PC12 Name____________________________

Logarithms

Chapter Notes

Assignment List Key

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	1. Graphing Logarithmic Functions	Mickelson Page 213 #6, 7, 8, 9
	2. Logarithms	Mickelson Page 211 #1-4 (left side), 5, 13
	3. Log Laws	Mickelson Page 221 #1-3 (left side)
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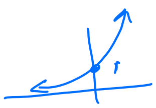
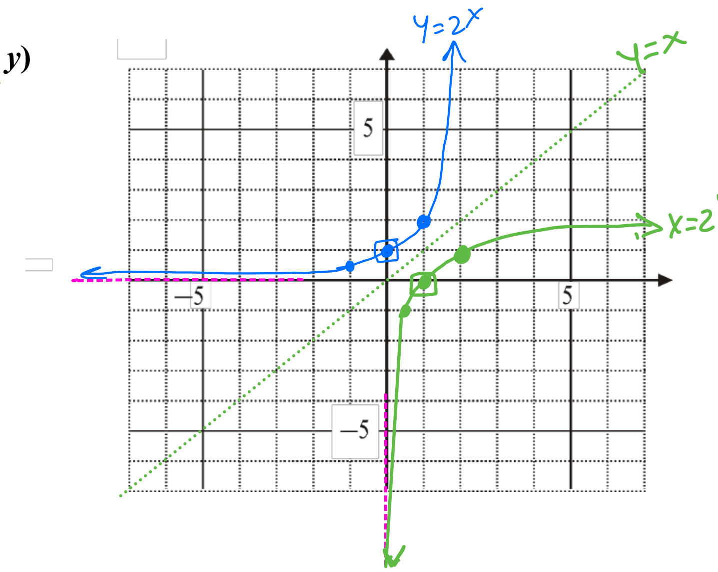
Logarithms Day 1: Graphing Logarithmic Functions

Graphing the Inverse of an Exponential Function

(switch x and y)

Use the key points for the Exponential function to determine the key points and graph for its inverse:

When $b > 1$ 



Exponential	**Inverse (switch x and y)**
$y = 2^x$ ($b=2$)	$\boxed{x = 2^y}$
Key Points:	
$(-1, \tfrac{1}{2})$	$(\tfrac{1}{2}, -1)$
$(0, 1)$	$(1, 0)$
$(1, 2)$	$(2, 1)$
HA $y=0$	VA $x=0$
Domain $x \in \mathbb{R}$	Domain $x > 0$
Range $y > 0$	Range $y \in \mathbb{R}$

switch!

But what do we call this new function?

Logarithms

The inverse of an exponential function is another function called a **logarithm.**

> Greek:
> 'logos'– word/speech/logic
> 'arithmos' – numbers

Exponential	**Inverse** (switch x and y, exponential form)	**Logarithmic form** (solve for y – call it a "log")
$y = b^x$	$x = b^y$	$y = \log_b x$

"answer" (x) — base (b) — how to get "y" out of exponent? ⇒ — exponent (y) — "answer" (x) — base b

Note: $b > 0,\ b \neq 1$

Formal Definition: A **logarithm** of a number is the exponent (y) to which a fixed value (b) must be raised in order to get that number (x)

So for the inverse graph above, $x = 2^y$ can be written as $y = \log_2 x$

Transformations of Logarithms

A logarithm, like any other function, can be transformed using the principles associated with transforming a function. The transformation will always be in relation to the basic graph, $y = \log_b x$ which has the following key points and basic shape:

$y = \log_b x$

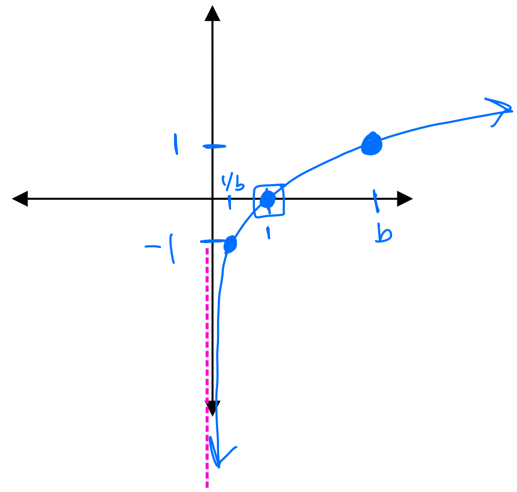
Key Points:

(1/b , −1)

(1 , 0)

(b , 1)

VA x = 0



Example 1: Sketch the function

$$y = -\log_3(x-2)$$

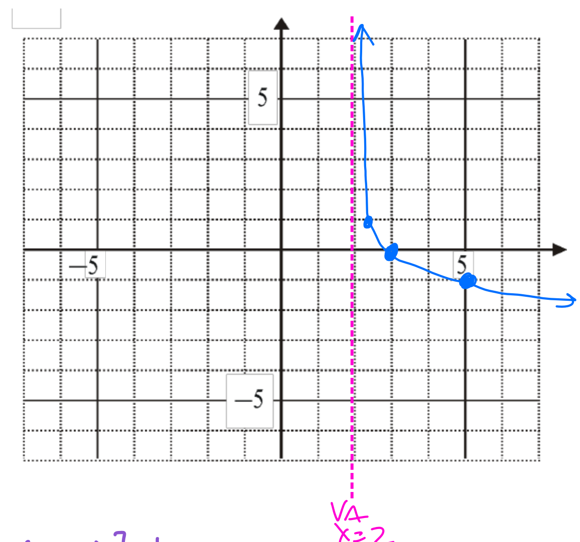
a = −1 (vertical reflection)

h = 2 (horiz. transl. +2)

basic graph:
y = log₃ x

k.p. → x(−1) → +2 →

(1/3, −1) →	(1/3, 1) →	(2 1/3, 1)
(1, 0) →	(1, 0) →	(3, 0)
(3, 1) →	(3, −1) →	(5, −1)
VA x = 0 →	x = 0 →	x = 2



Example 2: Sketch the function

$y = \log_2(2-x) + 1$ * rearrange into y = af[b(x−h)]+k

$y = \log_2[-(x-2)] + 1$ ← up 1

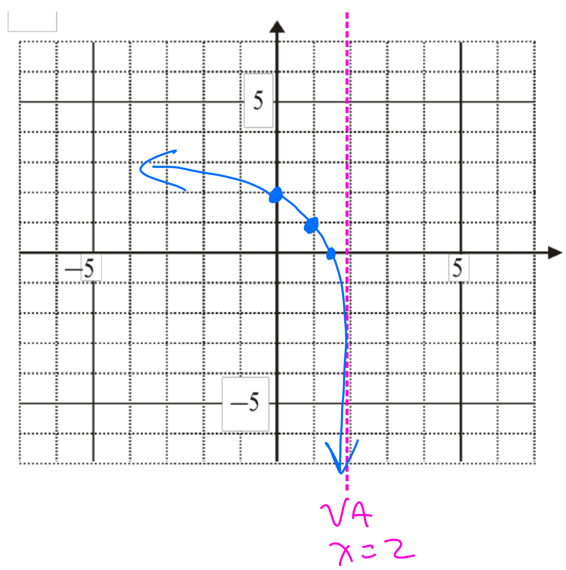
horiz. reflection

2 right

basic graph:
y = log₂ x

k.p. x(−1) → +2 +1

(1/2, −1) →	(−1/2, −1) →	(1.5, 0)
(1, 0) →	(−1, 0) →	(1, 1)
(2, 1) →	(−2, 1) →	(0, 2)
VA x = 0 →	x = 0 →	x = 2



Assignment: p. 213 #6, 7, 8, 9

Logarithms Day 2: Logarithms

Recall:

The inverse of an exponential function is another function called a **logarithm.**

Greek:
'logos'– word/speech/logic
'arithmos' – numbers

Inverse in exponential form | Logarithmic form

$$x = b^y \quad \Leftrightarrow \quad y = \log_b x$$

Note: $b > 0, \; b \neq 1$

Formal Definition: A **logarithm** of a number (x) is the exponent (y) to which a fixed value (b) must be raised in order to get that number (x)

Example 1: Change from exponential to logarithmic form

i) $3^3 = 27$ exp b "answer" $3 = \log_3(27)$

ii) $10^4 = 10000$ $4 = \log_{10} 10000$

Note: when it's base 10, we don't always write the base!

Try p.211 #1-2

Example 2: Determine the numerical value

Rewrite in exponential form... then use "change of base" to solve, or just evaluate.

i) $\log 1000 = y$

"default" base 10

$10^y = 1000$
$10^y = 10^3$
$\boxed{y = 3}$

ii) $\cancel{f(x)}\, y = \log_{1/3} 27$

$\left(\frac{1}{3}\right)^y = 27$
$(3^{-1})^y = 3^3$
$3^{-y} = 3^3$ $\therefore -y = 3$ $\therefore \boxed{y = -3}$

iii) $\log_6 x = 3$

$6^3 = x$
$\boxed{216 = x}$

iv) $\log_7 (x+2) = 3$

$7^3 = (x+2)$
$343 = x + 2$
$-2 \qquad -2$
$\boxed{341 = x}$

v) $\log_{17}(17^{381}) = y$

$17^y = 17^{381}$
$\therefore \boxed{y = 381}$

Try p.211 #3-4

Logarithmic Domains

Since the inverse of an exponential function is a logarithmic function, the domain of a logarithmic function is the range of the corresponding exponential function.

$y = \log_b \boxed{x}$ $\{x \in R | \boxed{x} > 0\}$ Remember, the base is: $b > 0,\ b \neq 1$

Example 3: Determine the domain of the following:

i) $y = -\log_3(\boxed{x-2})$

$\boxed{x-2} > 0$

$x > 2$

ii) $y = \log_{x+2}(\boxed{x-1})$

$\boxed{x-1} > 0$

$x > 1$

$b > 0$

$x+2 > 0$

$x > -2$

$b \neq 1$

$x+2 \neq 1$

$x \neq -1$

$x > 1$ "covers" all there restrictions

Try p.212 #5

Inverse Log Functions

Finding the inverse of a log function always refers to the relationship between logarithms and exponential functions.

$$\log_b x = y \quad \Leftrightarrow \quad b^y = x$$

Steps to find the inverse:

1. Reverse $x \leftrightarrow y$
2. Isolate the power or the logarithm
3. Switch: exponential form $\leftrightarrow$ log form
4. Solve for y

Example 4: Determine the inverse of the following

i) $f(x) = 6^{3x+2} - 4$

$y = 6^{3x+2} - 4$

inv: $x = 6^{3y+2} - 4$

isolate power: $x + 4 = 6^{3y+2}$

rewrite: $\log_6(x+4) = 3y+2$

solve for y: $y = \dfrac{\log_6(x+4) - 2}{3}$

ii) $y - 3 = \log_6(2x - 5)$

inv: $x - 3 = \log_6(2y-5)$ (log already alone)

rewrite: $6^{x-3} = 2y - 5$

solve for y: $\dfrac{6^{x-3} + 5}{2} = y$

Try p.214 #13

Assignment ~~p.211 #1-5, 13~~

p.211 #1-4 (left), 5, 13

Logarithms Day 3: Log Laws

A logarithm is just the inverse of an exponential function.

Just like there are established and proven rules for exponents, there are established and provable rules for logarithms.

Inverse property:

- $\log_b b^x = x$
- $b^{\log_b x} = x$

Rules for Logarithms (The Log Laws) **Note: $b > 0,\ b \neq 1$**

- $\log_b 1 = 0$

$b^0 = 1$

- $\log_b b = 1$

$b^1 = b$

Product Rule: $\log_b xy = \log_b x + \log_b y$

$a = \log_b x$

$x = b^a$

$c = \log_b y$

$y = b^c$

$xy = b^a \cdot b^c$

$xy = b^{a+c}$

$\log_b xy = a + c$

$\log_b xy = \log_b x + \log_b y$

Example: Simplify $\log_2 5 + \log_2 7$

$\log_2 (5 \cdot 7)$

$\boxed{\log_2 35}$

Quotient Rule: $\log_b \frac{x}{y} = \log_b x - \log_b y$

$a = \log_b x$

$x = b^a$

$c = \log_b y$

$y = b^c$

$\frac{x}{y} = \frac{b^a}{b^c}$

$\frac{x}{y} = b^{a-c}$

$\log_b \frac{x}{y} = a - c$

$\log_b \frac{x}{y} = \log_b x - \log_b y$

Example: Simplify $\log_2 24 - \log_2 8$

$\log_2 \frac{24}{8}$

$\boxed{\log_2 3}$

Power Rule: $\log_b x^n = n \cdot \log_b x$

$a = \log_b x$

$x = b^a$

$(b^a)^n = x^n$

$b^{an} = x^n$

$an = \log_b x^n$

$n \log_b x = \log_b x^n$

Example: Simplify $\log_2 16$

$\log_2 2^4$

$4 \log_2 2$

$4(1)$

4

Change of Base: $\log_b a = \frac{\log_c a}{\log_c b}$

$y = \log_b a$

$a = b^y$

$\log_x a = \log_x b^y$

$\log_x a = y \log_x b$

$y = \frac{\log_x a}{\log_x b}$

$\log_b a = \frac{\log_x a}{\log_x b}$

Example: Find $\log_3 7$ to 3 decimal places

$\log_3 7 = \frac{\log 7}{\log 3}$

$= 1.771$

Example 1: Write $\log \frac{25}{9}$ in terms of $\log 3$ and $\log 5$

$\log 25 - \log 9$

$\log 5^2 - \log 3^2$

$\boxed{2\log 5 - 2\log 3}$

Example 2: Find the exact value of $\log_3 27\sqrt{3}$

$\log_3 27 + \log_3 \sqrt{3}$

$\log_3 3^3 + \log_3 3^{1/2}$

$3\log_3 3 + \frac{1}{2}\log_3 3$

$3(1) + \frac{1}{2}(1)$

$3 + \frac{1}{2}$

$7/2$

Example 3: Find the exact value of $\log_4 64^3$

$\log_4 (4^3)^3$

$\log_4 4^9$

$9\log_4 4$

$9(1)$

9

Example 4: Find the exact value of $\log_{\frac{1}{4}} \frac{16^3}{2^{-3}}$

$\log_{\frac{1}{4}} \frac{2^{4\cdot 3}}{2^{-3}}$

$\log_{\frac{1}{4}} \frac{2^{12}}{2^{-3}}$

$\log_{\frac{1}{4}} 2^{15}$

$\frac{\log 2^{15}}{\log 1/4}$

$\frac{15\log 2}{\log 2^{-2}}$

$\frac{15\cancel{\log 2}}{-2\cancel{\log 2}} = -\frac{15}{2}$

Example 5: Expand the following $\log_7 x^2 y \sqrt[3]{z}$

$\log_7 x^2 + \log_7 y + \log_7 \sqrt[3]{z}$

$2\log_7 x + \log_7 y + \log_7 z^{1/3}$

$\boxed{2\log_7 x + \log_7 y + \frac{1}{3}\log_7 z}$

Assignment p. 221 #1-3 (left side)

Logarithms Day 4: Simplifying Logarithms

Simplifying Logarithmic Functions:

1. Understand rules #1-6 from yesterday.

2. Do not make up your own rules for logarithms. Common mistakes:

$log(A+B) \neq \log A + \log B$ $\frac{log\ a}{log\ b} \neq \log a - \log b$ $(log\ x)^2 \neq 2\log x$

3. Know how to change from exponential form to logarithmic form, and vice versa.

$$y = log_b\, x \quad \Leftrightarrow \quad b^y = x$$

4. Look for exponential/power relationships between b and x in $\log_b x$.

Example 1: Simplify $3\log_2 x - 2\log_2 3$

$\log_2 x^3 - \log_2 3^2$

$\boxed{\log_2 \frac{x^3}{9}}$

Example 2: Simplify $7^{2log_7 5}$

$7^{\log_7 5^2}$

$= 5^2$

$= 25$

Example 3: Simplify $\frac{\log x^4 + \log x^5}{\log x^7 - \log x^4}$

$\frac{\log x^4 \cdot x^5}{\log \frac{x^7}{x^4}}$

$\frac{\log x^9}{\log x^3}$

$\frac{9 \log x}{3 \log x}$

$\frac{9}{3}$

3

Example 4: Simplify $\frac{3}{2}\log 9x^4 - \frac{1}{3}\log y^6$

$\log (9x^4)^{3/2} - \log (y^6)^{1/3}$

$\log (3^2 x^4)^{3/2} - \log y^2$

$\log 3^3 x^6 - \log y^2$

$\log \frac{3^3 x^6}{y^2}$

$\log \frac{27x^6}{y^2}$

Example 5: Simplify $\frac{1}{\log_a x} - \frac{1}{\log_b x}$

$$\frac{\log_a a}{\log_a x} - \frac{\log_b b}{\log_b x}$$

$$\log_x a - \log_x b$$

$$\log_x\left(\frac{a}{b}\right)$$

Example 6: Simplify $(\log_3 10)\underbrace{(\log 45 - \log 5)}$ no law!

$$= (\log_3 10)\left(\log \frac{45}{5}\right)$$

$$= (\log_3 10)(\log_{10} 9) \quad \text{base 10!}$$

$$= \frac{\log 10}{\log 3} \cdot \log 3^2$$

$$= \frac{\log 10}{\cancel{\log 3}} \cdot (2 \cancel{\log 3})$$

$$= 2 \log 10 \qquad \log_{10} 10 = 1$$

$$= 2$$

Example 7: If $a = \log 5$ and $b = \log 3$, what is $\log_3 45$ in terms of a and b?

★ notice bases are different

$$\log_3 45 = \frac{\log(45)}{\log(3)}$$

$$= \frac{\log(9 \cdot 5)}{\log 3}$$

$$= \frac{\log(3^2 \cdot 5)}{\log(3)}$$

$$= \frac{\log 3^2 + \log 5}{\log 3}$$

$$= \frac{2 \log 3 + \log 5}{\log 3}$$

$\log 3 = b$

$\log 5 = a$

$$= \boxed{\frac{2b + a}{b}}$$

Assignment p. 222 #4, 5 and p. 228 #2a-d (Extension: p. 228 #2e-i)

(left side)

Logarithms Day 5: Solving Equations I

Using the rules we have established for logarithmic functions, we can now start to solve logarithmic equations. Remember, an equation has an equal sign, so you can "find x".

Be careful of logarithmic equations, though: the solutions must be a part of the domain of that function.

✓ check at end!

$$y = \log_b x \qquad \text{Note: } b > 0,\ b \neq 1,\ x > 0$$

Steps for Solving Logarithmic Equations:

If NO constant exists in the equation	**If a constant exists in the equation**
1. Combine logs on each side into single logs with a common base. **Aim for: single log = single log**	1. Bring all the logs to one side and combine into a single log. The constant(s) are on the other side of the equation. **Aim for: single log = constant**
2. Compare the insides of the logs using the rule $\log_b x = \log_b y \iff x = y$	2. Rewrite in exponential form using $\log_b x = y \iff b^y = x$
3. Solve the resulting equation for the unknown variable.	3. Solve the resulting equation for the unknown variable.
4. Reject any extraneous root(s) using: $y = \log_b x$; $b > 0,\ b \neq 1$ and $x > 0$	4. Reject any extraneous root(s) using: $y = \log_b x$; $b > 0,\ b \neq 1$ and $x > 0$

constant in eq'n.

Example 1: Solve for x: $\log_9(x-5) = 1 - \log_9(x+3)$

① $\log_9(x-5) + \log_9(x+3) = 1$

$\log_9[(x-5)(x+3)] = 1$

② $9^1 = (x-5)(x+3)$

③ $9 = x^2+3x-5x-15$

$0 = x^2-2x-24$

$0 = (x-6)(x+4)$

$\therefore x=6,\ x=-4$

④ Check: $\boxed{x=6}$ OK

but $x \neq -4$ causes log(negative)

Example 2: Solve for x: $\log_3(x-2) + \log_3 10 = \log_3(x^2+3x-10)$ [no constant]

① $\log_3[(x-2)\cdot 10] = \log_3(x^2+3x-10)$

② $10(x-2) = x^2+3x-10$

③ $10x-20 = x^2+3x-10$

$0 = x^2-7x+10$

$0 = (x-5)(x-2)$

$x=5,\ x \neq 2$

④ 5 checks out but 2 causes $\log(2-2)$ $\log(0)$ undefined

$\boxed{x=5}$

Steps for solving Exponential Equations:

If you can change to the same base (we did this earlier in the chapter!) e.g. $9^{x+2} = 3^{x-1}$	**If you CANNOT change to the same base** e.g. $5^{x+2} = 2 \cdot 3^{2x-1}$
1. Change all powers to the same base	1. Simplify if possible, then take the log of both sides (base 10)
2. Simplify into a single power on each side **Aim for: single power = single power**	2. Use log laws to move exponents in front of each log.
3. Compare exponents using the rule $b^x = b^y \quad \Leftrightarrow \quad x = y$	3. Expand out the logarithms; move logs containing variables to one side, and logs without variables to the other.
4. Solve the resulting equation.	4. Isolate and solve for the unknown (usually by factoring out the variable). Simplify to a single log if possible.
	5. Determine a numerical value of the single log (if possible)

can't get same base!

Example 3: Solve for x: $5^{x+2} = 2 \cdot 3^{2x-1}$

① $\log\left(5^{x+2}\right) = \log\left(2 \cdot 3^{2x-1}\right)$

② $(x+2)\log 5 = \log 2 + \log 3^{2x-1}$

$(x+2)\log 5 = \log 2 + (2x-1)\log 3$

③ $x\log 5 + 2\log 5 = \log 2 + 2x\log 3 - \log 3$

$x\log 5 - 2x\log 3 = \log 2 - \log 3 - 2\log 5$

④ $x(\log 5 - 2\log 3) = \log 2 - \log 3 - 2\log 5$

$x = \dfrac{(\log 2 - \log 3 - 2\log 5)}{(\log 5 - 2\log 3)}$

$2\log 5 = \log 5^2$ or $= \log 25$

$2\log 3 = \log 3^2$ or $= \log 9$

(type in calculator)

⑤ $x = 6.17$

Assignment: Pg 227 #1, 4

Logs Day 6 : Solving Equations II

Solving Log Equations: with a twist

Expressing equations in terms of stated or defined variables

Sometimes we have a question that only contains variables and we need to solve the logarithmic equation in terms of the stated variables. This is just a slight variation on what we have already looked at, just with no numbers.

aim for "Single log = single log"

Example ~~4~~ 1: Solve for A in terms of B and C: $2\log A - 3\log B = 2\log C$

$\log A^2 - \log B^3 = \log C^2$

$\log\left(\frac{A^2}{B^3}\right) = \log(C^2)$

$\therefore \frac{A^2}{B^3} = C^2$

$A^2 = B^3C^2$

$(A^2)^{1/2} = (B^3)^{1/2}(C^2)^{1/2}$

$\boxed{A = B^{3/2}C}$

Solving Log Equations when the bases don't match (change base!

Example ~~5~~ 2: Solve for x: $3\log_9 x + \log_3 x = 5$

$3\left(\frac{\log_3 x}{\log_3 9}\right) + \log_3 x = 5$

$3\frac{\log_3(x)}{\log_3(3^2)} + \log_3(x) = 5$

$\log_3 3^2 = 2$

inverse!

$\frac{3\log_3(x)}{2} + \log_3(x) = 5$

clear fractions

$3\log_3(x) + 2\log_3(x) = 10$

$5\log_3(x) = 10$

$\log_3 x = 2$

Rewrite in exponent form:

$3^2 = x$

$\boxed{9 = x}$

change of base. Get x out of base!

Example ~~6~~ 3: Solve for x: $3\log_8 x - 2\log_x 8 = -5$

$$3\log_8 x - 2\left(\frac{\cancel{\log_8 8}^{1}}{\log_8 x}\right) = -5$$

Need to "clear fractions" but easy to get confused.

$$3\log_8 x - \frac{2}{\log_8 x} = -5$$

Let $a = \log_8 x$

$$3a - \frac{2}{a} = -5$$

now clear fractions!

$$3a^2 - 2 = -5a$$

$$3a^2 + 5a - 2 = 0$$

$$(3a-1)(a+2) = 0$$

$$a = \frac{1}{3} \qquad a = -2$$

Replace back for $a = \log_8 x$

$$\log_8 x = \frac{1}{3} \qquad \log_8 x = -2$$

$$8^{1/3} = x \qquad 8^{-2} = x$$

$$\boxed{2 = x \qquad \frac{1}{64} = x}$$

check: ✓

Assignment: p. 229 #3, 5, 7

Logarithms Day 7: Applications

Using the rules we have established for exponential and logarithmic functions, we can now start to apply these to real life situations.

Recall from previous chapter:

Compound Interest	Discrete Growth	Continuous Growth
$A = P\left(1+\frac{r}{n}\right)^{nt}$	$F = I(r)^{\frac{t}{p}}$	$F = Ie^{rt}$

The Natural Base (e) and the Natural Log (ln)

→ 2.71828...

The natural base (e) is so useful, that its inverse has its own name:

The inverse of e^x is $\log_e x$ which we call $\ln x$

"ln" is "the natural log" (log base "e") and is pronounced "lawn"

$\ln e^x = x$

Just like there is a $\boxed{\text{log}}$ button on your calculator, you will also be able to find a $\boxed{\text{ln}}$ button.

Example 1: Eric inherits $10 000 and invests it in a guaranteed investment certificate (GIC) at 6 %. How long will it take to be worth $15 000 it is compounded a) monthly b) continuously?

a) Compounded monthly:

$A = P\left(1+\frac{r}{n}\right)^{nt}$

A = 15000
P = 10000
r = 0.06
n = 12
t = ?

$15000 = 10\,000\left(1+\frac{0.06}{12}\right)^{12t}$

$\frac{15000}{10\,000} = \frac{10\,000(1.005)^{12t}}{10\,000}$

$1.5 = (1.005)^{12t}$

$\log(1.5) = \log(1.005)^{12t}$

$\frac{\log 1.5}{12\log(1.005)} = \frac{12t \cdot \log(1.005)}{12 \cdot \log(1.005)}$

$6.7746 = t$

a) 6.77 years

b) Compounded Continuously:

F = 15000
I = 10 000
r = 0.06
t = ?

$F = Ie^{rt}$

$\frac{15\,000}{10000} = \frac{10\,000(e)^{0.06t}}{10\,000}$

$1.5 = e^{0.06t}$

$\ln(1.5) = \ln e^{0.06t}$

$\frac{\ln 1.5}{0.06} = \frac{0.06t}{0.06}$

$6.757 = t$

$6.76 = t$

b) 6.76 years

Example 2: A major earthquake of magnitude 8.3 is 120 times as intense as a minor earthquake. Determine the magnitude, to the nearest tenth, of the minor earthquake.

or:

$$\frac{10^{8.3}}{10^x} = 120$$
$$10^{8.3-x} = 120$$
$$\log_{10}(120) = 8.3 - x$$
$$2.079 = 8.3 - x$$
$$x = 6.2208$$

$E_1 = 10^{8.3}$
$E_2 = 10^x$

$$\frac{E_1}{E_2} = 120$$
$$\frac{10^{8.3}}{10^x} = 120$$
$$\frac{10^{8.3}}{120} = 10^x$$
$$1\,662\,718.596 = 10^x$$

(Rewrite as $\log_{10}$)

$$\log_{10}(1\,662\,718.596) = x$$
$$6.2208 = x$$

The minor earthquake measures 6.2 on the Richter scale.

Example 3: The half-life of carbon-14 is 5730 years. A bone sample is found to have 49.5% of the C-14 remaining. Determine the age of the bone.

$$F = I(r)^{t/p}$$

$F = 49.5$ (%)
$I = 100$ (%)
$r = \frac{1}{2}$
$p = 5730$ half-life
$t = ?$

$$F = I(r)^{t/p}$$
$$\frac{49.5}{100} = \frac{100}{100}\left(\frac{1}{2}\right)^{\frac{t}{5730}}$$
$$0.495 = \left(\frac{1}{2}\right)^{t/5730}$$
$$\log(0.495) = \log\left(\left(\frac{1}{2}\right)^{t/5730}\right)$$
$$\log 0.495 = \frac{t}{5730} \cdot \log(1/2)$$

$$t = \frac{5730 \cdot \log 0.495}{\log(\frac{1}{2})}$$
$$t = 5813.08$$

The bone is 5813 years old.

Example 4: The population of a fish in a lake is increasing at a rate of 3.5% per year. How long will it take the fish population to double?

$$F = Ie^{rt}$$

$\left.\begin{array}{l} F = 2 \\ I = 1 \end{array}\right\}$ "doubles"
$r = 0.035$
$t = ?$

$$2 = 1 \cdot e^{0.035t}$$
$$2 = e^{0.035t}$$
$$\ln 2 = \ln e^{0.035t}$$
$$\frac{\ln 2}{0.035} = \frac{0.035t}{0.035}$$
$$19.804 = t$$

It will take 19.8 years.

Assignment: p. 236 #1-11