Chapter 4 - Linear Relations

4.1 - Patterns

Can you predict the next number in the sequence?

- 6, 11, 16, 21, _____

- -7, -10, -13, -16, _____

But what will the 50th or 1000th number in the sequence be? For this type of problem, we turn to the subject of linear equations.

As its name suggests, *linear equations* centre around two main ideas:

1. The ___________ of our problems (that they go up by a constant difference every time).
2. That we can write a general _________ that can explain any at-hand relationship.

Let's come up with a method to create such equations. Interestingly, we only need two pieces of information:

1) The ______________________. Which is initially known as "*d*" but will be referred to later on as our "slope" or "*m*".
2) Where our pattern officially ________. In a sense, "where did your numbers start even before you took that first step." This will be referred to later as the "*y*-intercept" or "*b*".

Eg. Write the general equation for the above patterns and determine the value of the 1000th term.

a) 5, 10, 15, 20, …

Step (n)	**0**	1	2	3	4	1000
Value (t_n)						

Where did we officially start (when n = 0, what was its value?) ________
How much are we going up by each time? _________
Formula:

b) -7, -10, -13, -16, …

Step (n)	**0**	1	2	3	4	1000
Value (t_n)						

Where did we officially start (when n = 0, what was its value?) ________
How much are we going up by each time? _________
Formula:

Application problem:
Blackbird Cabs is Victoria's new premium cab company. They charge an initial $3.50 for a pickup, followed by 25¢ for every kilometre driven.

a) Write an equation relating cost to kilometres driven.
b) What is the cost of taking Blackbird from Parliament to the Airport (a distance of 28km)
c) If $15.75 was charged to a customer for their taxi ride, how many kilometres were driven?

In summary:

We start our investigation of linear equations through patterns using letters such as "*n*" (for number or location of a term) and " t_n " (for the value of the term itself); but we are heading toward our discussions on graphing which instead use the letters "*x*" (for the number or location of a term) and "*y*" (for the value of the term itself).

General Form Equation for **Linear Patterns**:

General Form Equation for **Graphing Linear Functions**:

Transitioning from patterns to graphing:

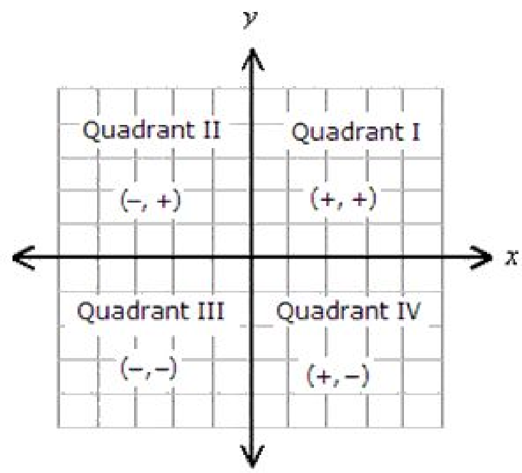
Patterns	Graphing	Application
$d =$	$m =$	Tells you the rate of change.
$b =$	$b =$	Tells you where you officially start
$n =$	$x =$	The step or location of a number
t or $t_n =$	$y =$	The value of the said location

4.2 Linear Systems

Graphs are a fundamental aspect of mathematics and play an integral part of the secondary school curriculum. They may feel overwhelming at first, but the longer you spend working with them the easier they will become.

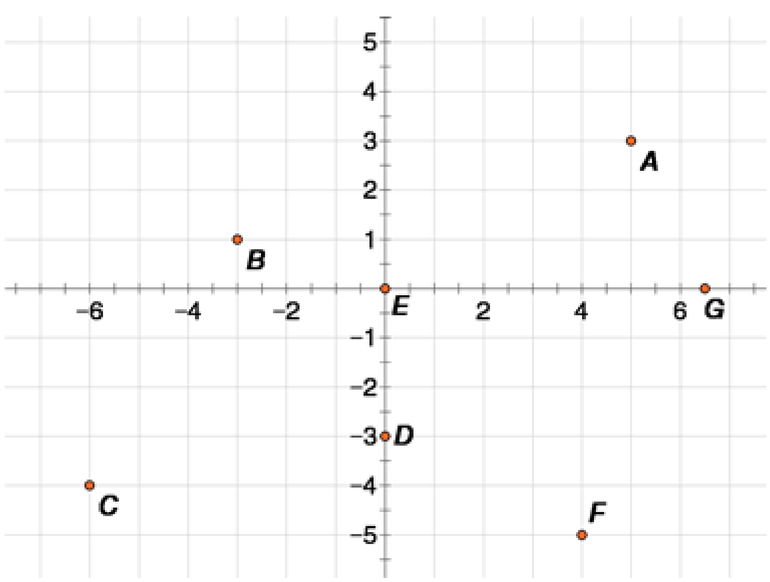
Coordinate System

The coordinate system uses a pair of numbers (coordinates) to uniquely determine the position of geometric elements. "Ordered pairs" are made up of one *x* and one *y* value, they are written as ***(x,y)***. A unique ordered pair will correspond to a unique point on a graph.



Note that the *x*-coordinate is always written first!

State the ordered pair for the corresponding letters.



A:

B:

C:

D:

E:

F:

G:

*Remember: The ordered pairs (4, -5) and (-5, 4) plot different points.
What quadrant is (4, -5) in? _____ What quadrant is (-5, 4) in? _____

Graphing-equations

Graphing-equations, in mathematics, are special formulas that have been specifically designed so that they can be read and then graphed without any extra computations.

Take the vertex formula for graphing parabolas. It looks like this:

$y = (x - 4)^2 + 1$

- The exponent of 2 tells us we have a parabolic shape
- The 4 and 1 tell us the parabola starts at (4,1)

What happens when x=2?

Our course will focus on much simpler functions -- straight line functions known as Linear Equations.

Linear Equations

Linear Equations are ones that denote_______________. They have a degree of one and will be the main focus for the remainder of our course.

The Slope-Intercept Formula: $y = mx + b$

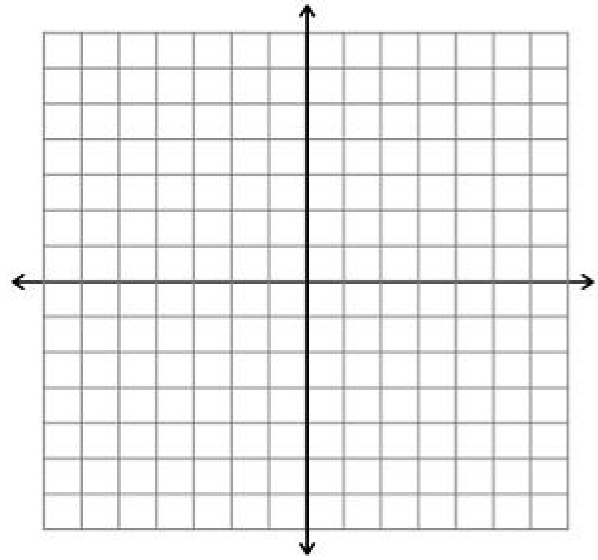
- $m =$
- $b =$
- x & y = placeholder for inputting values and their subsequent output

There are two methods for graphing equations. You must know how to use both.

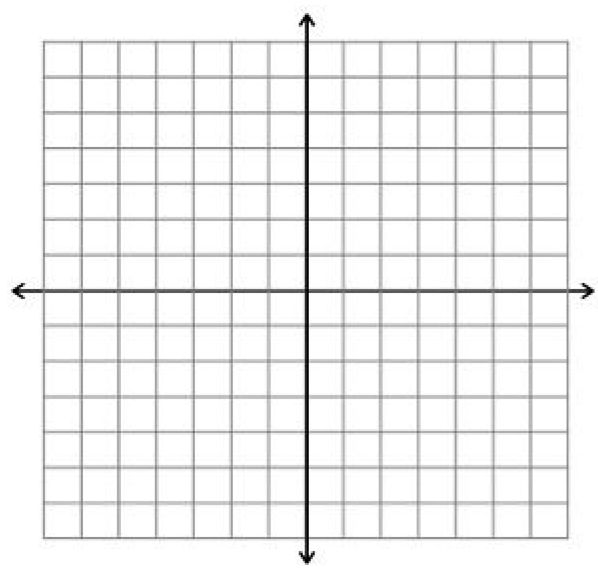
Method 1) Direct Reading from the Slope-Intercept Formula
Step 1: Make sure your equation is in the form of ______________
Step 2: ________ the b-value directly on the y-axis
Step 3: ________ from this point vertically, then horizontally, by the value of your slope. Repeat at least once.
Step 4: ________ the dots to create a line. Draw _________ on both ends to demonstrate that the line goes on forever.

Method 2) Calculate your points using a Table of Values
Step 1: _______ a table (x on the left, space for work in the middle, y on the right)
Step 2: _______ three x values and write them in your x column
Step 3: _______ for y with respect to the three chosen values
Step 4: _______ the pairs of points and connect the dots.

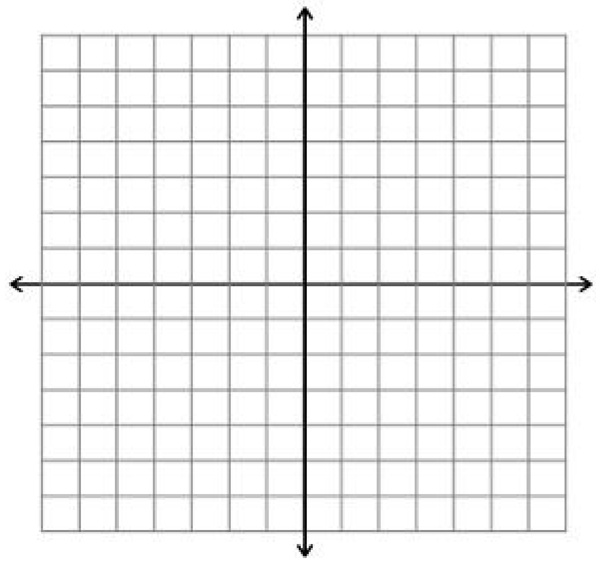
Use Method 1 to graph $y = \frac{1}{2}x - 3$
Use Method 2 to confirm your points



Use Method 2 to graph $y = -2x$
Use Method 1 to confirm.



Use either method to graph
$3x + 4y = 1$



Extra thoughts:

- Read graphs from left to right (not from the origin).
    - From L→ R: if the line goes up, we say it has a positive slope.
    - From L→ R: if the line goes down, we say it has a negative slope.
- A line that goes through the x-axis is known as the x-intercept. The x-axis has a y value of 0, so we say that the x-intercept will be at the point $(x, 0)$.
- A line that goes through the y-axis is known as the y-intercept. The y-axis has an x value of 0, so we say that the y-intercept will be at the point $(0, y)$.
- A horizontal line has a slope of 0. A vertical line has a slope of ∞ (undefined=$\varnothing$).

4.3 Graphing Equations in the Form $Ax + By = C$

In the previous section, we learned how to graph equations that were in "slope-intercept form" ______________. In this section, we learn how to graph equations that are set up differently, like ones that are in "standard form" ___________________.

Why was slope-intercept form useful: It allowed us to graph our equation because it told us where the *y*-intercept was, and then what the slope from the point was.

Why is standard form useful: It allows you to easily solve for the *x*-intercept and *y*-intercept.

<u>Graphing a Linear Equation in Standard Form</u> ($Ax + By = C$)

Step 1: Draw a table of values.

Step 2: You will then pick three points.

Point 1: Set x=0 (to find the _______________)

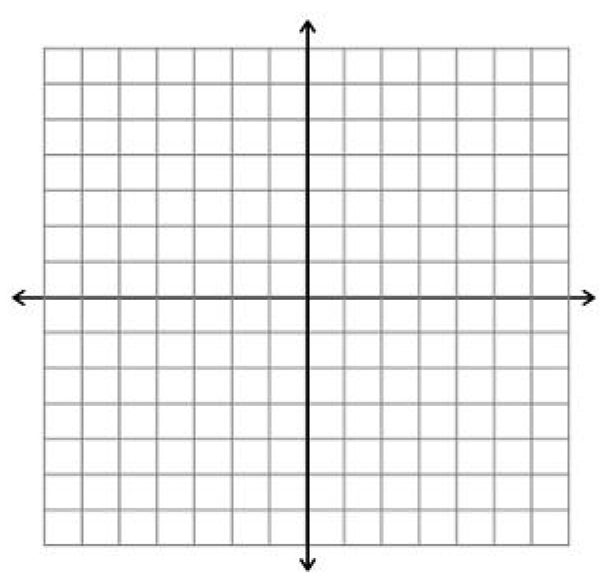
Point 2: Set y=0 (to find the _______________)

Point 3: Set x=# (pick an easy x value to confirm your line)

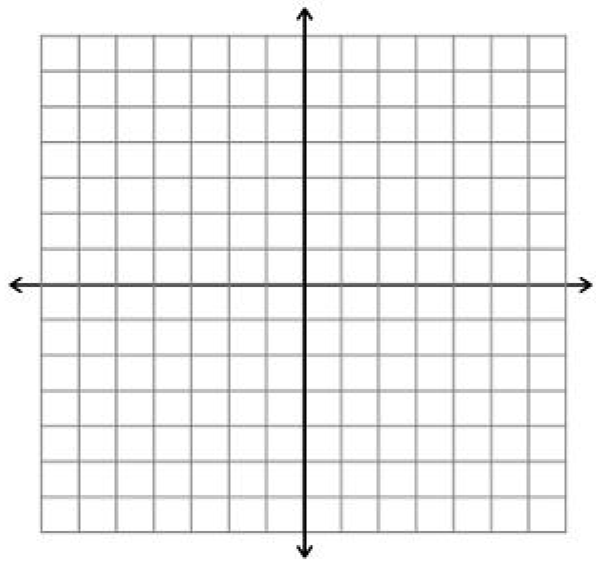
Step 3: Solve for the corresponding values.

Step 4: Plot the found coordinates.

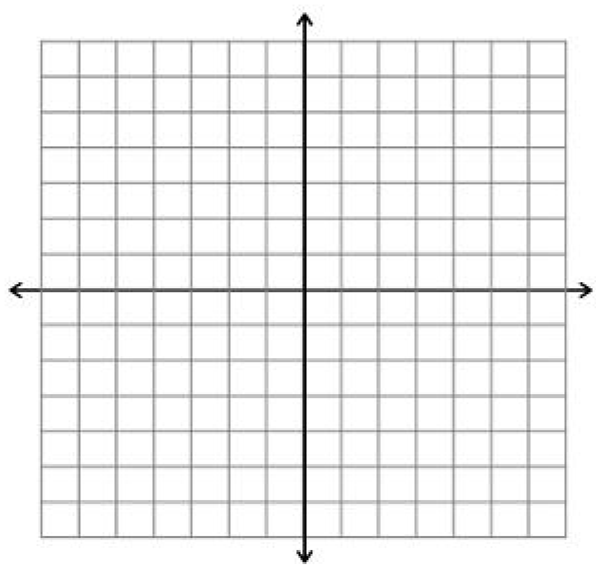
Example: Graph $2x + 3y = 6$



Graph $4x - \frac{1}{2}y = 4$



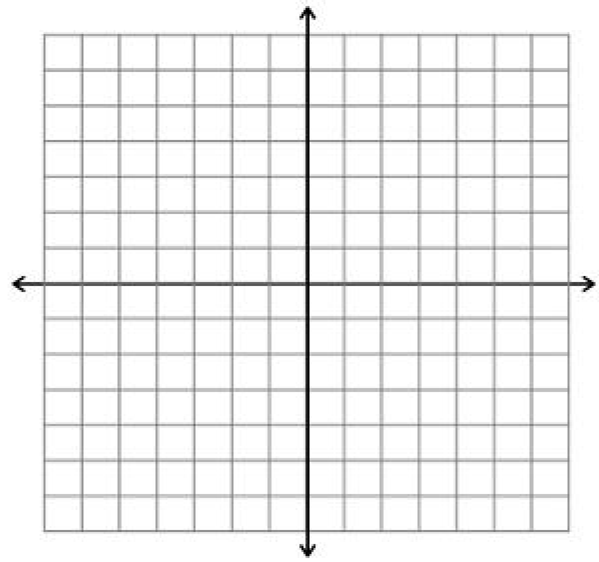
<u>Horizontal or Vertical Line Equations</u>



Take the provided **horizontal line** that is located at 4 spots up from the origin.

1) It has the formula of $y = 4$. Because it is located four units up, and there are infinite possibilities of what x can be. Any point $(x,4)$ is a solution to the equation.
2) Because the line does not move vertically what-so-ever, it has a slope of 0.
3) As it runs parallel to the x-axis, it does not have an x-intercept.
4) It has a y-intercept of (0,4).
5) Here's what a table of values would look like for a horizontal line:

Take the provided **vertical line** that is located at 2 spots right of the origin.



1) It has the formula of $x = 2$. Because it is located two units right, and there are infinite possibilities of what y can be.Any point $(2, y)$ is a solution to the equation.
2) Because the line does not move horizontally what-so-ever, it has a slope of ∞.
3) As it runs parallel with the y-axis, it does not have a y-intercept.
4) It has an x-intercept of (2,0).
5) Here's what a table of values would look like for a vertical line:

In Summary

The graph $y = \#$ is a horizontal line with a y-intercept of $(0, \#)$. It has a slope of 0.

The graph $x = \#$ is a vertical line with a x-intercept of (#, 0). It has an undefined slope (∞).

4.4 - Matching Equations of Graphs

In this section we will work on matching linear equations to their corresponding graphs -- and vice versa.

<u>Equation → Graph</u>

Method 1: Confirm the line using points.

Step 1) Look at the line, and select a coordinate that is on the graph.

Step 2) Plug in the coordinates for the x & y variables for one of your equations.

Step 3N) If the LS $\neq$ RS then the point is not a solution, and the equation does not match the graph -- discard this equation as a possible solution

Step 3Y) If the LS = RS then the point is a solution of the equation and may match the line.

Step 4) Input another point into the working equation to confirm the line. You may need to do this up to three times to verify the equation.

Method 2: Use the slope-intercept formula to verify the line.

Step 1) Convert your formula so it is the form of $y = mx + b$

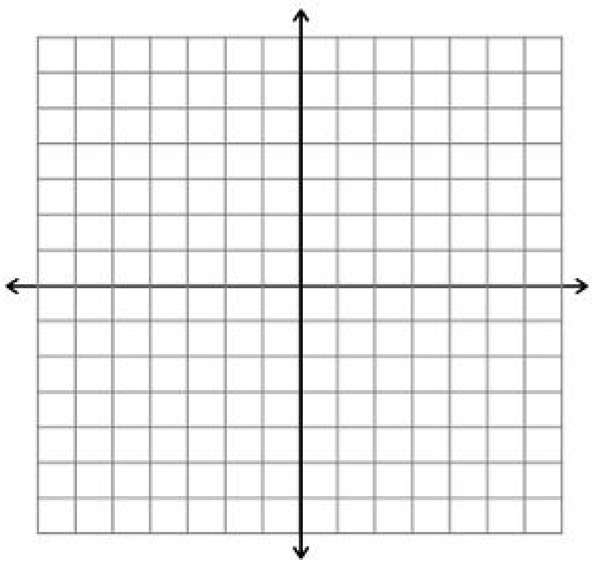
Step 2) Observe the slope (m). Is it positive or negative? What is the $\frac{rise}{run}$?

Step 3) Observe the location of the y-intercept (b).

Step 4) There will only ever by one line that satisfies both of these values. Choose the formula that has the correct m and b values.

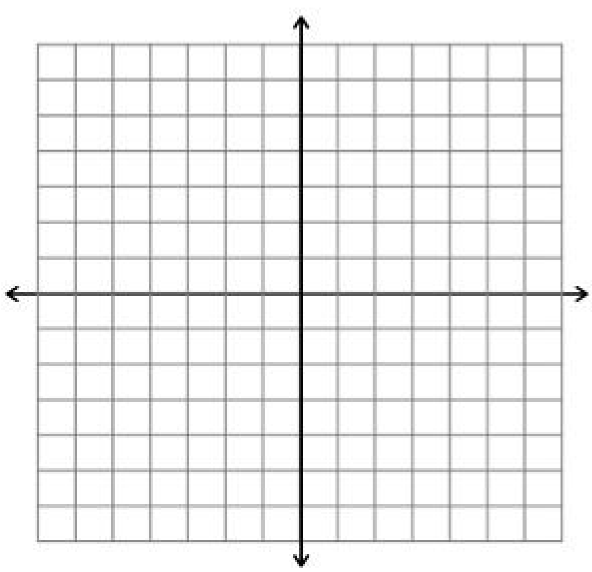
i) Which one of the equations match the graph?

a) $y = \frac{3}{2}x + 3$ b) $y = \frac{-2}{3}x - 3$ c) $y = \frac{-2}{3}x + 3$



ii) Which one of the equations match the graph?

a) $y = \frac{-1}{2}x + 1.5$ b) $3x - 5y = 7.5$ c) $5x - 3y = 4.5$



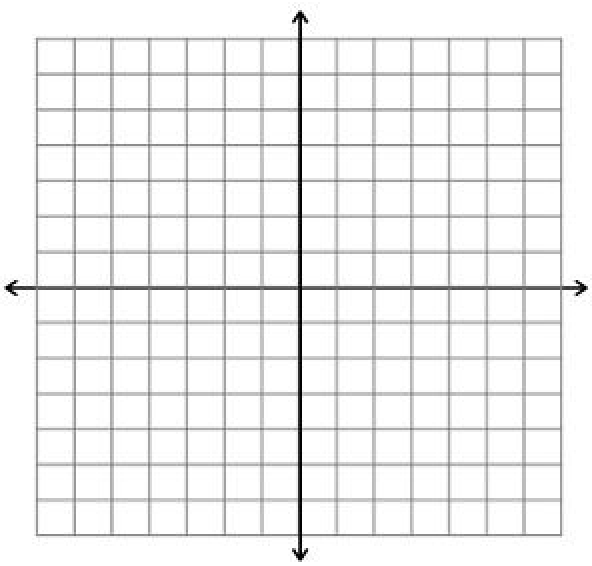
Graph → Equation

What does the equation of a line look like? ________________

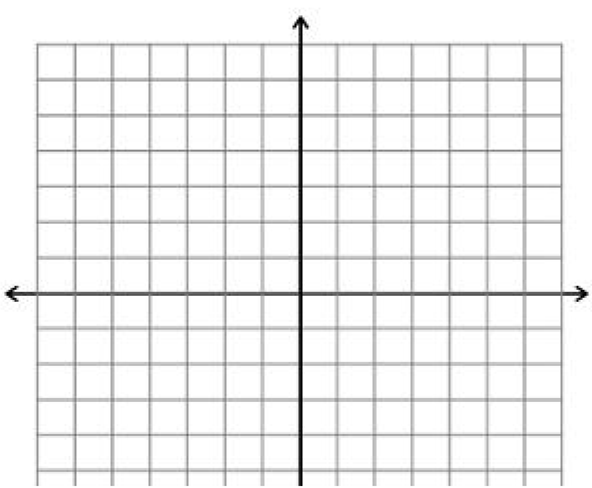
So to create the equation from a graph, we, again, only need two pieces of information:

- Slope =
- y-intercept =

Both of these values can be read directly from the graph, and can be confirmed with any point from the function.



Sometimes, if you have non-conventional units, you may need to use the slope-formula ($\frac{y_2 - y_1}{x_2 - x_1}$) to determine the value of the slope.



Notes: