<u>Chapter 1 - Square Roots, Powers, and Exponent Laws</u>
1.1 - The Real Number System

In math, it is important to be able to discuss different types of numbers. Some are positive (____), some negative (____) , some can be written as fractions (____), and some can't be written as fractions (____). Also, what about numbers that are neither really positive nor negative, like the number 0? They all must be classified into _____.

<u>Number Sets:</u>

First are the _____ (a.k.a the counting numbers).

They start at "1" and go up one-at-a-time. _____

Natural Numbers Symbol: _____

Before we can discuss the negatives, we must include ____. This inclusion of 0 creates a whole new set. Known as the _____.

_____

Whole Number Symbol: _____   or _____

Now we can include the negative numbers. This new set of numbers, which houses negatives, zero, and positives is called the set of _____.

_____

Integers Symbol: _____

Notice that until now, every number we have observed goes up or down one-at-a-time. But obviously that is not always the case. The _____ _____ can now be introduced. These are numbers that are either <u>terminating</u>, or <u>repeating</u>; they are numbers that <u>can</u> be written as fractions.

Rational Numbers Symbol: _____

Rational Number Examples:

Note A: fractions <u>can not</u> have denominators equal to 0. Eg. _____

Note B: fractions with denominators equal to 1 are often re-written without a

denominator at all. Eg. $\dfrac{3}{1} = 3, \ -\dfrac{.4}{1} = -.4$

Note C: Both the numerator and denominator in a fraction can be integers.

Note D: There are an infinite number of numbers, but there are more rational numbers than there are integers, and more integers than natural! Why is that?

Some numbers are unable to be expressed as fractions because they do not _____, or _____. They are known as _____

Irrational Number Symbol: _____

Irrational Number Examples:

All of the numbers listed above, make up the ultimate set, known as the set of _____.

Real Number Symbol: _____

BONUS: All the numbers you can think of probably fall under the set of Real Numbers. If a number falls outside of the Real-Number Set, it is known as an _____. Imaginary numbers, most importantly, allow us to solve negative roots! You won't be tested on this in grade 9.

Imaginary Number Symbol:

Examples:

Activity: Place the following numbers into the appropriate zones:
$$\{0.121, 18, -1, 0, \pi\}$$

VENN DIAGRAM

REAL NUMBER SYSTEM

RATIONAL

INTEGERS

WHOLE

NATURAL
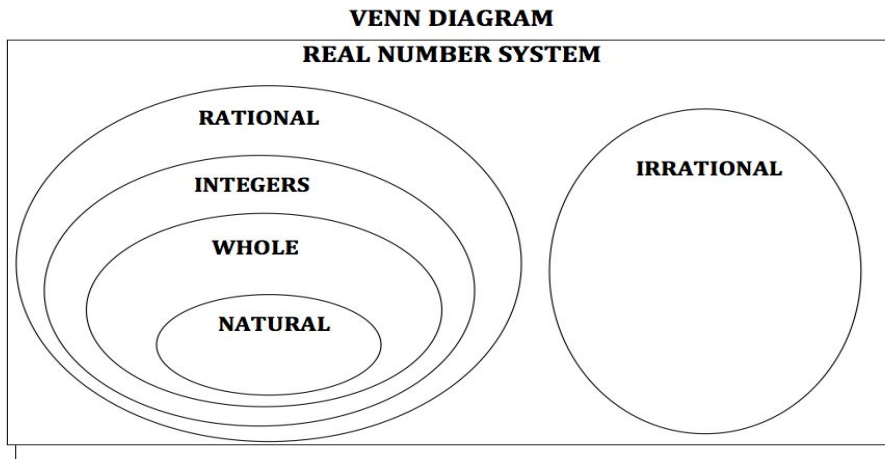
IRRATIONAL

True or *False*: All Rational Numbers are Whole Numbers.

True or *False*: All Natural Numbers are Integers.

1.2 - Square Roots

When a number is multiplied by itself, we say the number has been "squared."
Eg:

The reverse process of squaring, which finds the original number that was multiplied by itself, is "square-rooting."

Here is the full cycle:

The symbol $\sqrt{}$ is known as a radical sign. It indicates the operation of square rooting a number.
In full it looks like this:

$$\sqrt[2]{36}$$

We have investigated numbers like 9, 16, 25, and 36. These numbers are known as perfect-squares as they have rational square roots - they will pop up often on your math journey so it is important to be able to recognize them. Perfect squares are relatively uncommon, most numbers are non-perfect -- we will investigate these in the next section.

Perfect-Squares worth knowing:

$1^2 =$	$2^2 =$	$3^2 =$	$4^2 =$	$5^2 =$	$6^2 =$
$7^2 =$	$8^2 =$	$9^2 =$	$10^2 =$	$11^2 =$	$12^2 =$
$13^2 =$	$14^2 =$	$15^2 =$	$20^2 =$	$25^2 =$	$100^2 =$

Examples:

a)

b)

c)

d)

e)

f)

g)

h)

i)

j)

k)

1.3 - Square Roots of Non-Perfect Squares

As seen in the previous section, perfect squares are numbers that have rational square roots. In this section, we will explore non-perfect squares, numbers with irrational roots. For example, the square root of 10 produces an irrational number (one that does not terminate or repeat); because of this, finding the exact value of non-perfect squares is an impossible task, we will instead learn how to approximate such values.

Question: Of the numbers from 1 to10, how many are perfect squares?
Answer:

Example: Using a calculator, find $\sqrt{10}$.

Approximating Square Roots without a Calculator

To approximate the square root of a non-perfect number, we must have the knowledge of which perfect-squares surround that number -- this will tell us the whole number value of our answer. The relative position of our non-perfect number will provide us with the appropriate decimal approximation.

Example: Without a calculator, approximate the $\sqrt{5}$ to the nearest tenth.

Example: Approximate $\sqrt{30}$ to the nearest hundredth.
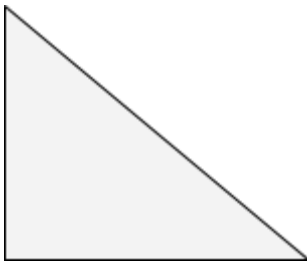
The Pythagorean Theorem

The Pythagorean Theorem is one of the most fundamental formulas found in common mathematics and has been applied for thousands of years. The Pythagorean Theorem describes the relationship between the sides of any right triangle.
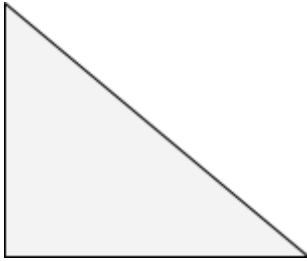
Formula:
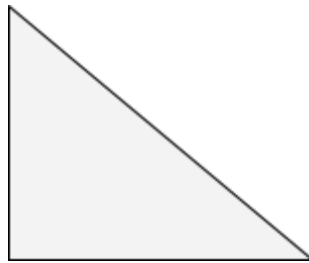
Examples: Solve for *x* to the nearest tenth, in each of the following triangles.

   a)

b)

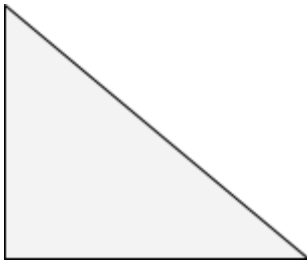c)

There are a few special triangles that have whole numbers for all three of their sides -- examples include triangles with side lengths of 3, 4, and 5, or 5, 12, and 13, or 33, 56, 65. Such perfect whole triangles are known as *Pythagorean Triples*. Interestingly, multiples of any of these triples also hold. For instance, since 3, 4, 5 is a Pythagorean triple, so is _____ and _____ and so on!

1.4 - Defining a Power

Any number that is added to itself a bunch of times can be simplified by the use of multiplication. For example, $4 + 4 + 4 + 4 + 4 =$ _____.

But what about repeated multiplication? For that, we can use exponents or powers. For example, $4 \times 4 \times 4 \times 4 \times 4 =$ _____ and is read "4 to the power of 5" or "4 raised to the 5th power."

Exponential Notation

$$4^5 \qquad\qquad -3^2 \qquad\qquad (-5)^8$$

Base =   Base =   Base =

Exponent =   Exponent =   Exponent =

Answer =   Answer =   Answer =

The Product of Negative Numbers

When working with exponents it is important to know:

1) Whether the base is positive or negative

2) Whether the exponent is even or odd

Remember that:

1) (+) times a (+) is a (+). Rule: All positive $\rightarrow$ positive.

2) (−) times a (−) is a (+). Rule: Even number of negatives $\rightarrow$ positive.

3) (−) times a (+) is a (−). Rule: Odd number of negatives $\rightarrow$ negative.

The brackets will inform you if a base is positive or negative. In general,

Rule: $(-x)^n \rightarrow$ base is _____   Eg.
Rule: $-x^n \rightarrow$ base is _____   Eg.

This is due to the fact that in BEDMAS (or GEMA) the Exponent acts on the variable before the Multiplication of the negative does. If the variable is within brackets, the exponent works on everything within; but if there are no brackets, the exponent only works on the variable itself, with the multiplication of the negative coming after. It is very common to quickly glance at a question and misinterpret the base, changing your solution significantly, take your time to ensure proper comprehension of its correct value.

Example: Determine if the following solutions are positive or negative.

a) $(-1)^2 \rightarrow$ _____   b) $-2^4 \rightarrow$ _____

c) $(-3)^7 \rightarrow$ _____   d) $-(-5)^2 \rightarrow$ _____

e) $4^9 \rightarrow$ _____   f) $-(-1)^9 \rightarrow$ _____

Summary: Given $x > 0$,

$(-x)^{even} =$ $(-x)^{odd} =$ $-x^{odd \ or \ even} =$

One, Zero, and Negatives as Exponents

Fact: any variable to the power of one is itself. Eg.

Fact: any variable to the power of zero is one. Eg.

Fact: negative exponents become positive if written on the opposite side of the division bar. Eg.

Observe the following pattern:

$2^4 =$

$2^3 =$

$2^2 =$

$2^1 =$

$2^0 =$

$2^{-1} =$

$2^{-2} =$

$2^{-3} =$

$2^{-4} =$

Summary for exponents of 0 and 1:

$a^1 = a,$ for any number a

$a^0 = 1,$ for any non-zero a

Note: $0^0 = undefined$

Evaluate

a) $95^0 =$

b) $8^1 =$

c) $(\frac{3}{7})^0 =$

d) $(-4)^0 =$

e) $-4^0 =$

f) $0^0 =$

g) $(a+b)^0 =$

h) $a^0 + b^0 =$

i) $a + b^0 =$

j) $(ab)^0 =$

k) $ab^0 =$

l) $(2(x^0))^1 =$

1.5 - Orders of Operations

Take the statement:

$$20 - 3(2 + 3)^2 \times 2$$

While there are multiple ways of simplifying the problem, only one will lead to the correct solution. The proper order of operations is defined by the following acronym:

B

E   **G**

D   **E**

M   **M**

A   **A**

S

With the following rules:
1) Work starting from the innermost brackets
2) When there is a tie in hierarchy, move from left to right.

So to solve the above problem: $20 - 3 \times (2 + 3)^2 \times 2$

Unit 1 - Square Roots, Powers, and Exponent Laws

Simplify:

a) $3 + 5^2 - 2$

b) $(-3 + 2) + 8 \div 2 \times 4$

c) $(2 - 5) + 4^2 - 10 \div 5$

d) $3 + 5 - (-1) \times 7$

e) $(4 - 1)^2 + 15 \div 3 - 1$

f) $\frac{5+7}{4} - 1 + 3^2 \times 2$

g) $10 - (3 + 4 \div 2)^2 + 15$

h) $\frac{5 \times 2}{4 - 3} - 8 + (-1)^2 - 4$

1.6 - Exponent Laws

In the following sections we will investigate how exponents interact with one another.

Multiplying with Exponents - "The Product Rule"

The Product Rule: When exponents of the same base are multiplied, the exponents are added together.

Take the expression:

The expression can be elongated:

Therefore:

Notice: The base does not change!

Common Mistakes:

a) $2^3 \times 2^4 = 4^7$ Error: Multiplying the bases. Should be left unchanged.

b) $2^3 \times 2^4 \neq 2^{12}$ Error: Multiplying the exponents. Should be added.

Examples: Simplify, but do not evaluate.

a) $3^6 \cdot 3^5$

b) $(4^2)(4^3)(4)$

c) $2^3 \cdot 3^2 \cdot 2^5$

d) $5^2 \times 5^3 + 5^4$

Dividing with Exponents - "The Quotient Rule"

The Quotient Rule: When exponents of the same base are divided, the exponents are subtracted from one another (the denominator is subtracted from the numerator).

Take the expression:
The expression can be elongated:

Therefore:
Notice: The base does not change!

Common Mistakes:

a) $\frac{5^{12}}{5^2} = 1^{10}$ Error: Dividing the bases. Should be left unchanged.

b) $\frac{5^{12}}{5^2} = 5^6$ Error: Dividing the exponents. Should be subtracted.

Examples: Simplify, but do not evaluate.

e) $3^6 \div 3^5$

f) $(4^2)(4^3) \div (4)$

g) $2^3 \div 2^5 \cdot$

h) $(5^2) \div (5^3 \times 5^4)$

Zero Exponent Proof

Earlier we learned that anything to the power of zero equals one. For the reason this is true we can turn to the quotient rule for an elegant proof.

Take the examples:

$\frac{5}{5} =$ $\qquad\qquad$ $\frac{x}{x} =$ $\qquad\qquad$ $\frac{x^4}{x^4} =$

But from the quotient rule, we can see that:

And since,

Combined Operations

We can now do more intricate examples now that we have discussed the product rule and the quotient rule.

a) $(-3)^7 \cdot (3)^3$

b) $(-3)^8 \div 3^3$

c) $3^8 - 3^3 \cdot 3^4$

d) $\frac{(-2)^6}{2^3 \times 4} + (3)(3^5)$

1.7 - Power Rules

Exponents of exponents- "The Power Rule"

The Power Rule: When raising a power by an exponent, the exponents are multiplied together, the base is left unchanged.

Take the expression:

The expression can be elongated:

Therefore:

Notice: The base does not change!

Examples: Simplify, but do not evaluate.

i) $(2^4)^2$

j) $(x^5)^3$

k) $(\frac{1}{x^2})^6$

Raising a Product or Quotient to a Power

The power rule can also be applied to both products and quotients. But be careful, it can not be applied to sums or differences. We will observe these here.

What we can <u>not</u> do:

$$(x+y)^2 \neq x^2 + y^2$$

$$(x-y)^2 \neq x^2 - y^2$$

What we <u>can</u> do:

<u>Raising a Product to a Power:</u>

Rule:

Example:

<u>Raising a Quotient to a Power</u>

Rule:

Example:

Summary:

	Example	Expanded	Simplified	Exponent Rule
Multiplication	$a^3 \times a^5$	$a \cdot a \cdot a \times a \cdot a \cdot a \cdot a \cdot a$	a^8	$a^m \cdot a^n = a^{m+n}$
Division	$\dfrac{a^6}{a^2}$	$\dfrac{\cancel{a} \cdot \cancel{a} \cdot a \cdot a \cdot a \cdot a}{\cancel{a} \cdot \cancel{a}}$	a^4	$\dfrac{a^m}{a^n} = a^{m-n}$
Power Law	$\left(a^2\right)^4$	$a \cdot a \times a \cdot a \times a \cdot a \times a \cdot a$	a^8	$\left(a^m\right)^n = a^{m \times n}$
Power of a Product	$\left(a^2 b\right)^3$	$a^2 b \cdot a^2 b \cdot a^2 b$	$a^6 b^3$	$\left(a^x b^y\right)^m = a^{xm} b^{ym}$
Power of a Quotient	$\left(\dfrac{a}{b^3}\right)^5$	$\dfrac{a}{b^3} \cdot \dfrac{a}{b^3} \cdot \dfrac{a}{b^3} \cdot \dfrac{a}{b^3} \cdot \dfrac{a}{b^3}$	$\dfrac{a^5}{b^{15}}$	$\left(\dfrac{a^x}{b^y}\right)^m = \dfrac{a^{xm}}{b^{ym}}$

Notes: