PC12 Name______________________________

Transformations

Chapter Notes

Key

Assignment List

Date	Lesson	Assignment
	1. Basic Graphs	Memorize basic graphs (there will be a quiz on these) &
	2. Functions and Relations	Mickelson Page 53 #1-3
	3. Vertical and Horizontal Translations	Transformations Supplemental Practice Handout #1-3
	4. Reflections, Expansions, and Compressions	Transformations Supplemental Practice Handout #4-6
	5. Combinations of Transformations	Transformations Supplemental Practice Handout #7-9
	6. Inverse Graphs & Equations	Transformations Supplemental Practice Handout #10-12 & Mickelson Pg. 90 #1,3,4,6 Optional: pg. 90 #8 (can graph by hand or Desmos)
		Practice Test
		Review
		Transformations Test

Transformations Day 1: Basic Graphs

* Identify 3 key points. (Highlighted in yellow)

For each of the given equations, sketch the graph by first creating a table of values.

- Give the domain & range and the equations of any asymptotes.
- State if the relation is a function, one-to-one function, or neither.

} Revisit after "day 2" notes.

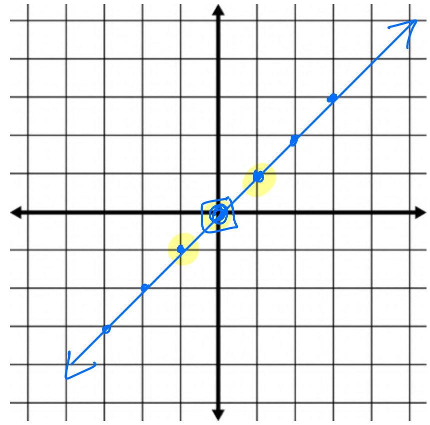
Polynomials

(Linear)

1. $y = x$

x	y $= (x)$
−3	−3
−2	−2
−1	−1
$\boxed{0}$	$\boxed{0}$
1	1
2	2
3	3

(Line)



D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R}\}$

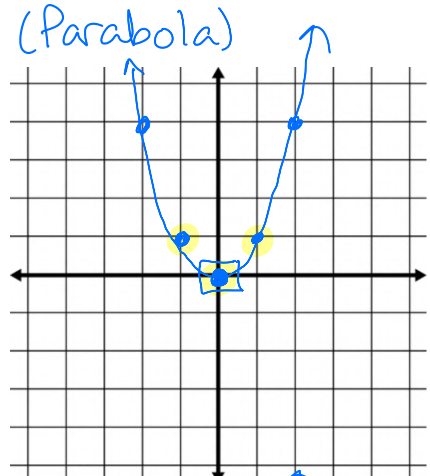
one-to-one function

(Quadratic)

2. $y = x^2$

x	y	$= (x)^2$
−3	9	$= (-3)^2 = 9$
−2	4	$= (-2)^2 = 4$
−1	1	$= (-1)^2 = 1$
$\boxed{0}$	$\boxed{0}$	$= (0)^2 = 0$
1	1	$= (1)^2 = 1$
2	4	$= (2)^2 = 4$
3	9	$= (3)^2 = 9$

(Parabola)



D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R} \mid y \geq 0\}$

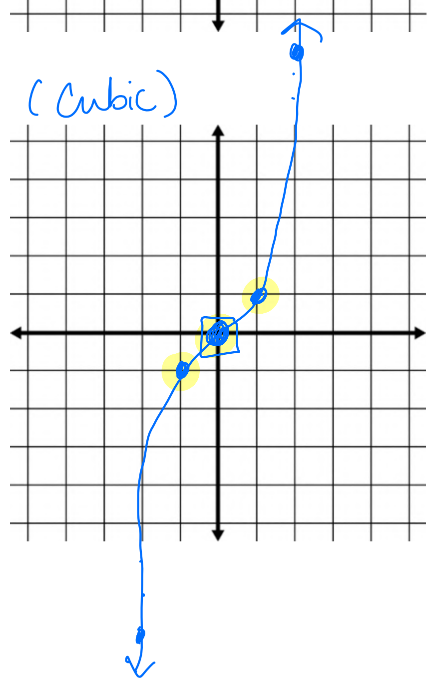
function

(cubic)

3. $y = x^3$

x	y	$= (x)^3$
−3	−27	$= (-3)^3 = -27$
−2	−8	$= (-2)^3 = -8$
−1	−1	$= (-1)^3 = -1$
$\boxed{0}$	$\boxed{0}$	$= (0)^3 = 0$
1	1	$= (1)^3 = 1$
2	8	$= (2)^3 = 8$
3	27	$= (3)^3 = 27$

(Cubic)



D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R}\}$

one-to-one function

NEW!

The Absolute Value Function: $y = |x|$

order of operations.

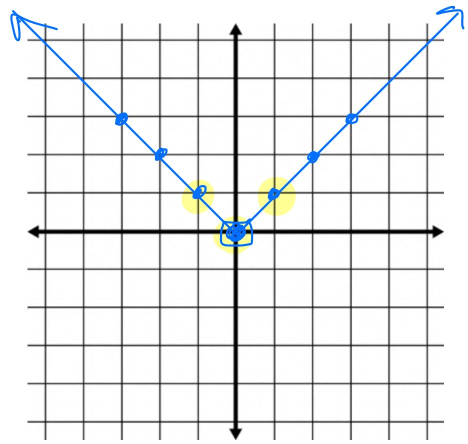
★ important! they are Brackets. Remember BEDMAS.

What do the absolute value brackets | | do? They guarantee that the "output" of the bracket will be positive.

$|5| =$ 5 $\qquad\qquad$ $|-6| =$ 6

4. $y = |x|$

x	y = |x|	
−3	3	= |−3| = 3
−2	2	= |−2| = 2
−1	1	= |−1| = 1
0	0	= |0| = 0
1	1	= |1| = 1
2	2	= |2| = 2
3	3	= |3| = 3



D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R} \mid y \geq 0\}$

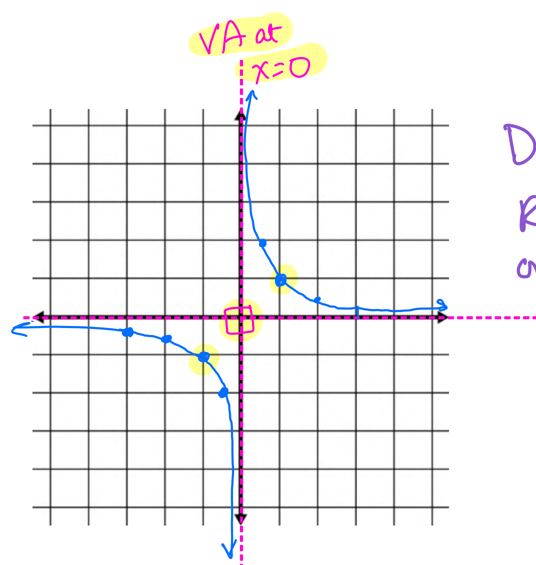
function

The Reciprocal Function $y = \frac{1}{x}$

What are the restrictions on x? $x \neq 0$ (can't divide by zero)

5. $y = \frac{1}{x}$

x	y = $\frac{1}{(x)}$	
−3	$-\frac{1}{3}$	= $\frac{1}{(-3)}$
−2	$-\frac{1}{2}$	= $1/(-2)$
−1	−1	= $1/(-1)$
0	?	= $1/(0)$ ⚠ undefined. VA x=0 HA y=0
1	1	= $1/(1)$
2	$\frac{1}{2}$	= $1/(2)$
3	$\frac{1}{3}$	= $1/(3)$
$\frac{1}{2}$	2	
$-\frac{1}{2}$	−2	



HA at y=0

D: $\{x \in \mathbb{R} \mid x \neq 0\}$

R: $\{y \in \mathbb{R} \mid y \neq 0\}$

one-to-one function

The Square Root Function: $y = \sqrt{x}$

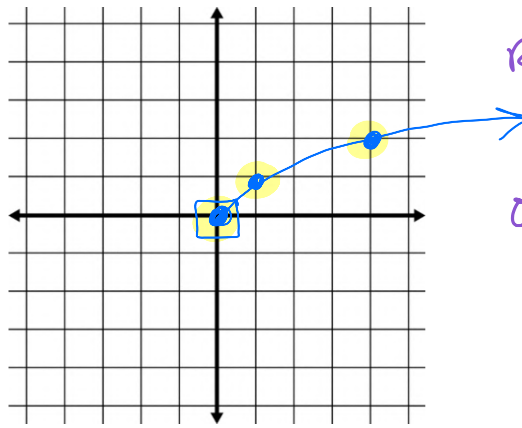
"whatever is under the root"

What are the restrictions on x? x can't be negative / x must be greater than or equal to zero

$x \geq 0$

6. $y = \sqrt{x}$

x	y	$= \sqrt{(x)}$
-2	?	$= \sqrt{-2}$ = undefined
-1	?	$= \sqrt{-1}$ = undefined
$\boxed{0}$	$\boxed{0}$	$= \sqrt{0} = 0$
1	1	$= \sqrt{1} = 1$
2	~ 1.4	$= \sqrt{2} = 1.414\ldots$
4	2	$= \sqrt{4} = 2$
9	3	$= \sqrt{9} = 3$
16	4	$= \sqrt{16} = 4$

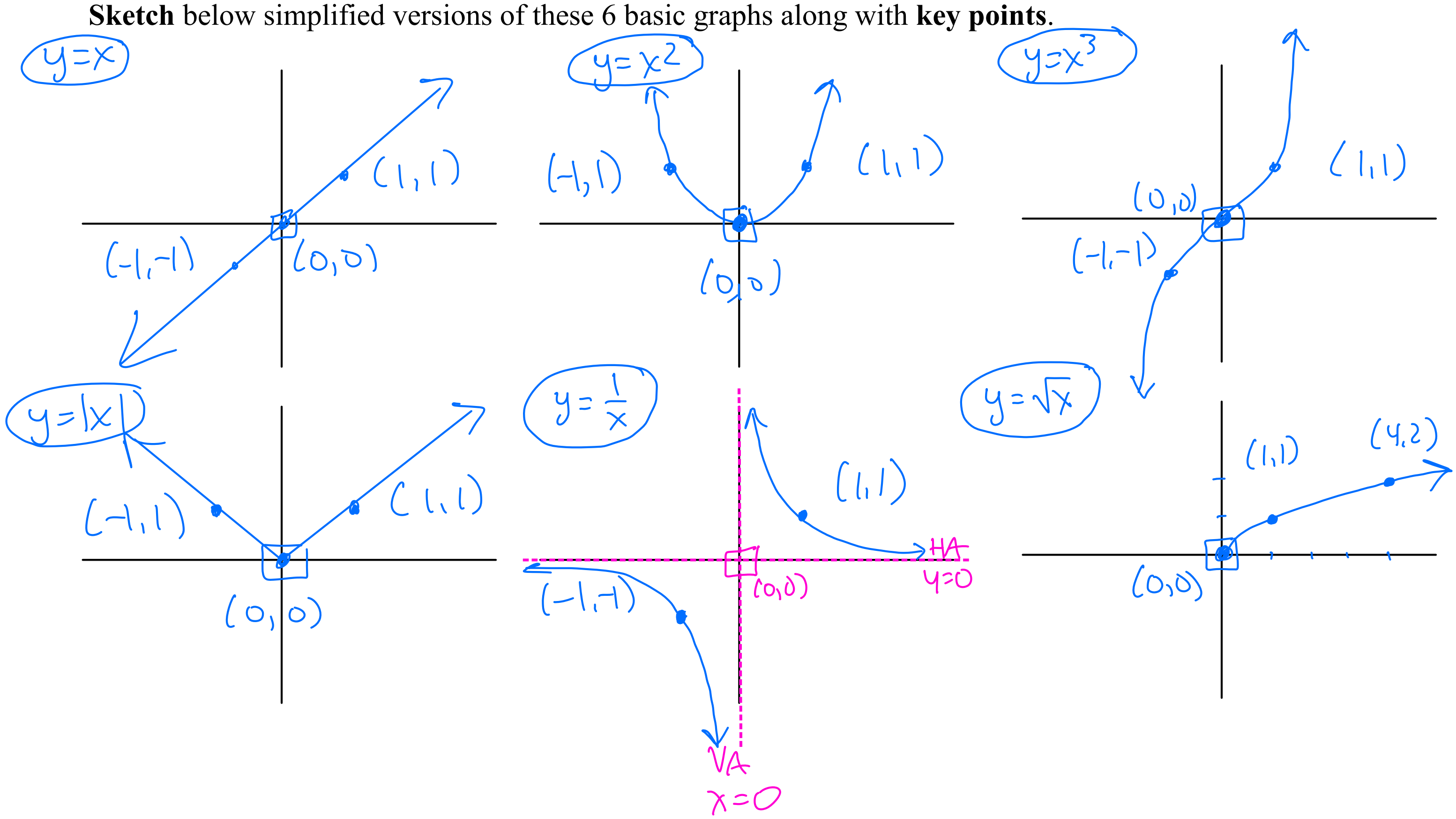


$D: \{x \in \mathbb{R} \mid x \geq 0\}$

$R: \{y \in \mathbb{R} \mid y \geq 0\}$

One-to-one function

Sketch below simplified versions of these 6 basic graphs along with **key points**.



Assignment: **Memorize** these simplified sketches **and** their key points!

$\underbrace{\text{these simplified sketches}}$ basic shape + $\approx$ 3 key points / features

Transformations Day 2: Functions and Relations

Definitions:

Domain - the set of all possible (x) values of a relation.

Range - the set of all possible (y) values of a relation.

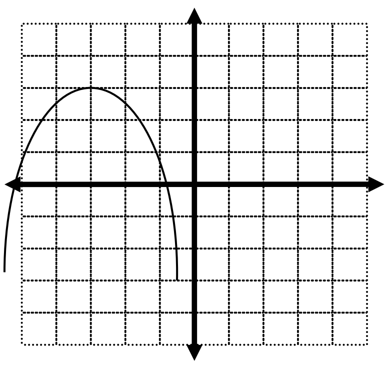
Relation - a set of ordered pair(s)

Function - a relation in which each domain (x) value is paired with only one unique range (y) value.

Vertical line test - an equation defines y as a <u>function</u> of x if and only if every vertical line in the coordinate plane intersects the graph of the equation only once.

Example 1: Determine the domain/range of the following graphs and whether they are a function/relation

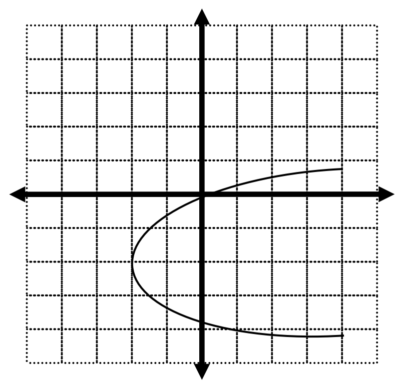
set notation



D: $\{x \in \mathbb{R}\}$

R: $\{y \in \mathbb{R} \mid y \leq 3\}$

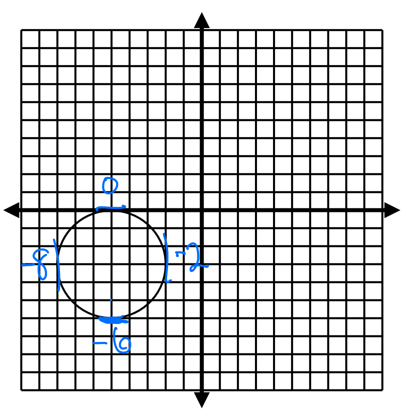
function



D: $\{x \in \mathbb{R} \mid x \geq -2\}$

R: $\{y \in \mathbb{R}\}$

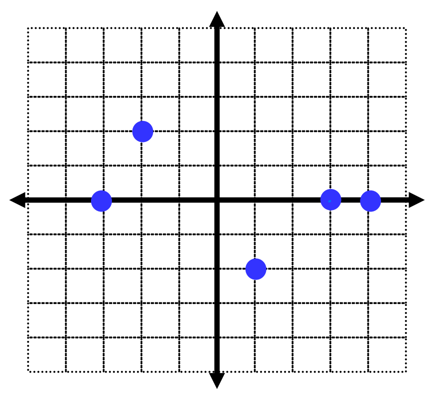
relation



$\{x \in \mathbb{R} \mid -8 \leq x \leq -2\}$

$\{y \in \mathbb{R} \mid -6 \leq y \leq 0\}$

relation



$\{x \mid x = -3, -2, 1, 3, 4\}$

$\{y \mid y = -2, 0, 2\}$

function

Restrictions on the domain of a functions:

1. Cannot have a negative number inside an even root. $f(x)=\sqrt{3-x}$

$3-x \geq 0$

$3 \geq x$

$\boxed{x \leq 3}$

2. Cannot have zero in a denominator $f(x)=\dfrac{8}{x-6}$ $x \neq 6$

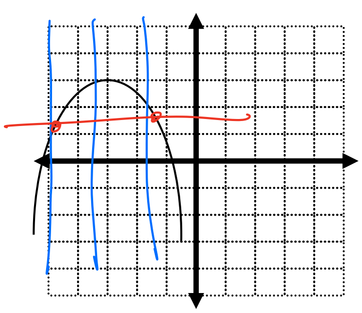
One–to–One Function

A one-to-one function is a function in which every single value of the domain is associated with only one value in the range, and vice-versa.

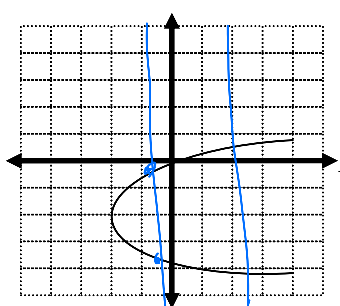
Horizontal Line Test - for a one-to-one function:

A function, f(x), is a one-to-one function of x if and only if every horizontal line in the coordinate plane intersects the function only once at most.

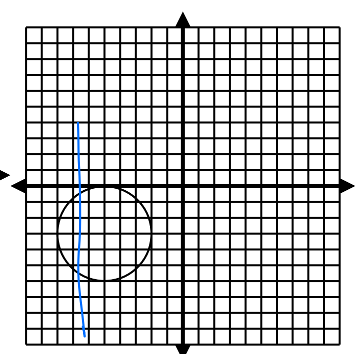
Example 2: Determine whether the following relations are functions, one-to-one functions or neither



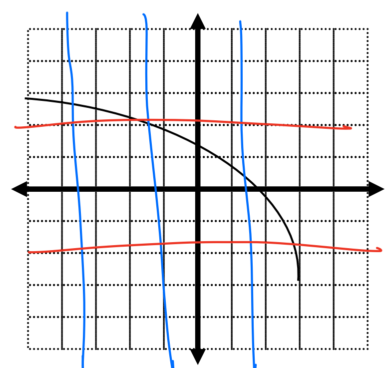
function



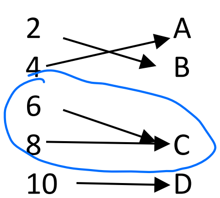
neither



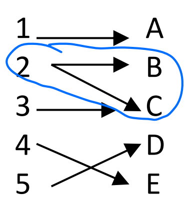
neither



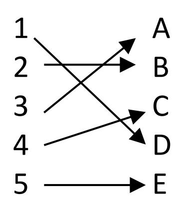
one to one function



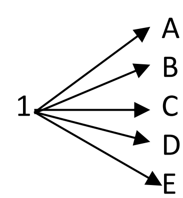
function



neither



one to one function



neither

Transformations Day 3 Vertical and Horizontal Translations

Vertical translations **(shifting the graph up or down)** form: $y = f(x-h) + \boldsymbol{k}$

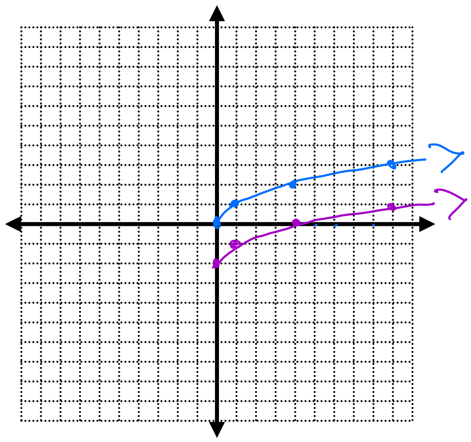
Square Root

$y = \sqrt{x} - 2$

$y=\sqrt{x}$

$y=\sqrt{x}-2$

x	y
0	-2
1	-1
4	0
9	1



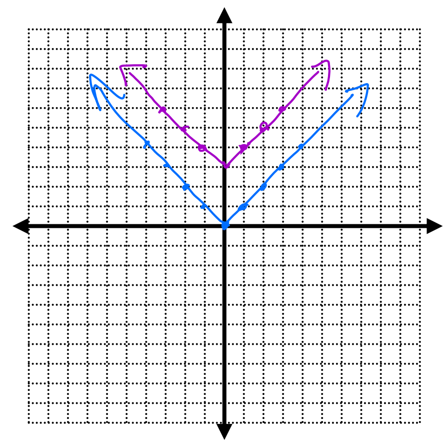
Absolute Value

$y = |x| + 3$

$y=|x|$

$y=|x|+3$

x	y
-3	6
-2	5
-1	4
0	3
1	4
2	5
3	6

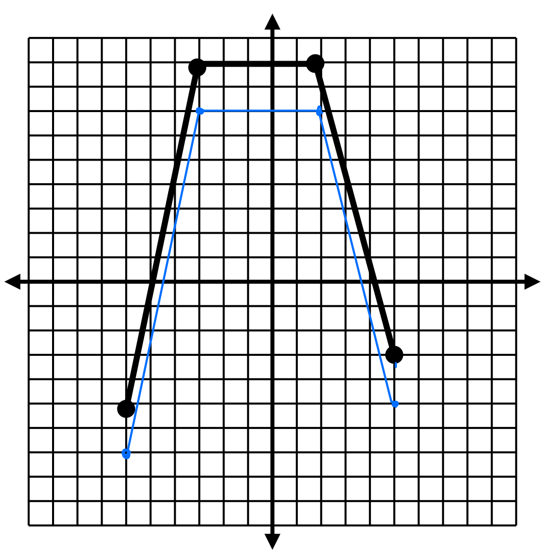


So for $y = f(x-h) + \boldsymbol{k}$

If $\boldsymbol{k}$ is positive → vertical translation up k units

If $\boldsymbol{k}$ is negative → vertical translation down k units

Example 1: Given the graph of $y = f(x)$ below, describe the transformation applied graph $y = f(x) - 2$, and map the coordinates of the image points.



Vertical translation down 2 units

$(-6,-5) \rightarrow (-6,-7)$

$(3,9) \rightarrow (3,7)$

$(2,9) \rightarrow (2,7)$

$(5,-3) \rightarrow (5,-5)$

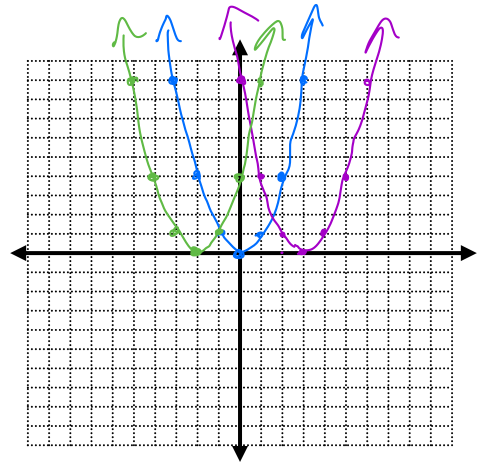
General $(x,y) \rightarrow (x,y-2)$

Horizontal Translations **(shifting the graph left or right) form:** $y = f(x - \boldsymbol{h})$

Graph: $y = x^2$; $y = (x-3)^2$; $y = (x+2)^2$

V(3,0) horizontal translation 3 right

V(−2,0) horizontal translation 2 left



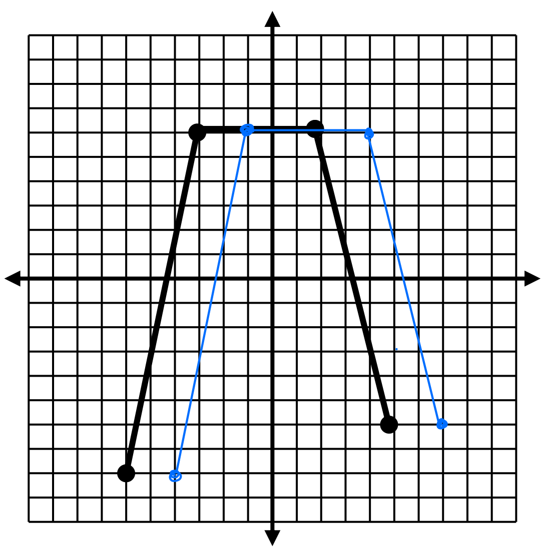
For $y = f(x - \boldsymbol{h})$

If $\boldsymbol{h}$ is positive → horizontal trans 3 right

If $\boldsymbol{h}$ is negative ← horiz trans 3 left

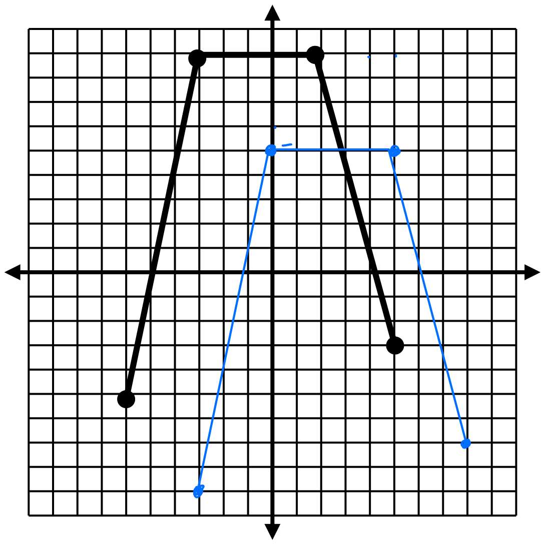
Example 2: Given the graph of $y = f(x)$ below, describe the transformation applied graph $y = f(x-2)$, and map the coordinates of the image points.

h=2 ∴ horizontal translation 2 to right



	+2
(−6, −8)	(−4, −8)
(−3, 6)	(−1, 6)
(2, 6)	(4, 6)
(5, −6)	(7, −6)
(x, y)	(x+2, y)

Example 3: Given the graph of $y = f(x)$ below, describe the transformation applied graph $y = f(x - 3) - 4$, and map the coordinates of the image points.



horizontal trans +3
vertical trans −4

$(x, y) \rightarrow (x+3, y-4)$

Example 5: Given the function $y = f(x)$ below, describe the transformation applied to each of the functions below.

a) $y = f(x + 3) - 2$

- horizontal translation −3
- vertical translation −2

b) $y = f(x - 5) + 6$

- horizontal translation +5
- vertical translation +6

c) $y + 1 = f(x + 5)$

$y = f(x+5) - 1$

- horizontal translation −5
- vertical translation −1

d) $y - 4 = f(x - 7) - 7$

+4 +4

$y = f(x-7) - 3$

- horizontal translation +7
- vertical translation −3

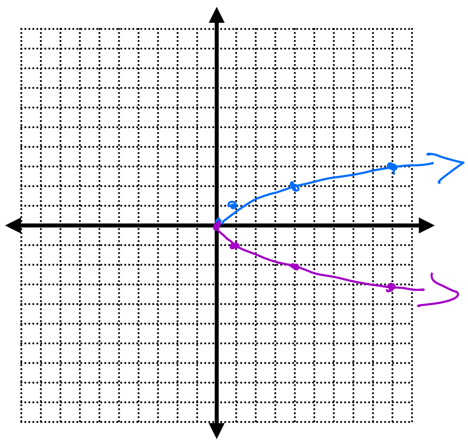
Transformations Day 5 Reflections, Expansions and Compressions

$y = \sqrt{x}$

Reflections in the Coordinate Axis (flipping the graph over the x or y axis)

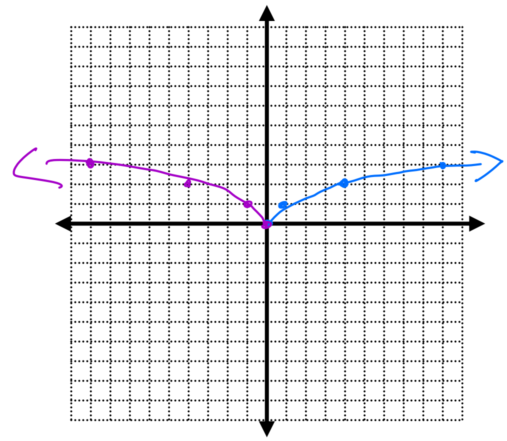
$y = -\sqrt{x}$

x	y
0	0
1	-1
4	-2
9	-3



$y = \sqrt{-x}$

x	y
0	0
-1	1
-4	2
-9	3

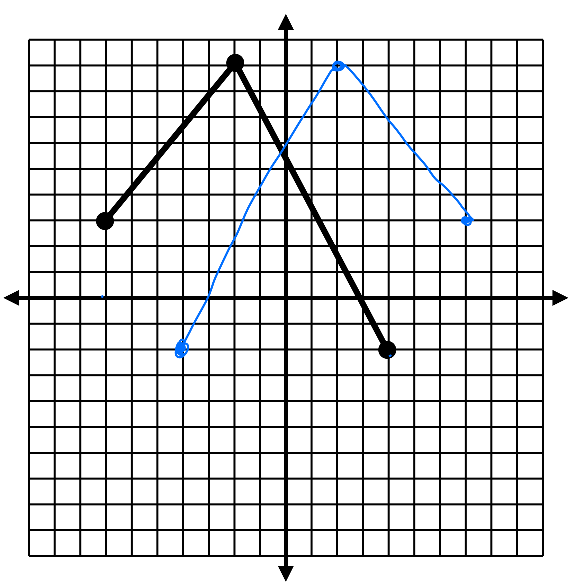


So, for $y = f(x)$

If $y = -f(x)$ → reflection across x-axis (vertical reflection)

If $y = f(-x)$ → reflection across y-axis (horizontal reflection)

Example 1: Given the graph of $y = f(x)$ below, describe the transformation applied to the graph $y = f(-x)$, and map the general coordinates of the image points.



$(-7, 3) \to (7, 3)$

$(-2, 9) \to (2, 9)$

$(4, -2) \to (-4, -2)$

$(x, y) \to (-x, y)$

Reflection across y-axis

Vertical Expansions/Compressions (stretching the graph in y direction)

Graph: $y = \sqrt{x}$ $y = 2\sqrt{x}$ $y = \frac{1}{2}\sqrt{x}$

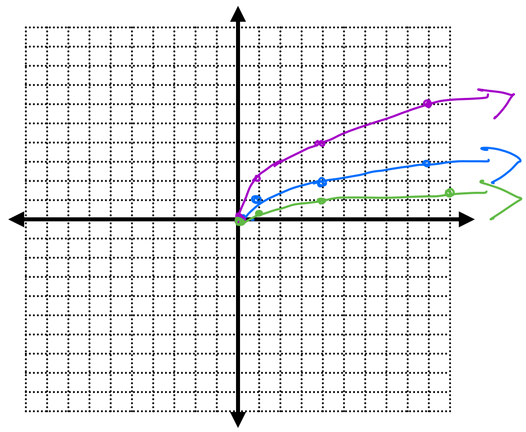
x	y
0	0
1	1
4	2
9	3

x	y
0	0
1	2
4	4
9	6

expansion

x	y
0	0
1	1/2
4	1
9	1 1/2

Compression

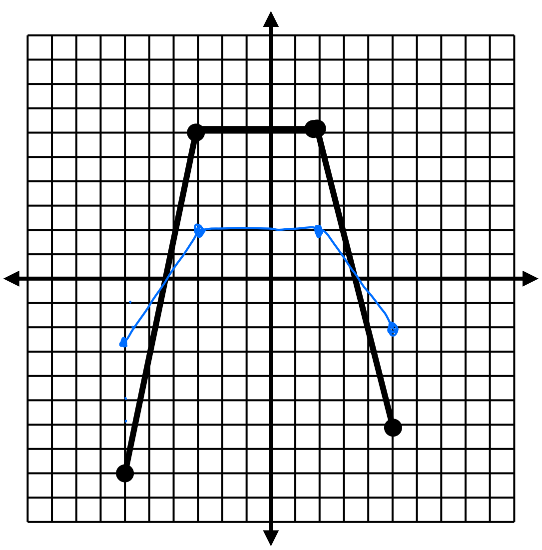


For $y = af(x)$

If |**a**| > 1 → vertical expasion by a factor of |a|

If |**a**| < 1 → vertical compression by a factor of |a|

Example 2: Given the graph of $y = f(x)$ below, describe the transformation applied to the graph $y = \frac{1}{3}f(x)$, and map the general coordinates of the image points.



Vertical compression bafo 1/3

$(-6, -8) \rightarrow (-6, -8/3)$

$(-3, 6) \rightarrow (-3, 2)$

$(2, 6) \rightarrow (2, 2)$

$(5, -6) \rightarrow (5, -2)$

$(x, y) \rightarrow (x, \frac{1}{3}y)$

Horizontal Compressions/Expansions **(stretching the graph in x direction)**

Graph: $y = x^2$

Compression

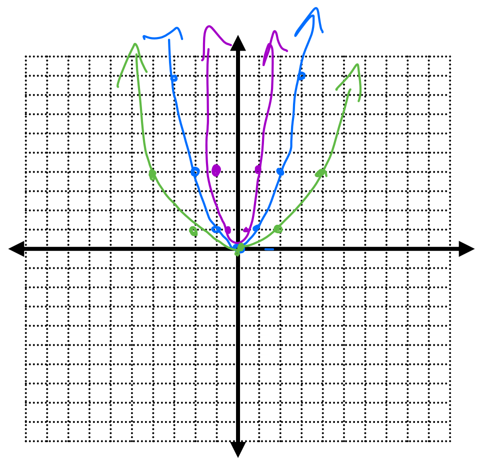
$y = (2x)^2$

x	y
-1	4
$-\frac{1}{2}$	1
0	0
$\frac{1}{2}$	1
1	4

expansion

$y = \left(\frac{1}{2}x\right)^2$

x	y
-4	4
-2	1
0	0
2	1
4	4

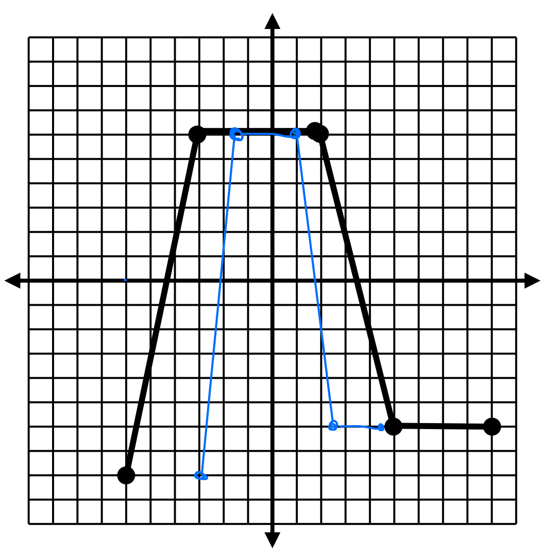


For $y = f(bx)$

If $|\mathbf{b}| > 1$ → horizontal compression by a factor of $\frac{1}{|b|}$

If $|\mathbf{b}| < 1$ → horizontal expansion by a factor of $\frac{1}{|b|}$

Example 3: Given the graph of $y = f(x)$ below, describe the transformation applied to the graph $y = f(2x)$, and map the general coordinates of the image points.



horizontal compression by a factor of $\frac{1}{2}$

$(-6,-8) \to (-3,-8)$

$(-3,6) \to (-1\frac{1}{2},6)$

$(2,6) \to (1,6)$

$(5,-6) \to (2.5,-6)$

$(9,-6) \to (4.5,-6)$

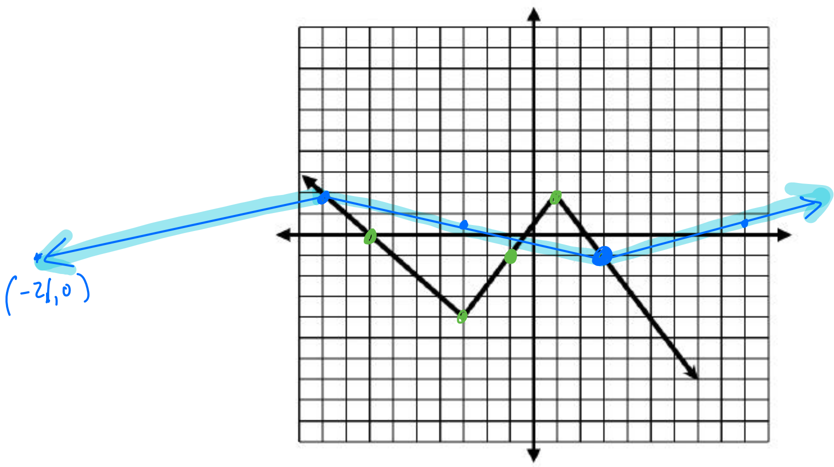
$(x,y) \to \left(\frac{1}{2}x, y\right)$

Example 4: Given the graph of $y = f(x)$, sketch the graph of $y = -\frac{1}{2} f\left(\frac{1}{3}x\right)$

$a = -\frac{1}{2}$ $b = \frac{1}{3}$

- vertical reflection
- vertical compression by a factor of $\frac{1}{2}$
- horizontal expansion by a factor of 3

$(x, y) \to (3x, -\frac{1}{2}y)$



key points: ×3 ×$-\frac{1}{2}$

$(-7, 0) \to (-21, 0)$

"valley" $(-3, -4) \to (-9, 2)$ now a "peak" because we flipped it vertically

$(-1, -1) \to (-3, 0.5)$

"peak" $(1, 2) \to (3, -1)$ now a "valley"

$(3, -1) \to (9, -0.5)$

Assignment: Transformations Supplemental Practice Handout #4-6

Transformations Day 5: Combined Transformations

y alone!

All of the transformations we have performed can be summarized as follows: "h" value is in a bracket with "1x"

$$y = f(x) \text{ transforms to } y = af\left[b(x-h)\right]+k$$

note: must be factored out

Reflections: $a < 0$, reflection in the x-axis
$b < 0$, reflection in the y-axis
f^{-1} reflection across the line y = x

* "inside the function" with x ⇒ horizontal change (do opposite)

Expansion: $|a| > 1$, vertical expansion by a factor of $|a|$
$|b| < 1$, horizontal expansion by a factor of $\frac{1}{|b|}$

Compression: $|a| < 1$, vertical compression by a factor of $|a|$
$|b| > 1$, horizontal compression by a factor of $\frac{1}{|b|}$

* "outside the function" ⇒ vertical change (do what it says)

Translation: $k > 0$, vertical translation k units up
$k < 0$, vertical translation k units down
$h > 0$, horizontal translation h units right
$h < 0$, horizontal translation h units left

When combining transformations, the $\underbrace{\text{reflections/expansions/compressions}}_{\text{multiply/divide}}$ must occur before the translations.

before add/subtract

(Like BEDMAS!)

Can remember with "CERT"

Example 1: Given the function $y = f(x)$ below, describe the transformation applied to each of the functions below.

a) $a=-2$ $b=1$ $h=4$ $k=6$

$y = -2f(x-4)+6$

- vertical reflection
- vertical expansion bafo 2
- vertical translation +6 (up 6)
- horizontal translation +4 (right 4)

b) must factor out

$a=1$ $b=-3$ $h=-4$ $k=-4$

$y = f(-3x-12)-4$

$y = f[-3(x+4)]-4$

- horizontal reflection
- horizontal compression bafo $\frac{1}{3}$
- horizontal translation −4 (left 4)
- vertical translation −4 (down 4)

c) → get y alone

$2y+10 = f(5-x)$ −10

$2y = f(5-x)-10$ ÷2

$y = \frac{1}{2}f(-x+5)-5$ Re-order

$y = \frac{1}{2}f[-(x-5)]-5$ factor

$a = \frac{1}{2}$ → vertical compr bafo $\frac{1}{2}$

$b = -1$ → horiz. reflection

$h = 5$ → horiz. transl +5

$k = -5$ → vert. transl. −5

$(x, y) \to (\overset{\times \frac{1}{b}}{} , \overset{\times a}{}) \to (\overset{+h}{} , \overset{+k}{})$

Example 2: Given point P (– 4, 2) on $y = f(x)$ find the new location for P on:

$y = -f\left(\frac{x}{3}\right) + 2$ (h=0)

a=-1 (x by -1)

b=1/3 (x by 3)

k=2 (add 2)

$(-4, 2) \xrightarrow{} (\overset{\times 3}{-12}, \overset{\times(-1)}{-2}) \xrightarrow{} (\overset{\rightarrow}{-12}, \overset{+2}{0})$

Example 3: If (-3, 4) is a point on the graph $y = f(x)$ what must be the point on the graph $y + 7 = -5f(2x - 4) + 3$.

factor

−7 −7

$y = -5f[2(x-2)] - 4$

a = -5

b = 2 (× ½)

h = 2

k = -4

$(-3, 4) \to (\overset{\times \frac{1}{2}}{-1\frac{1}{2}}, \overset{\times(-5)}{-20}) \to (\overset{+2}{\frac{1}{2}}, \overset{-4}{-24})$

Example 4: Graph the following functions:

y = |x|

a) $y = -\frac{1}{2}|x - 3| + 4$ notice "vertex" at (h,k) ☺

a = -½ h = 3 k = 4

key points:

$(-1, 1) \to (\overset{\rightarrow}{-1}, \overset{\times -\frac{1}{2}}{-\frac{1}{2}}) \to (\overset{+3}{2}, \overset{+4}{3\frac{1}{2}})$

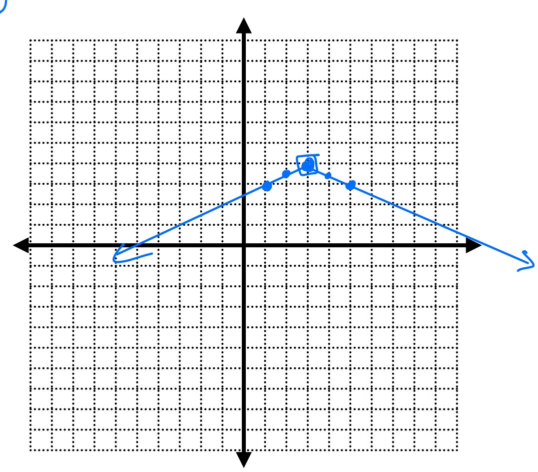
$(0, 0) \to (0, 0) \to (3, 4)$

$(1, 1) \to (1, -\frac{1}{2}) \to (4, 3\frac{1}{2})$

extras:

$(-2, 2) \to (-2, -1) \to (1, 3)$

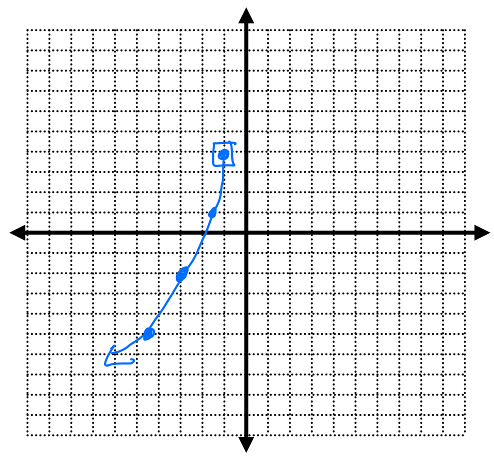
$(2, 2) \to (2, -1) \to (5, 3)$



x by $-\frac{1}{2}$

$a=-3$ $b=-2$ $h=-1$ $k=4$

b) $h(x) = -3\sqrt{-2(x+1)} + 4$



$y=\sqrt{x}$

keypoints

$x \cdot -\frac{1}{2}$ $\quad$ $\times -3$ $\quad$ -1 $\quad$ $+4$

$(0,0) \rightarrow (0,0) \rightarrow (-1,4)$

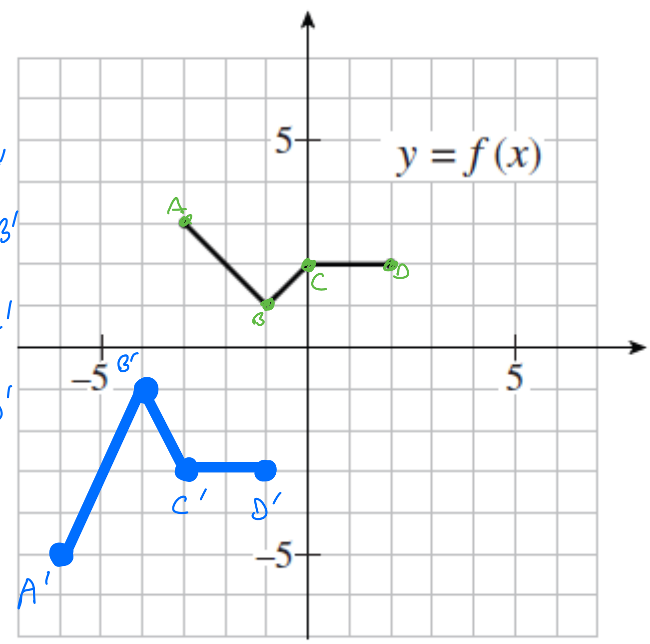
$(1,1) \rightarrow (-\frac{1}{2},-3) \rightarrow (-1.5,1)$

$(4,2) \rightarrow (-2,-6) \rightarrow (-3,-2)$

$(9,3) \rightarrow (-4.5,-9) \rightarrow (-5.5,-5)$

$a=-2$ $\quad$ $b=1$ $\quad$ $h=-3$ $\quad$ $k=1$

Example 5: Given the graph of $y = f(x)$, sketch the graph $y = -2f(x+3)+1$



keypoints

$\rightarrow$ $\quad$ $x(-2)$ $\quad$ -3 $\quad$ $+1$

A $(-3,3) \rightarrow (-3,-6) \rightarrow (-6,-5)$ A'

B $(-1,1) \rightarrow (-1,-2) \rightarrow (-4,-1)$ B'

C $(0,2) \rightarrow (0,-4) \rightarrow (-3,-3)$ C'

D $(2,2) \rightarrow (2,-4) \rightarrow (-1,-3)$ D'

* optional to label points. Sometimes it helps to picture how to "connect the dots".

Assignment: Transformations Supplemental Practice Handout #7-9

Transformations Day 6: Inverse Graphs & Equations

The Machine Metaphor of Functions

Functions are often thought of as a machine that takes an input value and produces an output value. For example, if the point (3,5) is on the graph of $y = f(x)$ then we know that when we stick a _3__ into the machine a _5___ comes out.

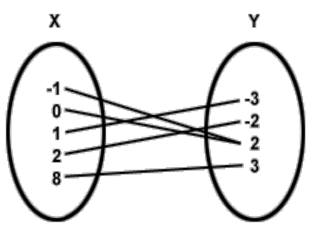
$3 \rightarrow \boxed{f(x)} \rightarrow 5$

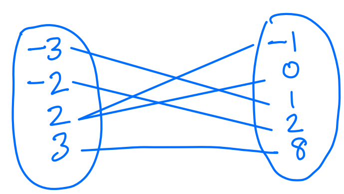
This allows us to imagine another machine which always does the **opposite** of what f does. We call this the **inverse** of $f(x)$ and can use the notation $f^{-1}(x)$.

Another way of thinking of this is that whatever _$f(x)$_ does, _$f^{-1}(x)$_ "undoes."

$3 \rightarrow \boxed{f(x)} \rightarrow 5$ $\qquad$ $5 \rightarrow \boxed{f^{-1}(x)} \rightarrow 3$

1. Given the domain-range map for $y = f(x)$, draw a domain-range map for its inverse:



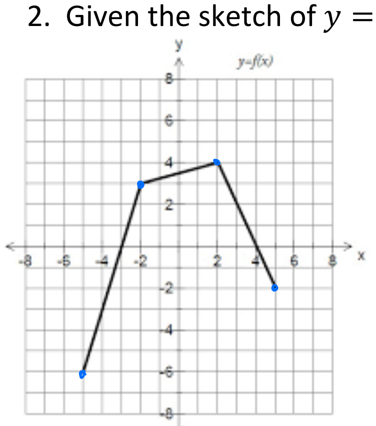


Is $f(x)$ a function? _yes_ is its inverse a function? _no_ Is $f(x)$ a one-to-one function? _no_

Notice the inverse this gives us another way to think about one-to-one functions.

$y = f(x)$ will be a one-to-one function if it is a function and its ***inverse*** *is a function.*

2. Given the sketch of $y = f(x)$, use a table of values to create a sketch of its inverse:



x	y
-5	-6
-2	3
2	4
5	-2

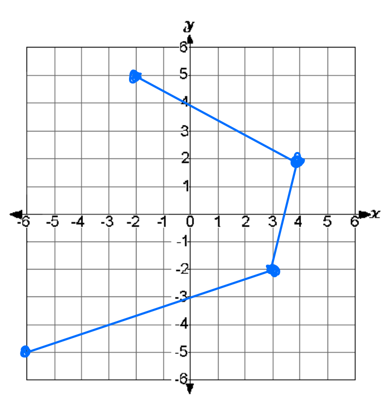
Inverse Pts

(-6, -5)

(3, -2)

(4, 2)

(-2, 5)



To Graph an Inverse

1) Find key points on the **original** graph
2) **Interchange *x* and *y* values** to get points on the **inverse** graph
3) Plot the **inverse**

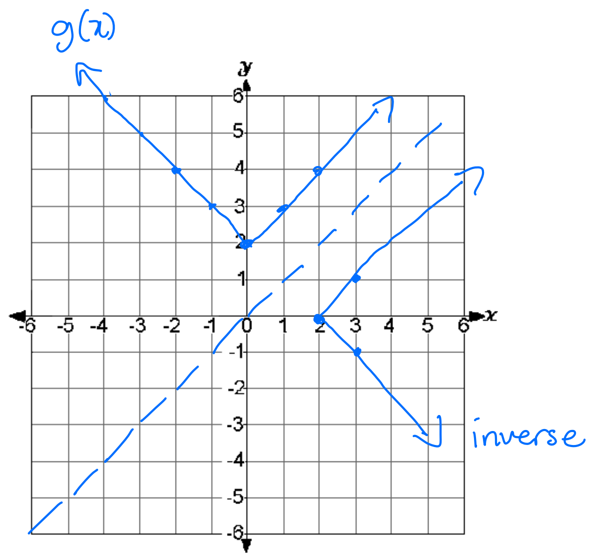
3. Sketch $g(x) = |x| + 2$ and its inverse on the graph below.

$y = |x| + 2$ Basic graph up 2

$(-1, 1) \to (-1, 3)$ $x \leftrightarrow y$ $(3, -1)$

$(0, 0) \to (0, 2)$ $(2, 0)$

$(1, 1) \to (1, 3)$ $(3, 1)$



Now sketch in the line ***y* = *x***. What do you notice about how the two graphs relate?

y=x is like a mirrow for the graph and its inverse

The Inverse is a Reflection across the Line $y = x$

In example 3, you can see that the inverse graph is a reflection of the original across the line ***y* = *x***. This may help you visualize your inverses, but it is usually easiest to graph them by interchanging *x* and *y* values of key points.

Terminology & Notation Options

We can ask about inverses in a variety of ways. The following four questions would all be asking you to do the ***exact same thing:***

Sketch the graph of the inverse of $y = f(x)$	(states "inverse")
Sketch the graph of $y = f^{-1}(x)$	(uses inverse notation $f^{-1}(x)$)
Sketch the graph of $x = f(y)$	(interchanges *x* and *y*)
Sketch the reflection of $y = f(x)$ over the line $y = x$	(the inverse is the reflection across *y* = *x*)

Note: we only use the notation $f^{-1}(x)$ if the inverse is also a function.

To Find the Inverse of an Equation

To find the inverse graph, we interchange *x* and *y*. To find the inverse of an equation, we do the same!

1) Write the original equation (use "*y*" instead of *f(x)*.)
2) Interchange *x* and *y*
3) Solve for *y*

(If the inverse is also a function, you can rewrite with the notation $f^{-1}(x)$. Otherwise, keep "*y*".)

4. a. Determine the equation for the reflection of $g(x) = (x+2)^2 - 1$ over the line $y = x$.

① $y = (x+2)^2 - 1$

② $x = (y+2)^2 - 1$

③ $x+1 = (y+2)^2$

$\pm\sqrt{x+1} = y+2$

$\pm\sqrt{x+1} - 2 = y$

$\boxed{y = \pm\sqrt{x+1} - 2}$

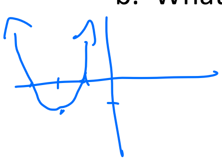
What is this asking you to do?

Find the equation of the inverse

How do you do that?

- switch x and y
- solve for y

b. What is the domain of $g(x)$? What is the domain of the inverse?



$\{x \in \mathbb{R}\}$

$\{y \in \mathbb{R} \mid y \geq -1\}$

$\boxed{\text{Inverse}}$

switch x and y!

R: $\{y \in \mathbb{R}\}$

D: $\{x \in \mathbb{R} \mid x \geq -1\}$

c. Sketch the graphs of $y = g(x)$ and $x = g(y)$.

$y = (x+2)^2 - 1$

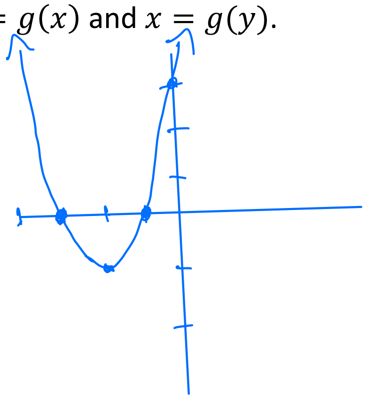
$-2 \quad -1$

$(-1, 1) \rightarrow (-3, 0)$

$(0, 0) \rightarrow (-2, -1)$

$(1, 1) \rightarrow (-1, 0)$

$(2, 4) \rightarrow (0, 3)$



Inv Pts

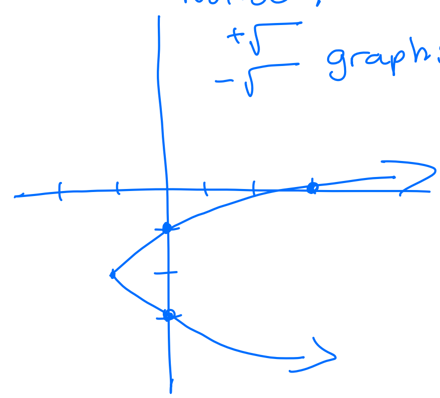
$(0, -3)$

$(-1, -2)$

$(0, -1)$

$(3, 0)$

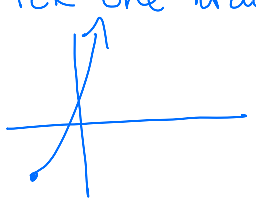
Notice: you can see $+\sqrt{}$ $-\sqrt{}$ graphs



d. Is $g(x)$ a function? __yes__ Is the inverse a function? __no__ Is $g(x)$ a one-to-one function? __no__

e. What is a restriction that could be put on the graph of $g(x)$ that would make the inverse a function?

Pick one "branch" of $g(x)$ e.g. right side



$\{x \in \mathbb{R} \mid x \geq -2\}$

then the inverse will be a function!

5. Determine the equation for the inverse of $h(x) = \sqrt{x+3}$

*(Note you will need to **restrict** the domain of your **answer** to avoid including half of the parabola not in the original function. Use sketches to help you do this.)*

$y = \sqrt{x+3}$

$x = \sqrt{y+3}$

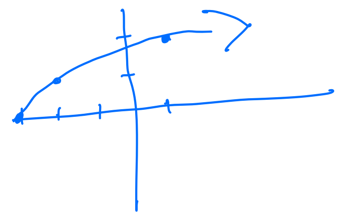
$x^2 = y+3$

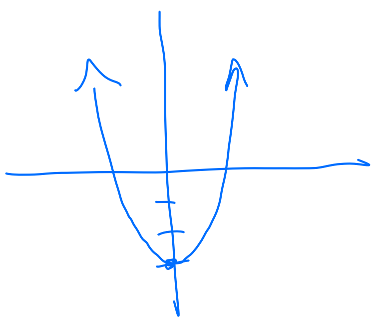
$x^2 - 3 = y$

$y = x^2 - 3$

D: $\{x \in \mathbb{R} \mid x \geq -3\}$

R: $\{y \in \mathbb{R} \mid y \geq 0\}$





If we don't restrict the domain, we get an extra piece. We only want right branch so $x \geq 0$

$$\boxed{f^{-1}(x) = x^2 - 3, \quad x \geq 0}$$