sin 114.50016}{\sin 38}$$

$$c = 11.824$$

information necessary to use the sine law

For oblique triangles, what is the minimum information needed in order to use the sine law to find new information?

Think of Angle A/B/C and side a/b/c as partners

To use the sin law you must know one Full set of partners at at least one half set.

8.6 – The Cosine Law

For right triangles, the trigonometric ratios sine, cosine, and tangent can be used to find unknown sides and angles. For oblique triangles, **sine law** and **cosine law** must be used.

An effective way to work with oblique triangles is to imagine the angle and its opposite side as 'partners'. Thus, angle *A* and side *a* are partners, <*B* and *b* are partners, and <*C* and *c* are partners.

all half set
at least 2 sides

In order to use the sine law, you must know one full set of partners and half of another set. If you know only half of each set of the three partners, at least two of which are sides, you must use **cosine law**.

Example 1 – For each oblique triangle, state which law you would use.

a) *x* =30cm, *y* =28cm, *z* =32cm

- no full sets
- 2+ sides
- cos law

b) <*C* =27°, *a* =17m, *c* =13m

- one full set
- one half set

sin law

c) <*J* =41°, *k* =16cm, *p* =14cm

- no full sets
- 2+ sides

cos law

d) <C =27°, <B =46°, a =120m + <A

2 angles can use 180 – to find 3rd

- secretly have one full set
- at least one half set

sin law

e) <A =35°, <B =50°, <C =95°

- no full sets
- no sides

If you have ONLY angles, then you don't have enough info to solve

cosine law

The **cosine law** describes the relationship between the cosine of an angle and the lengths of the three sides of any triangle.

angle across

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Cosine law can also be written as $a^2 = b^2 + c^2 - 2bc\cos A$ **OR**

$$b^2 = a^2 + c^2 - 2ac\cos B$$

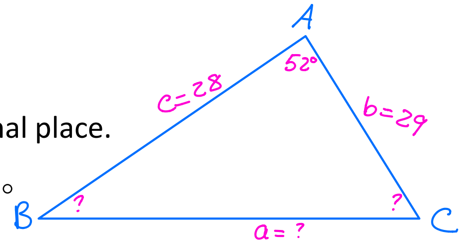
(deriving cosine law:

see p.299 of your workbook)

using
cosine law
& sine law

$A = 52°$	$a = 25.0$
$B = 66.1°$	$b = 29cm$
$C = 61.9°$	$c = 28cm$

Example 2 – Solve the triangle. Round answers to one decimal place.



need cos law

b = 29cm, *c* = 28cm, and <*A* = 52°

① find side a, use cos law

$\boxed{a}$ $a^2 = b^2 + c^2 - 2bc\cos A$

$a = \sqrt{29^2 + 28^2 - 2(29)(28)(\cos 52)}$

BEDMAS

$a = 25.0033$

② find ∠B, now can use sin law

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$29 \times \frac{\sin B}{29} = \frac{\sin 52}{25.0033} \times 29$$

$$B = \sin^{-1}\left(\frac{29 \times \sin 52}{25.0033}\right)$$

$$B = 66.1°$$

③ find ∠C

$180° - 52° - 66.1° =$

$\angle C = 61.9°$

Example 3 – Solve the triangle. Round answers to one decimal place.

a = 14m, *b* = 18m, *c* = 22m → cos law

It is also helpful to know what the cosine law looks like rearranged for an angle:

angle you're looking for

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$A = 39.4°$	$a = 14m$
$B = 54.7°$	$b = 18m$
$C = 85.9°$	$c = 22m$

① $\boxed{\angle C}$, cos law

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$\cos^{-1}$ $\cos^{-1}$

$$C = \cos^{-1}\left(\frac{(14^2 + 18^2 - 22^2)}{(2 \times 14 \times 18)}\right)$$

$$C = 85.9°$$

② ∠A sin law

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$14 \times \frac{\sin A}{14} = \frac{\sin 85.9}{22} \times 14$$

$$A = \sin^{-1}\left(\frac{14 \times \sin 85.9}{22}\right)$$

$$A = 39.4$$

③ ∠B

$180 - 85.9 - 39.4 = 54.7°$