PC12 Name______________________________

Exponents

Chapter Notes

Assignment List

Key

Date	Lesson	Assignment
	1. Exponent Laws	Exponent Rules Practice Handout and Mickelson Page 203 #1
	2. Solving Exponential Equations	Solving Exponential Equations Handout and Mickelson Page 203 #2 (Ans. 2b) $x = -2,1$)
	3. Graphing Exponential Functions	Mickelson Page 204 #4-7
	4. Applications of Exponential Functions	Mickelson Page 206 #9
		Practice Test
		Review
		Exponents Test

Exponents Day 1: Exponent Laws

Review of Exponent Laws

	Law:	**Example(s):**		
"Anything to the power of 0 is 1"	$x^0 = 1$	$5^0 = 1$	base exponent $(-124)^0 = 1$	base=6 exp. $-6^0 = -1$
Product Rule	add exponents $x^m \cdot x^n = x^{m+n}$	$x^3 \cdot x^2 = x^{3+2} = x^5$		
Quotient Rule	subtract exp. $\frac{x^m}{x^n} = x^{m-n}$	$\frac{y^6}{y^2} = y^{6-2} = y^4$		
"Power of a power"	multiply exp. $(x^m)^n = x^{mn}$	$(b^5)^2 = b^{(5)(2)} = b^{10}$		
"Power of a product"	$(xy)^m = x^m y^m$	$(-3x)^4 = (-3)^4(x)^4 = 81x^4$		
"power of a quotient"	$\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$	$\left(\frac{x}{6}\right)^2 = \frac{x^2}{6^2} = \frac{x^2}{36}$		
Negative exponents "flip base"	$x^{-m} = \frac{1}{x^m}$	$5^{-1} = \frac{1}{5^1} = \frac{1}{5}$	$(-3)^{-2} = \frac{1}{(-3)^2} = \frac{1}{9}$	
	$\left(\frac{x}{y}\right)^{-m} = \left(\frac{y}{x}\right)^m$	$\left(\frac{1}{2}\right)^{-1} = \left(\frac{2}{1}\right)^1 = 2$	$\left(\frac{5}{7}\right)^{-2} = \left(\frac{7}{5}\right)^2 = \frac{7^2}{5^2} = \frac{49}{25}$	
Rational Exponents: bottom of fraction ⇒ index of root	$(x)^{\frac{1}{n}} = \sqrt[n]{x}$	$(x)^{\frac{1}{2}} = \sqrt[2]{x} = \sqrt{x}$	$(-27)^{\frac{1}{3}} = \sqrt[3]{-27} = -3$	
	$(x)^{\frac{m}{n}} = \sqrt[n]{x}^{\,m}$ or $\sqrt[n]{x^m}$ whichever is easier ☺	$(x)^{\frac{3}{4}} = \sqrt[4]{x^3}$ or $\sqrt[4]{x}^{\,3}$	$(9)^{\frac{3}{2}} = \left(\sqrt[2]{9}\right)^3 = (3)^3 = 27$	

Warning!

Note: $(x \pm y)^m \xcancel{\neq} x^m \pm y^m$

Common error: $(x+3)^2 \neq x^2 + 9$

Instead, write bracket twice ☺

$(x+3)^2 = (x+3)(x+3)$

$= x^2 + 6x + 9$ ✓

easier to see FOIL!

Simplifying Exponential Expressions using Exponent Laws

Example 1: Simplify $$\frac{(-4a^{-5}b^{3})^{-2}}{a^{-1}b^{2}}$$

$$= \frac{(-4)^{-2}(a^{-5})^{-2}(b^{3})^{-2}}{a^{-1}\ b^{2}}$$

$(a^{-5})^{-2} = a^{(-5)(-2)} = a^{10}$

$(b^{3})^{-2} = b^{(3)(-2)} = b^{-6}$

$$= \frac{(-4)^{-2}\ a^{10}\ b^{-6}}{a^{-1}\ b^{2}}$$

a^{10--1}

b^{-6-2}

$$= (-4)^{-2}\ a^{11}\ b^{-8}$$

$(-4)^{-2} = \frac{1}{(-4)^{2}}$

$$= \frac{a^{11}}{(-4)^{2}\ b^{8}}$$

$$= \boxed{\frac{a^{11}}{16b^{8}}}$$

Simplifying Exponential Expressions by first Finding a Common Base

bases are different .. no exponent laws ... yet!

Example 2: Simplify $$4^{2x}\cdot 8^{3-x}$$

multiply

multiply (to whole bracket)

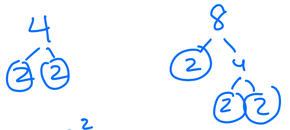
$$= (2^{2})^{2x}\cdot(2^{3})^{(3-x)}$$

$$= 2^{4x}\cdot 2^{9-3x}$$

$$= 2^{4x\ +\ 9-3x}$$

$$= \boxed{2^{x+9}}$$

These are both base 2:



$4=2^{2}$

$8=2^{3}$

Example 3: Simplify $$\frac{(9^{x-3})^{3}\,(3^{2x})}{27^{x-5}}$$

$$= \frac{(3^{2})^{(3x-9)}\ (3)^{2x}}{(3^{3})^{(x-5)}}$$

add

$$= \frac{3^{6x-18}\ 3^{2x}}{3^{3x-15}}$$

$$= \frac{3^{6x-18\ +\ 2x}}{3^{3x-15}}$$

subtract

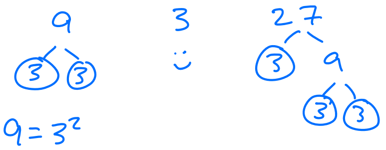
$$= \frac{3^{8x-18}}{3^{3x-15}}$$

$$= 3^{(8x-18)-(3x-15)}$$

$$= 3^{8x-18-3x+15}$$

$$= \boxed{3^{5x-3}}$$

These are all base 3:



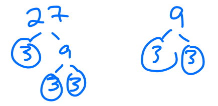
$9=3^{2}$

$27=3^{3}$

Example 4: Simplify

$$\frac{27^{2x+1}\cdot 9^{1-3x}}{\left(\frac{1}{3}\right)^{3x-2}}$$

These are all base 3:



$27 = 3^3$ $9 = 3^2$ $\frac{1}{3} = 3^{-1}$

$$= \frac{(3^3)^{(2x+1)}\cdot(3^2)^{(1-3x)}}{(3^{-1})^{(3x-2)}}$$

$$= \frac{3^{6x+3}\cdot 3^{2-6x}}{3^{-3x+2}}$$

} add

$$= \frac{3^{6x+3+2-6x}}{3^{-3x+2}}$$

$$= \frac{3^5}{3^{-3x+2}}$$

subtract

$$= 3^{5-(-3x+2)}$$

$5-(-3x+2) = 5+3x-2 = 3x+3$

$$= \boxed{3^{3x+3}}$$

Example 5: Simplify

$$\frac{3x^5\cdot 2^{4x} + 6x^4\cdot 2^{4x}}{x^2+2x}$$

Warning! No rules for add/subtract... what else can we try?

Notice: x^2+2x
we can factor! $x(x+2)$

Try looking for other common factors!

$$= \frac{3x^5\cdot 2^{4x} + 6x^4\cdot 2^{4x}}{x^2+2x}$$

common 2^{4x}

gcf: $3x^4$

$$= \frac{2^{4x}(3x^5+6x^4)}{x^2+2x}$$

gcf: x

$$= \frac{2^{4x}\cdot 3x^4(x+2)}{x\,(x+2)}$$

Reduce!

$\frac{x^4}{x} = x^3$

$$= \frac{2^{4x}\cdot 3x^{\cancel{4}3}\cancel{(x+2)}}{\cancel{x}\,\cancel{(x+2)}}$$

$$= \boxed{2^{4x}\cdot 3x^3}$$

Assignment:

Exponent Rules Practice Handout and Mickelson Page 203 #1

Exponents Day 2: Solving Exponential Equations

Solving Exponential Equations

To solve an exponential equation, use the same principles of simplifying expressions to get a common base on either side of the equation. If the bases on either side are equal, then the exponents must also be equivalent.

This allows us to use $x^m = x^n \iff m = n, \quad when\ x \neq -1,\ 0,\ 1$

Need "single power" on each side, with same base.

Example 1: Solve $4^{3-x} = 8^{x+1}$

Steps:

① get same base

② simplify each side so you have single power = single power

③ "compare exponents"

④ solve

① $(2^2)^{3-x} = (2^3)^{x+1}$

② $2^{6-2x} = 2^{3x+3}$

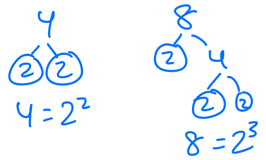
③ $\therefore\ 6-2x = 3x+3$

$\ {-3}\ \ {+2x} \qquad {+2x}\ \ {-3}$

④ $\frac{3}{5} = \frac{5x}{5}$

$$\boxed{3/5 = x}$$

common base: 2



$4 = 2^2$

$8 = 2^3$

Example 2: Solve $\dfrac{(3^{2x})^3(9^{x-1})^2}{(27^{x-3})^4} = 81^5$

① $\dfrac{(3)^{6x}(3^2)^{2x-2}}{(3^3)^{4x-12}} = (3^4)^5$

② $\dfrac{3^{6x} \cdot 3^{4x-4}}{3^{12x-36}} = 3^{20}$

add

$\dfrac{3^{10x-4}}{3^{12x-36}} = 3^{20}$

subtract

$3^{-2x+32} = 3^{20}$

③ $\therefore\ -2x+32 = 20$

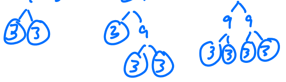
$\ {-32} \qquad {-32}$

④ $\frac{-2x}{-2} = \frac{-12}{-2}$

$$\boxed{x = 6}$$

common base: 3

③ $9 = 3^2$ $\quad 27 = 3^3$ $\quad 81 = 3^4$



Careful! We do know how to solve √ equations... but we have to remember to CHECK solutions.

Example 3: Solve $9^{\sqrt{x-2}} = 3^{x-2}$

common base: 3

① $(3^2)^{\sqrt{x-2}} = 3^{x-2}$

② $3^{2\sqrt{x-2}} = 3^{x-2}$

③ ∴ $2\sqrt{x-2} = x-2$

④ Solve: ( )^2 both sides

$(x-2)^2=(x-2)(x-2)$

$\sqrt{(x-2)}^2=(x-2)$

$(2\sqrt{x-2})^2 = (x-2)^2$

$2^2\cdot\sqrt{(x-2)}^2 = (x-2)(x-2)$

$4(x-2) = x^2-2x-2x+4$

$4x-8 = x^2-4x+4$

$-4x \quad +8 \qquad -4x \quad +8$

$0 = x^2-8x+12$

$0 = (x-6)(x-2)$

∴ $x=6$ or $x=2$

Check!

$x=6$

$9^{\sqrt{x-2}} = 3^{x-2}$

$9^{\sqrt{6-2}}$	3^{6-2}
$9^{\sqrt{4}}$	3^4
9^2	81
81	
LS=	RS

✓

$x=2$

$9^{\sqrt{x-2}} = 3^{x-2}$

$9^{\sqrt{2-2}}$	$=3^{2-2}$
$9^{\sqrt{0}}$	3^0
9^0	1
1	
LS=	RS

✓

$\boxed{\text{Final answer: } x=6,\ x=2}$

Extension Example: Solve $5^{-|x+3|} = \frac{1}{25}$

common base: 5

$\frac{1}{25} = \frac{1}{5^2} = 5^{-2}$

①② $5^{-|x+3|} = 5^{-2}$

③ ∴ $-|x+3| = -2$ (divide both sides by -1)

④ ∴ $|x+3| = 2$

To solve absolute value equations, consider both + and − cases

or

$x+3 = +2$

$-3 \qquad -3$

$x+3 = -2$

$-3 \qquad -3$

$\boxed{x=-1 \quad \text{or} \quad x=-5}$

Assignment: Solving Exponential Equations Handout and

Mickelson Page 203 #2abcef (Ans. 2b) $x = -2,1$) Extension : 2d

Exponents Day 3: Graphing Exponential Functions

Exponential Functions
An exponential function is any function that has a variable in the exponent and a positive base not equal to zero.

$$y = b^x \qquad \text{where } b > 0 \text{ , } b \neq 1$$

Exploration:

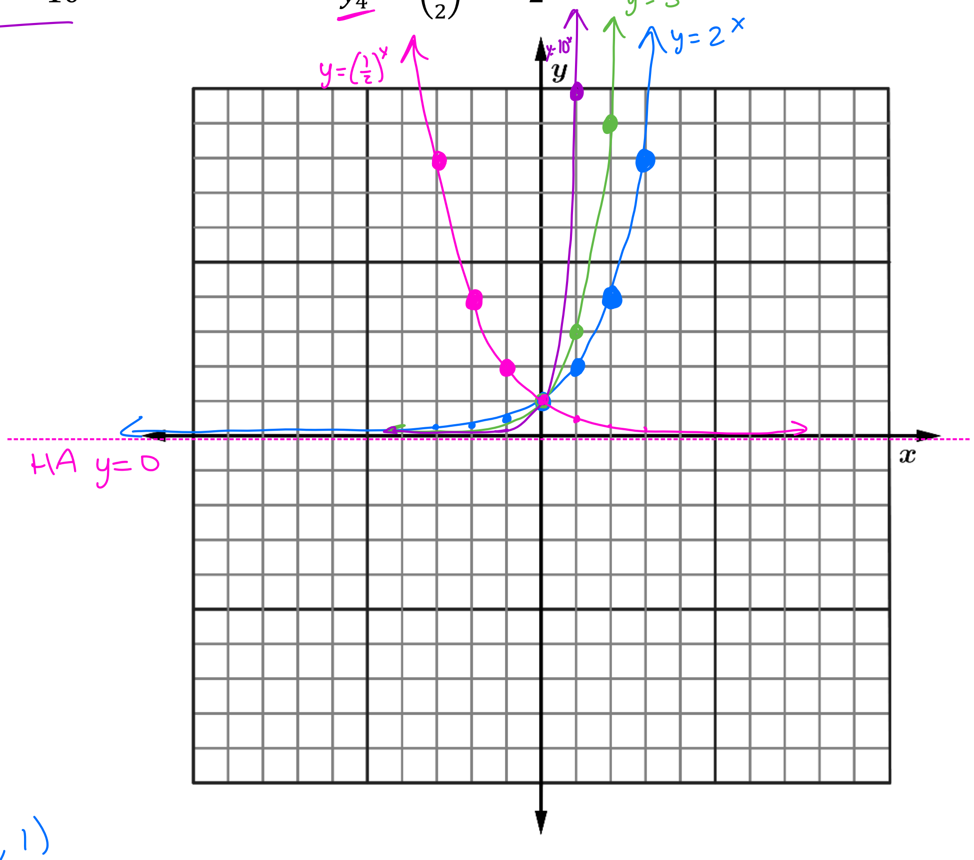
a) Use a table of values to graph $y_1 = 2^x$

b) Use graphing technology to graph the other functions (on the same grid) and determine the basic properties of any exponential function

base b > 1 $\qquad$ ***base*** $0 < b < 1$

$y_1 = 2^x$, $\quad y_2 = 3^x$, $\quad y_3 = 10^x$ $\qquad y_4 = \left(\frac{1}{2}\right)^x = 2^{-x}$

x	y	$= 2^{(x)}$
-3	$1/8$	$= 2^{(-3)} = \frac{1}{2^3} = \frac{1}{8}$
-2	$1/4$	$= 2^{(-2)} = \frac{1}{2^2} = \frac{1}{4}$
-1	$1/2$	$= 2^{-1} = \frac{1}{2}$
0	1	$= 2^0$
1	2	$= 2^1$
2	4	$= 2^2$
3	8	$= 2^3$



Common features:

All pass through the point: $(0, 1)$

Horizontal Asymptote: $y = 0$

Domain: $\{x \in \mathbb{R}\}$

Range: $\{y \in \mathbb{R} \mid y > 0$

Key Points: $(-1, 1/b)$

$(0, 1)$

$(1, b)$

HA $y = 0$

$y = b^{x} \rightsquigarrow y = a \cdot b^{x-h} + k$

Transformations of Exponential Functions

Example 1: Without technology, graph the following. State the domain, range, intercept(s) and asymptotes.

$h = -3$

$y = 2^{x+3} - 3$

$k = -3$

base is 2

$-3 \quad -3$

k.p.

$(-1, 1/b) \Rightarrow (-1, 1/2) \rightarrow (-4, -2.5)$

$(0,1) \Rightarrow (0,1) \rightarrow (-3, -2)$

$(1,b) \Rightarrow (1,2) \rightarrow (-2, -1)$

HA $y=0 \quad y=0 \rightarrow y=-3$

y-int (set $x=0$)

$y = 2^{x+3} - 3$

$y = 2^{(0+3)} - 3$

$y = 2^{3} - 3 = 8 - 3 = 5$

plot $(0,5)$

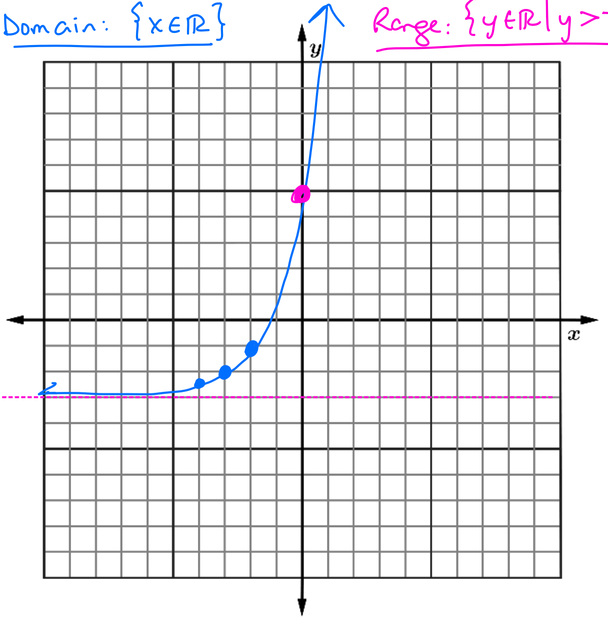
x-int can approximate until we learn logs!

$\approx (-1.2, 0)$

HA $y=-3$

Domain: $\{x \in \mathbb{R}\}$

Range: $\{y \in \mathbb{R} \mid y > -3\}$



Example 2: Without technology, graph the following. State the domain, range, intercept(s) and asymptotes.

$a = -1$

"no h"

$y = -\left(\frac{1}{2}\right)^{x} - 1$

$k = -1$

base is $\frac{1}{2}$

note: $\frac{1}{b} = \frac{1}{1/2} = 1 \times \frac{2}{1} = 2$

$\rightarrow \cdot(-1) \quad \rightarrow -1$

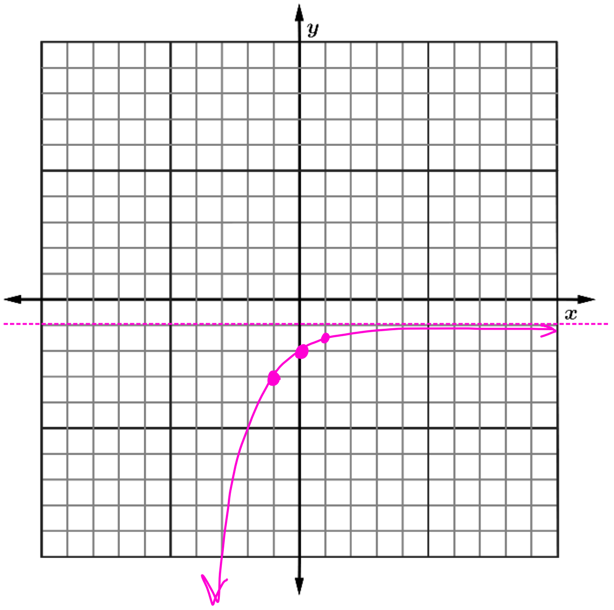
k.p.

$(-1, 1/b) \Rightarrow (-1, 2) \rightarrow (-1, -2) \rightarrow (-1, -3)$

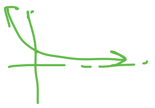
$(0,1) \Rightarrow (0,1) \rightarrow (0, -1) \rightarrow (0, -2)$

$(1,b) \Rightarrow (1, \frac{1}{2}) \rightarrow (1, -1/2) \rightarrow (1, -1.5)$

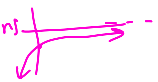
HA $y=0 \quad y=0 \rightarrow y=0 \rightarrow y=-1$



basic Shape for $\left(\frac{1}{2}\right)^{x}$ is



Vertical Reflection $(a=-1)$ means



Domain: $\{x \in \mathbb{R}\}$

Range: $\{y \in \mathbb{R} \mid y < -1\}$

x-int: none

y-int: $(0, -2)$

HA: $y = -1$

Assignment: Mickelson Page 204 #4-7

Exponents Day 4: Applications of Exponential Functions

Applications of Exponential Functions

There are a variety of situations where exponential functions can be applied to solve real life problems. Though these may seem like unique situations, they are all just variations of the basic exponential $y = a(b)^x$

$$\text{"End} = \text{Start}(\text{rate})^{\text{time}}\text{"}$$

Compound Interest: Interest calculated on principal invested and then added to the original investment

principal → original amount

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

where A = final Amount

P = Principal

r = rate as a decimal (per year)

n = number of compounding periods (per year)

t = time (in years)

/a = "per annum" = per year

Example 1: Find the interest earned if \$2500 is invested at 8.5%/a compounded semi-annually for 4 years

A = ?

P = 2500

r = 0.085

n = 2

t = 4

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$= 2500\left(1+\frac{0.085}{2}\right)^{(2)(4)}$$

$$= 2500(1.0425)^8$$

$$= 3487.77546$$

The final Amount will be \$ 3487.78.

★ Remember units

★ Round money to 2 decimal places

★ Re-read question.

$I = A - P$

$= 3487.78 - 2500$

$= 987.78$

$$\boxed{\text{The interest earned will be \$ 987.78.}}$$

Earthquakes/pH

The Richter scale and the pH scale are power of 10 scales, meaning in $y = b^x$, the base is 10.

Example 2: How many times more powerful is an earthquake measuring $\overset{E_1}{8.4}$ on the Richter scale compared to an earthquake measuring $\underset{E_2}{4.2}$?

"8.4" means $10^{8.4}$

so $\frac{E_1}{E_2} = \frac{10^{8.4}}{10^{4.2}} = 10^{8.4-4.2} = 10^{4.2} = 15\,848.9$

$\boxed{\text{It is 15 849 times more powerful.}}$

Growth and Decay

Discrete Growth/Decay –growth/decay occurs at a specified rate over a specific time period.

7.5 growth rate → r = 7.5

15% depreciation → r = 0.85 (100% - 15%)

5% increase → r = 1.05 (100% + 5%)

population doubles → r = 2,

population triples → r = 3,

radioactive half-life decay → r = $\frac{1}{2}$

$$F = I(r)^{\frac{t}{p}}$$

F = Final amount
I = initial amount
r = rate of growth/decay
t = total time
p = period of growth/decay

e.g. how long before something "doubles" or "halves"

Example 3: A photocopier is set to reduce an image by 8%. What is the copy size after 4 reductions?

100% - 8% = 92%

F = ?
I = 100%
r = 0.92
t = 4
p = 1

$F = I(r)^{t/p}$

$F = 100(0.92)^{4/1}$

$= 100(0.92)^4$

$= 71.639$

$\boxed{\text{The copy size will be 71.6\% of the original.}}$

Continuous Growth/Decay – growth/decay occurs at a continuously over time:

E.g. bacteria and plant growth, human population growth, radiation absorption and decay

$F = Ie^{rt}$

F = Final amount
I = initial amount
r = continuous rate of growth/decay per unit time*
t = total time

*Note: for continuous growth/decay rate: do not add to or subtract from 1. Increase is a positive r, and decrease is a negative r.

A note about *e*

$\boxed{e}$ is an important constant (like $\boxed{\pi}$!) that is useful in exponential functions.

$\boxed{e^x}$ you will find it on your calculator, usually as the base with a mystery exponent.

Try it! e^1 or e^1 = ? $e \simeq 2.71828$

Example 4: A bacterial population of 3000 grows at a rate of 2.3% per minute. How many bacteria exist after 4 hours?

continuously

F = ?
I = 3000
r = 0.023
t = 240

(t = 4 hours × $\frac{60\text{min}}{\text{hr}}$ = 240 minutes)
↑
need same units of ti

$$F = 3000(e)^{(0.023)(240)}$$
$$= 3000(e^{5.52})$$
$$= 748\,905.1116$$

★ Round to whole numbers for populations.

After 4 hours, there will be 748 905 bacteria.

Assignment: p. 206 #9