

PC12

Name: Key (for all questions) Date: _____

Polynomials Key Skills Review

A. FRACTIONS

Multiplying Fractions

- 1) Multiply the numerators (tops)
- 2) Multiply the denominators (bottoms)
- 3) Simplify/reduce the final fraction if possible.

Examples:

1. $\frac{10}{3} \times \frac{6}{5}$

$$= \frac{60}{15} \begin{matrix} \div 15 \\ \div 15 \end{matrix}$$

$$= \boxed{\frac{4}{1} \approx 4}$$

2. $\frac{1}{2} \left(\frac{10}{1} \right)$

$$= \frac{10}{2}$$

$$= \boxed{5}$$

3. $\frac{7}{2} \times \frac{3}{4}$

$$= \boxed{\frac{21}{8}}$$

Your turn!

1. $\frac{9}{2} \times \frac{4}{3}$

$$= \frac{36}{6}$$

$$= 6$$

2. $\frac{12}{1} \left(\frac{1}{3} \right)$

$$= \frac{12}{3}$$

$$= 4$$

3. $\frac{1}{2} \times \frac{1}{3}$

$$= \frac{1}{6}$$

4. $\frac{1}{2} \times \frac{5}{1}$

$$= \frac{5}{2}$$

5. $\frac{1}{2} \times \frac{5}{2}$

$$= \frac{5}{4}$$

6. $\frac{1}{2} \times 11$

$$= \frac{11}{2}$$

Adding and Subtracting Fractions

- 1) In order to add or subtract fractions, you must have a Common denominator.
 - a. If needed, multiply the top AND bottom of the fraction by the same number so that your denominators match.
- 2) Add (or subtract) the numerators (leave the denominators alone)
- 3) Simplify/reduce the final fraction if possible.

Examples:

$$1. \frac{2 \times \frac{1}{3} + \frac{1}{6}}$$

$$= \frac{\frac{2}{6} + \frac{1}{6}}$$

$$= \frac{3}{6} \div 3$$

$$= \frac{1}{2}$$

$$2. \frac{4 \times \frac{2}{5} - \frac{3 \times 5}{4 \times 5}}$$

$$= \frac{\frac{8}{20} - \frac{15}{20}}$$

$$= \frac{-7}{20}$$

$$3. \frac{4 \times 6 + \frac{3}{4}}$$

$$= \frac{24}{4} + \frac{3}{4}$$

$$= \frac{27}{4}$$

Your turn!

$$1. \frac{7 \cdot \frac{1}{4} + \frac{1 \cdot 4}{7 \cdot 4}}$$

$$= \frac{7}{28} + \frac{4}{28}$$

$$= \frac{11}{28}$$

$$2. \frac{3 \cdot \frac{2}{3} - \frac{1}{9}}$$

$$= \frac{6}{9} - \frac{1}{9}$$

$$= \frac{5}{9}$$

$$3. \frac{8 \cdot \frac{1}{5} + \frac{3 \cdot 5}{8 \cdot 5}}$$

$$= \frac{8}{40} + \frac{15}{40}$$

$$= \frac{23}{40}$$

$$4. \frac{2 \cdot \frac{1}{2} - \frac{1}{4}}$$

$$= \frac{2}{4} - \frac{1}{4}$$

$$= \frac{1}{4}$$

$$5. \frac{\frac{17}{11} + \frac{3 \cdot 4}{1 \cdot 11}}$$

$$= \frac{17}{11} + \frac{33}{11}$$

$$= \frac{50}{11}$$

$$6. \frac{9 \cdot \frac{5}{9} - \frac{4}{9}}$$

$$= \frac{45}{9} - \frac{4}{9}$$

$$= \frac{44}{9}$$

B. SOLVING LINEAR EQUATIONS

- 1) Expand brackets and clear fractions → to "clear fractions", multiply all TERMS by each denominator.
- 2) Move all variable terms to one side and all other terms to the other side
- 3) Collect like terms and divide to isolate variable

Terms are separated by add/subtract.

↳ same variable, same exponent

Examples:

1. $4x - 1 = 9$

$$\frac{4x}{4} = \frac{10}{4}$$

$$x = \frac{10}{4} \div 2$$

$$x = \frac{5}{2}$$

Fraction answers are better than decimals

Your turn!

1. $3x + 6 = 12$

$$\frac{3x}{3} = \frac{6}{3}$$

$$x = 2$$

2. $3(x + 2) - 24 = 2(4x - 3) + x$

$$3x + 6 - 24 = 8x - 6 + x$$

$$3x - 18 = 9x - 6$$

$$-6x = 12$$

$$x = -2$$

2. $7x - 3 = 18$

$$\frac{7x}{7} = \frac{21}{7}$$

$$x = 3$$

3. $\frac{1}{2}(x + 18) = \frac{4x}{5} - 3$

$$5(x + 18) = 4(2)x - 3(2)(5)$$

$$5x + 90 = 8x - 30$$

$$-3x = -120$$

$$x = 40$$

3. $2x + 100 = 200$

$$\frac{2x}{2} = \frac{100}{2}$$

$$x = 50$$

4. $5(x + 2) = x + 2(x - 3)$

$$5x + 10 = x + 2x - 6$$

$$5x + 10 = 3x - 6$$

$$\frac{2x}{2} = \frac{-16}{2}$$

$$x = -8$$

5. $\frac{3x}{2} - 3 = \frac{1}{3}(x + 5)$

$$(3)(\frac{3x}{2}) - 3(2)(3) = 2(x + 5)$$

$$9x - 18 = 2x + 10$$

$$\frac{7x}{7} = \frac{28}{7}$$

$$x = 4$$

C. GRAPHING LINEAR RELATIONS

Finding Slope of a Line

Slope is the steepness of the graph. We think of "rise over run."

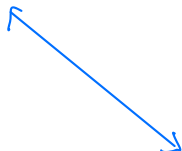
$$\text{slope} = m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Example: Sketch a line with a...

Positive Slope



Negative Slope



Slope of Zero



Undefined Slope



Finding x- and y- intercepts

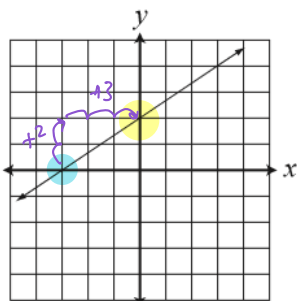
x-intercepts are where the graph crosses the x-axis (where height = 0, or $y = 0$).

y-intercepts are where the graph crosses the y-axis (where you are "zero over" or $x = 0$).

Examples:

Find and label the x- and y-intercepts and the slopes of the following graphs:

1.



x-intercept: $(-3, 0)$

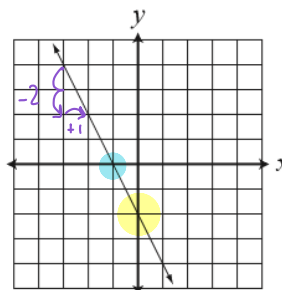
y-intercept: $(0, 2)$

Slope (m): $\frac{2}{3}$

up 2
right 3

Your turn!

2.



x-intercept: $(-1, 0)$

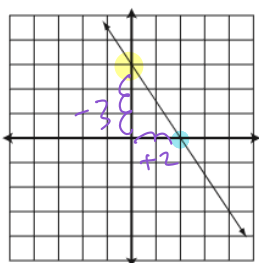
y-intercept: $(0, -2)$

Slope (m): $-\frac{2}{1}$

down 2
right 1

Find and label the x- and y-intercepts and slopes of the following graphs:

1.

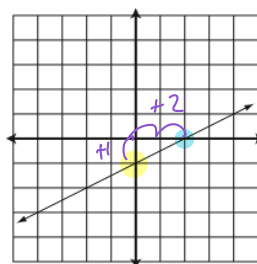


x-intercept: $(2, 0)$

y-intercept: $(0, 3)$

Slope (m): $-\frac{3}{2}$

2.



x-intercept: $(2, 0)$

y-intercept: $(0, -1)$

Slope (m): $\frac{1}{2}$

Graphing Linear Relations ($y = mx + b$)

If your Linear Relation is in $y = mx + b$ form (also called "slope-intercept" form), then

- m (the coefficient of x) is the slope
- and
- b (the constant term) is the y-intercept

To graph:

- 1) Rearrange into $y = mx + b$ if needed (make sure "y" is by itself on its own side of the equation)
- 2) Start on the y-axis at "b" -- plot the y-intercept, which is at $(0, b)$
- 3) Use the slope to "travel" (rise/run) to the next point and plot it
- 4) Plot a few points and connect the dots with a ruler!

Examples:

1. $y = -2x + 6$

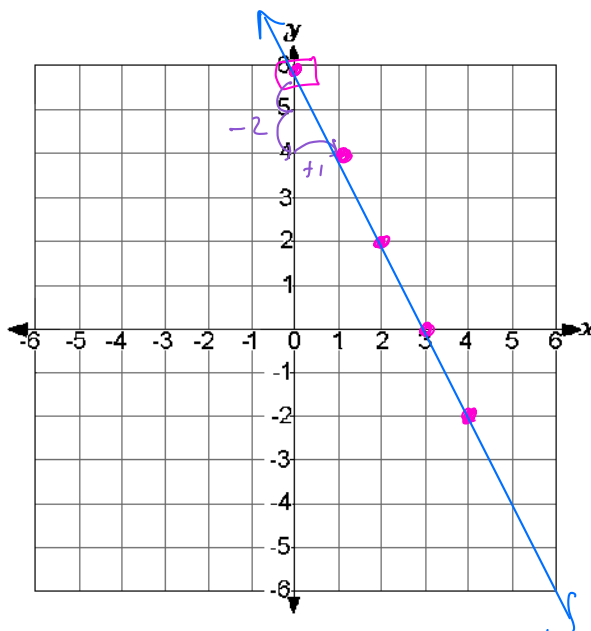
y-intercept: 6 or $(0, 6)$

Slope (m): -2 or $-\frac{2}{1}$

Plot $(0, 6)$ \square

travel $-\frac{2}{1}$ \leftarrow down 2
 $1 \leftarrow$ right 1
 until next point

connect the dots (remember arrows)



*rearrange first! $y = mx + b$

2. $x - 2y = 4$
 $-x$ $-x$

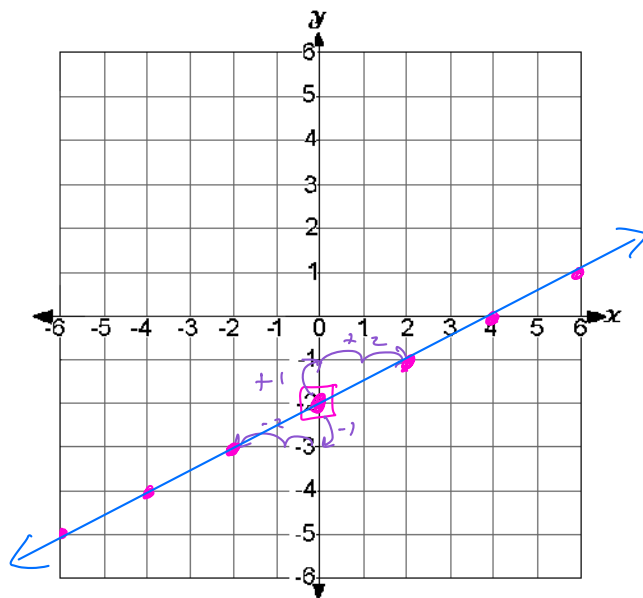
$$\frac{-2y}{-2} = \frac{-x + 4}{-2} \quad \frac{-x}{-2}$$

$$y = +\frac{1}{2}x - 2$$

y-intercept: -2

Slope (m): $\frac{1}{2}$ \leftarrow up 1
 $\frac{2}{2}$ \leftarrow right 2

(or $\frac{-1}{-2}$ \leftarrow down 1
 $\frac{-2}{-2}$ \leftarrow left 2)

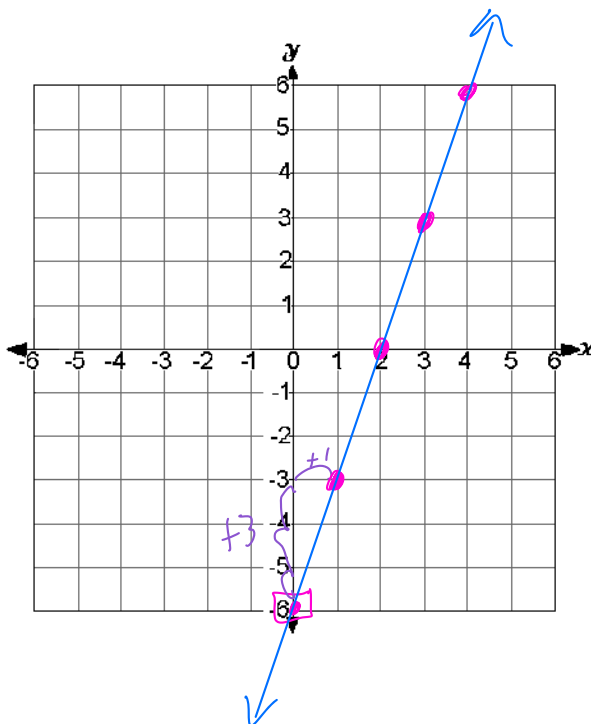


Your turn!

1. $y = 3x - 6$

y-intercept: -6

Slope (m): $\frac{+3}{1}$

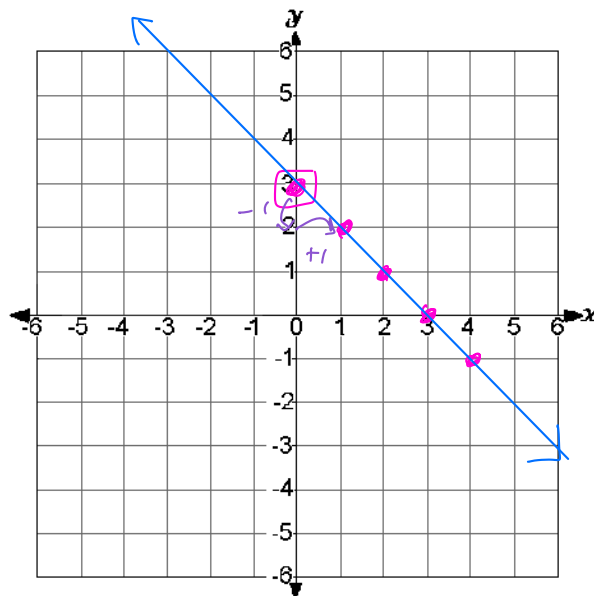


$$2. \quad \begin{array}{l} \overbrace{x + y = 3} \\ -x \quad -x \end{array}$$

$$y = -x + 3$$

$$\text{y-intercept: } \underline{\quad 3 \quad}$$

$$\text{Slope (m): } \underline{\quad \frac{-1}{1} \quad} \quad \begin{array}{l} \text{dn} \\ \text{rt} \end{array}$$



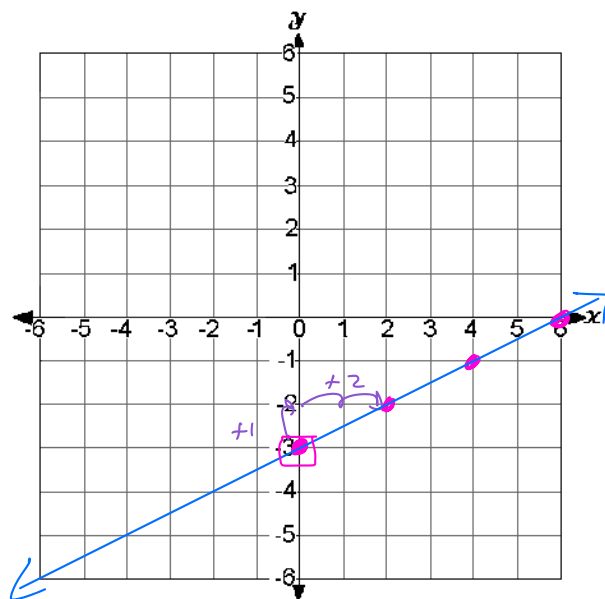
$$3. \quad \begin{array}{l} \overbrace{x - 2y = 6} \\ -x \quad -x \end{array}$$

$$\frac{-2y}{-2} = \frac{-x + 6}{-2}$$

$$y = \frac{1}{2}x - 3$$

$$\text{y-intercept: } \underline{\quad -3 \quad}$$

$$\text{Slope (m): } \underline{\quad \frac{+1}{2} \quad} \quad \begin{array}{l} \text{up} \\ \text{rt} \end{array}$$



D. FACTORING

- 1) Check for a Greatest Common Factor (GCF)
- 2) How many terms are there? If there are TWO terms, check if it's a DIFFERENCE of SQUARES
- 3) If there are THREE terms in the form $ax^2 + bx + c$, check if $a = 1$
- 4) If there are THREE terms in the form $ax^2 + bx + c$, and $a \neq 1$, you'll need a few extra steps
- 5) Check at the end to see if anything factors further
- 6) Optional: Expand out again (distribute/FOIL) to check that it's correct

Factoring a GCF

Look for numbers and/or variables that can "divide out" of EVERY term. (Terms are separated by add/subtract/equals signs).

Divide the common factor out of every term, write the common factor out front, and write the "leftovers" in a bracket.

Examples: Factor

1. $4x^2 + 12$

Think:
both \div by
4

GCF=4
write out
front

$$4(x^2 + 3)$$

Check:

$$4(x^2 + 3) \\ 4x^2 + 12 \quad \checkmark$$

Your turn! Factor

1. $-5x^2 + 10$

$$= -5(x^2 - 2)$$

2. $6x^3 + 5x^2$

Think:
 \div out
 x^2

$$\frac{6x^3}{x^2} + \frac{5x^2}{x^2} \\ 6x + 5$$

GCF:
 x^2

$$x^2(6x + 5)$$

Check:

$$x^2(6x + 5) \\ 6x^3 + 5x^2 \quad \checkmark$$

2. $4x^2 + 9x$

$$= x(4x + 9)$$

3. $-9x^2 + 12x + 3$

TIP: always divide out a negative as well if your highest power of x is negative!

$$-3(3x^2 - 4x - 1)$$

gcf: -3
 \div out of all
terms and
write out
front!

Check: $-3(3x^2 - 4x - 1)$

$$-9x^2 + 12x + 3 \quad \checkmark$$

3. $12x^3y^2 + 6x^2y^5 - 21xy^3$

$$= 3xy^2(4x^2 + 2xy^3 - 7y)$$

Factoring a Difference of Squares

If you have TWO terms, check if it's a "Difference of Squares" (a perfect square with a SUBTRACT and another perfect square). Something of the form:

$$a^2 - b^2$$

Which will always factor into $(a + b)(a - b)$

(these brackets are called "conjugates")

*Remember to check for a GCF first, and remember that you can always check by expanding out! Also, check if anything factors FURTHER.

Examples: Factor

1. $4x^2 - 25$

Set-up: $(\quad + \quad)(\quad - \quad)$

$$(2x + 5)(2x - 5)$$

$$\sqrt{4x^2} = 2x$$

$$\sqrt{25} = 5$$

same bracket, except with opposite sign!

2. $2t^5 - 162t$

$$2t(t^4 - 81)$$

$$2t(t^2 + 9)(t^2 - 9)$$

factors more!

$$2t(t^2 + 9)(t + 3)(t - 3)$$

3. $x^4 - 1$

$$(x^2 + 1)(x^2 - 1)$$

factors more!

$$(x^2 + 1)(x + 1)(x - 1)$$

Your turn! Factor

1. $x^2 - 9$

$$= (x + 3)(x - 3)$$

2. $8x^2 - 72$

$$= 8(x^2 - 9)$$

$$= 8(x + 3)(x - 3)$$

3. $25x^4 - 16$

$$= (5x^2 + 4)(5x^2 - 4)$$

Factoring a Trinomial when $a = 1$

If you have a trinomial of the form $x^2 + bx + c$, then

- 1) check to see if you can find the two numbers that

$$\underline{\quad} \times \underline{\quad} = c \text{ (multiply to the outside)}$$

$$\underline{\quad} + \underline{\quad} = b \text{ (add to the middle)}$$

- 2) If those numbers exist, then the trinomial factors into two brackets with those numbers "dropped in". Remember the + or - sign.

$$(x \underline{\quad})(x \underline{\quad})$$

*Remember to check for a GCF first, and remember that you can always check by expanding out!

Examples: Factor

$$\begin{array}{r} 7 \quad -4 \\ \underline{7} \times \underline{-4} = -28 \\ \underline{7} \quad \underline{-4} = 3 \end{array}$$

$$\begin{array}{r} -8 \quad -4 \\ \underline{-8} \times \underline{-4} = 32 \\ \underline{-8} \quad \underline{-4} = -12 \end{array}$$

$$\begin{array}{r} -11 \quad 6 \\ \underline{-11} \times \underline{6} = -66 \\ \underline{-11} \quad \underline{6} = -5 \end{array}$$

1. $x^2 + 9x + 20$

$$\begin{array}{r} 4 \quad 5 \\ \underline{4} \times \underline{5} = 20 \\ \underline{4} \quad \underline{5} = 9 \end{array}$$

Setup $\rightarrow (x \quad)(x \quad)$
 $= (x + 4)(x + 5)$

2. $x^2 + 3x - 28$

$$(x \quad)(x \quad) = (x + 7)(x - 4)$$

3. $z^2 - 12z + 32$

$$(z - 8)(z - 4)$$

4. $a^2 - 5a - 66$

$$(a - 11)(a + 6)$$

Your turn! Factor

1. $x^2 + 7x + 12$

$$\begin{array}{r} 3 \quad 4 \\ \underline{3} \times \underline{4} = 12 \\ \underline{3} \quad \underline{4} = 7 \end{array}$$

$$= (x + 3)(x + 4)$$

2. $x^2 - 9x + 8$

$$\begin{array}{r} -1 \quad -8 \\ \underline{-1} \times \underline{-8} = 8 \\ \underline{-1} \quad \underline{-8} = -9 \end{array}$$

$$= (x - 1)(x - 8)$$

3. $x^2 + 5x - 66$

$$\begin{array}{r} +11 \quad -6 \\ \underline{+11} \times \underline{-6} = -66 \\ \underline{+11} \quad \underline{-6} = 5 \end{array}$$

$$= (x + 11)(x - 6)$$

4. $x^2 - 6x - 27$

$$\begin{array}{r} -9 \quad 3 \\ \underline{-9} \times \underline{3} = -27 \\ \underline{-9} \quad \underline{3} = -6 \end{array}$$

$$= (x - 9)(x + 3)$$

Factoring a Trinomial when $a \neq 1$

If you have a trinomial of the form $ax^2 + bx + c$, and $a \neq 1$, then you can use EITHER the “ac method” or “decomposition”.

Example: Factor $3x^2 + 16x + 5$

$$a \times c = 3 \times 5 = 15$$

$$b = 16$$

ac method	Decomposition
<p>1) check to see if you can find the two numbers that</p> <p><i>Use 1 and 15 below ↓</i></p> $\left\{ \begin{array}{l} \underline{1} \times \underline{15} = 15 \text{ ac (multiply to the outside numbers multiplied together, a times c)} \\ \underline{1} + \underline{15} = 16 \text{ b (add to the middle)} \end{array} \right.$ <p>If those numbers exist, then you can use them for EITHER the “ac method” or “decomposition”.</p>	
<p>2) Write two helper brackets set up with x in each bracket and a fraction over “a”, like this:</p> $\left(x \quad \frac{\quad}{a}\right) \left(x \quad \frac{\quad}{a}\right)$ <p>Here, $a = \underline{3}$, so the helper brackets are</p> $\left(x \quad \frac{\quad}{3}\right) \left(x \quad \frac{\quad}{3}\right)$	<p>2) “split” the middle (b) term into two terms using the numbers you found in step 1 (that multiply to ac and add to b)</p> $ax^2 + \underline{\quad}x + \underline{\quad}x + c$ <p>Here, we would get</p> $3x^2 + \underline{1}x + \underline{15}x + 5$ <p style="text-align: center;"> $\underbrace{\hspace{1.5cm}}_{\text{gcf: } x} \quad \underbrace{\hspace{1.5cm}}_{\text{gcf: } 5}$ </p>
<p>3) Drop the numbers you found in step 1 (that multiply to ac and add to b) into the brackets “over a”</p> $\left(x + \frac{1}{3}\right) \left(x + \frac{15}{3}\right)$ <p style="text-align: center;"><i>Reduce</i></p>	<p>3) Group the first two terms together, and the last two terms, and take out a GCF from each pair:</p> $x(3x+1) + 5(3x+1)$ <p style="text-align: right;"><i>✓ match!</i></p>
<p>4) Reduce the fractions (if they reduce), and any fractions that do not reduce, take the bottom of the fraction and make it the coefficient of x:</p> $(3x+1)(x+5)$	<p>4) The <u>brackets should match</u>. They are a “common bracket” and can be <u>factored out</u>. Write the matching bracket once, and the “leftovers” in their <u>own bracket</u>:</p> $(3x+1)(x+5)$ <p style="text-align: center;"> <i>(matching bracket) (leftovers)</i> </p>

Same answer!

$$\begin{array}{r} 10 \quad -5 \\ \underline{-x \quad -} \\ 10 \quad + \quad -5 \\ \hline \end{array} = -50$$

$$\begin{array}{r} 10 \quad -5 \\ \underline{+ \quad -} \\ 10 \quad + \quad -5 \\ \hline \end{array} = 5$$

$$\begin{array}{r} -4 \quad -3 \\ \underline{x \quad -} \\ -4 \quad + \quad -3 \\ \hline \end{array} = 12$$

$$\begin{array}{r} -4 \quad -3 \\ \underline{+ \quad -} \\ -4 \quad + \quad -3 \\ \hline \end{array} = -7$$

Examples: Factor

1. $2x^2 + 5x - 25$

helper brackets:

$$\left\{ \begin{array}{l} (x - \frac{5}{2})(x - \frac{5}{2}) \\ (x + \frac{10}{2})(x - \frac{5}{2}) \end{array} \right.$$

reduce! move denom!

$$= \boxed{(x+5)(2x-5)}$$

Decomp. 2.

$$6y^2 - 7y + 2$$

$$6y^2 - 4y - 3y + 2$$

gcf: 2y gcf: -1

$$2y(3y-2) - 1(3y-2)$$

$$\boxed{(3y-2)(2y-1)}$$

* always take out a negative from the highest power of x!

Your turn! Factor

1. $3x^2 - 13x + 12$

$$\begin{array}{r} -9 \quad -4 \\ \underline{-x \quad -} \\ -9 \quad + \quad -4 \\ \hline \end{array} = 36$$

$$\begin{array}{r} -9 \quad -4 \\ \underline{+ \quad -} \\ -9 \quad + \quad -4 \\ \hline \end{array} = -13$$

(ac)

$$\left(x - \frac{9}{3} \right) \left(x - \frac{4}{3} \right)$$

$$= \boxed{(x-3)(3x-4)}$$

2. $2x^2 + 3x - 14$

$$\begin{array}{r} +7 \quad -4 \\ \underline{x \quad -} \\ +7 \quad + \quad -4 \\ \hline \end{array} = -28$$

$$\begin{array}{r} +7 \quad -4 \\ \underline{+ \quad -} \\ +7 \quad + \quad -4 \\ \hline \end{array} = 3$$

(ac)

$$\left(x + \frac{7}{2} \right) \left(x - \frac{4}{2} \right)$$

$$= \boxed{(2x+7)(x-2)}$$

decomp

$$3x^2 - 13x + 12$$

$$= \underbrace{3x^2 - 9x}_{\text{gcf: } 3x} - \underbrace{4x + 12}_{\text{gcf: } -4}$$

$$= 3x(x-3) - 4(x-3)$$

$$= \boxed{(x-3)(3x-4)}$$

decomp

$$2x^2 + 3x - 14$$

$$= \underbrace{2x^2 + 7x}_{\text{gcf: } x} - \underbrace{4x - 14}_{\text{gcf: } -2}$$

$$= x(2x+7) - 2(2x+7)$$

$$= \boxed{(2x+7)(x-2)}$$

E. SOLVING QUADRATIC EQUATIONS

Quadratic Equations can have 0, 1, or 2 solutions.

Get organized!

- 1) Move all terms to one side of the equation, leaving zero on the other side
- 2) If you can factor, factor! The SOLUTIONS are the "zeros of the brackets."
- 3) If you can't factor, use the quadratic formula:

For $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solving Quadratic Equations by Factoring

Examples: Solve by factoring

1. $x^2 = 13x + 48$
 $-13x - 48 \quad -13x \quad -48$

$$x^2 - 13x - 48 = 0$$

$$-16 \times 3 = -48$$

$$-16 + 3 = -13$$

$$(x - 16)(x + 3) = 0$$

the "zero" is the x-value that makes this bracket = 0
 $x - 16 = 0$
 $x + 3 = 0$
 $x = -3$

$x = 16$
 $\therefore x = -3, 16$

Your turn! Solve by factoring:

1. $x^2 - 5x + 6 = 0$

$$(x - 2)(x - 3) = 0$$

$$x = 2, x = 3$$

2. $6x^2 + 4x = 0$

$$2x(3x + 2) = 0$$

Think:

$$2(x)(3x + 2) = 0$$

$$x = 0$$

$$3x + 2 = 0$$

$$3x = -2$$

$$x = -\frac{2}{3}$$

$$\therefore x = 0, -\frac{2}{3}$$

3. $\frac{1}{2}x^2 + \frac{3}{8} = x$

Clear fractions! x all terms by 8

$$4x^2 + 3 = 8x$$

$$4x^2 - 8x + 3 = 0$$

$$(x - \frac{3}{4})(x - \frac{1}{4}) = 0$$

$$(x - \frac{3}{2})(x - \frac{1}{2}) = 0$$

$$-6 \times -2 = 12$$

$$-6 + -2 = -8$$

a ≠ 1 ac method. Use decomp if you prefer!

$$(2x - 3)(2x - 1) = 0$$

$$\therefore x = \frac{3}{2}, \frac{1}{2}$$

2. $x^2 + 7x + 10 = 0$

$$x^2 + 7x + 10 = 0$$

$$2 \times 5 = 10$$

$$2 + 5 = 7$$

$$(x + 2)(x + 5) = 0$$

$$x = -2, x = -5$$

3. $2x^2 = 27 - 15x$

$$2x^2 + 15x - 27 = 0$$

$$(2x - 3)(x + 9) = 0$$

$$x = \frac{3}{2}, x = -9$$

ac $(x + \frac{18}{2})(x - \frac{3}{2})$
 $(x + 9)(2x - 3)$
decomp $2x^2 + 18x - 3x - 27$
 $2x(x + 9) - 3(x + 9)$
 $(x + 9)(2x - 3)$

need $ax^2 + bx + c = 0$ first!

Solving Quadratic Equations with the Quadratic Formula

Example: Solve using the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(give exact answers AND decimal approx.)

1. $5x^2 + 2x = 6$

$$5x^2 + 2x - 6 = 0$$

$a=5$ $b=2$ $c=-6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(5)(-6)}}{2(5)}$$

$$x = \frac{-2 \pm \sqrt{4 + 120}}{10}$$

BEDMAS

$$x = \frac{-2 \pm \sqrt{124}}{10}$$

simplify radicals
 $\sqrt{124}$
 $= \sqrt{4 \cdot 31}$
 $= 2\sqrt{31}$

$$x = \frac{(-2 \pm 2\sqrt{31})}{(10)} \div 2$$

$$x = \frac{-1 \pm \sqrt{31}}{5}$$

Exact answers

$$x = \frac{-1 + \sqrt{31}}{5}$$

$$x = \frac{-1 - \sqrt{31}}{5}$$

$$x = 0.91355$$

$$x = -1.31355$$

Decimal Approx:

Your turn! Solve using the quadratic formula (give exact answers AND decimal approx.)

1. $2x^2 + 3 = 7x$

$$2x^2 - 7x + 3 = 0$$

$a=2$ $b=-7$ $c=3$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(3)}}{2(2)}$$

$$x = \frac{7 \pm \sqrt{49 - 24}}{4}$$

$$x = \frac{7 \pm \sqrt{25}}{4}$$

$$x = \frac{(7 \pm 5)}{4}$$

$$x = \frac{(7+5)}{4} \text{ or } x = \frac{(7-5)}{4}$$

$$x = \frac{12}{4}$$

$$x = \frac{2}{4}$$

$$x = 3 \quad x = \frac{1}{2}$$

Exact value (and no need for decimal approx.)

(This one would have factored!)

2. $x^2 - 10x = 9$

$$x^2 - 10x - 9 = 0$$

$a=1$ $b=-10$ $c=-9$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(-9)}}{2(1)}$$

$$x = \frac{10 \pm \sqrt{100 + 36}}{2}$$

$$x = \frac{10 \pm \sqrt{136}}{2}$$

$\sqrt{136}$
 $= \sqrt{4 \cdot 34}$
 $= 2\sqrt{34}$

$$x = \frac{(10 \pm 2\sqrt{34})}{(2)} \div 2$$

$$x = 5 \pm \sqrt{34} \leftarrow \text{exact value}$$

$$\begin{cases} x = 5 + \sqrt{34} = 10.83095 \\ \text{or} \\ x = 5 - \sqrt{34} = -0.83095 \end{cases}$$

decimal approx.

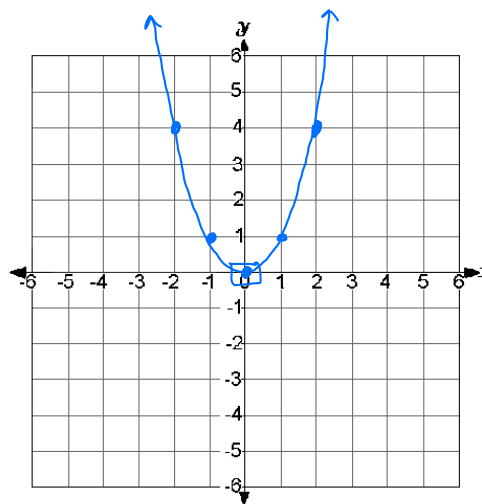
F. GRAPHING QUADRATIC FUNCTIONS $f(x) = ax^2 + bx + c$

Graphing the Basic Quadratic Function $y = x^2$

Use a table of values to generate the graph of $y = x^2$:

x	$y = (x)^2$
-3	$9 = (-3)^2 = 9$
-2	$4 = (-2)^2 = 4$
-1	$1 = (-1)^2 = 1$
0	$0 = (0)^2 = 0$
1	$1 = (1)^2 = 1$
2	$4 = (2)^2 = 4$
3	$9 = (3)^2 = 9$

vertex →



The shape generated by a quadratic function is called a PARABOLA

The VERTEX is where the graph "turns around". Here, (0, 0)

Direction of Opening: this one opens up!

Sketching Quadratic Functions $f(x) = ax^2 + bx + c$

$f(x)$ means "y"

In Math 11, we learned how to graph quadratic functions using the vertex and the "basic count."

We can also get a **quick sketch** of a quadratic function by finding its **x-intercepts**, and using the "a" value to determine whether the parabola opens up or down.

x-intercepts (when $y=0$)

Set $y=0$ and solve the quadratic equation (by factoring or quadratic formula)

The "solutions" are the x-intercepts of your graph!

a-value

- When a is positive, the parabola Opens up ↶ ↷
- When a is negative, the parabola Opens down ↷ ↶

y-intercept (when $x=0$)

Set $x=0$ and evaluate for y . (Optional)

To sketch:

- Find the x-intercepts (set $y=0$ and solve for x) of the graph and plot them
- Use "a" to decide if the parabola opens up or down
- Sketch the parabola going through the x-intercepts and opening in the correct direction
- (Optional – find and plot the y-intercept. Set x equal to zero and find y .)

Example: Sketch $y = -x^2 - 5x + 6$ using x-intercepts.

x-int: set $y=0$

$$y = -x^2 - 5x + 6$$

$$0 = -x^2 - 5x + 6$$

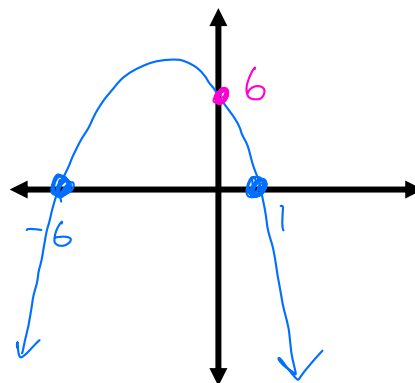
get: -1

$$0 = -1(x^2 + 5x - 6)$$

$$0 = -(x-1)(x+6)$$

$\therefore x=1 \quad x=-6 \rightarrow$ ① plot x-intercepts!
 $(-6, 0)$ and $(1, 0)$

$$\begin{aligned} -1 \times 6 &= -6 \\ -1 + 6 &= 5 \end{aligned}$$



y-int: set $x=0$

$$y = -x^2 - 5x + 6$$

$$y = -(0)^2 - 5(0) + 6$$

$$y = 6$$

- Sketch "down" parabola
- Label y-int $(0, 6)$ (optional)

To sketch:

- 1) Find the x-intercepts (set $y = 0$ and solve for x) of the graph and plot them
- 2) Use "a" to decide if the parabola opens up or down
- 3) Sketch the parabola going through the x-intercepts and opening in the correct direction
- 4) (Optional – find and plot the y-intercept. Set x equal to zero and find y .)

Your turn!

1. Sketch $y = x^2 + 2x - 8$ using x-intercepts.

x-int: (set $y=0$)

$$y = x^2 + 2x - 8$$

$$0 = x^2 + 2x - 8$$

$$0 = (x+4)(x-2)$$

$$\therefore x = -4 \quad x = 2 \quad \textcircled{1} \text{ Plot } (-4, 0) \quad (2, 0)$$

$a=1$ (positive) so parabola opens UP $\textcircled{2}$ Sketch parabola going UP

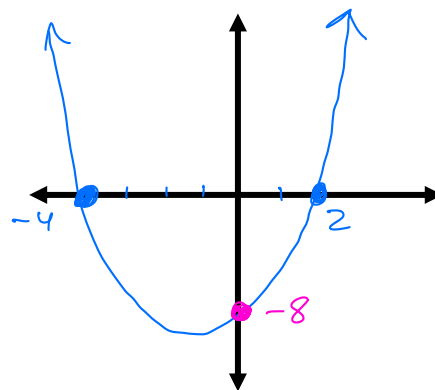
y-int (set $x=0$)

$$y = x^2 + 2x - 8$$

$$y = 0^2 + 2(0) - 8$$

$$y = -8$$

$\textcircled{3}$ Label y-int at $(0, -8)$



2. Sketch $y = -2x^2 - 9x + 5$ using x-intercepts.

x-int: (set $y=0$)

$$y = -2x^2 - 9x + 5$$

$$0 = -2x^2 - 9x + 5$$

$$0 = -(2x^2 + 9x - 5)$$

$$0 = -(2x - 1)(x + 5)$$

$$\therefore x = \frac{1}{2} \quad x = -5 \quad \textcircled{1} \text{ Plot } (-5, 0) \quad (0.5, 0)$$

$a = -2$ (a is negative, opens down) $\textcircled{2}$ Sketch down parabola

y-int (set $x=0$)

$$y = -2x^2 - 9x + 5$$

$$y = 5$$

$\textcircled{3}$ Label y-int $(0, 5)$

$$\begin{array}{r} 10 \cdot -1 = -10 \\ 10 \cdot -1 = 9 \end{array}$$

Decomp $2x^2 + 9x - 5$
 $2x^2 + 10x - 1x - 5$
 $2x(x+5) - 1(x+5)$
 $(x+5)(2x-1)$

