PC12 Name____________________________

Trigonometric Functions

Chapter Notes

Key

Assignment List

Date	Lesson	Assignment
	1. Angles in Standard Position and Radians	Mickelson Page 255 #1-8, 9-16
	2. Trigonometric Ratios of an Acute Angle	Mickelson Page 262 #1-6
	3. General and Special Angles	Mickelson Page 273 #1-10
	4. Exploration of Trig Functions	(In-class exploration)
	5. Graphing Trig Functions I	Mickelson Page 284 #1-4
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	7. Applications of Trig Functions	Mickelson Page 289 #1-12
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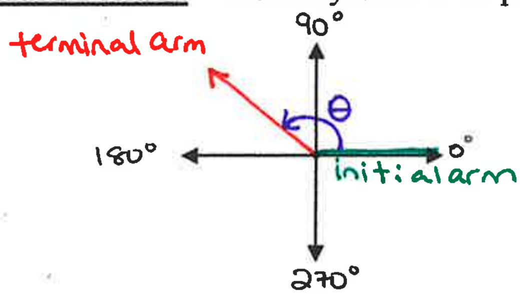
		Review
		Trigonometric Functions Test

Trigonometric Functions Day 1: Angles in Standard Position and Radians

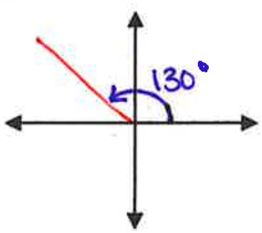
Measuring Angles:

Angles in Standard Position – initial ray/arm on the positive x –axis

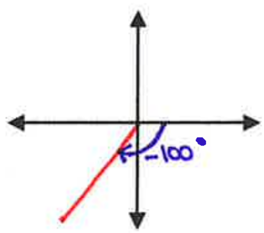


Degree Measure – 1/360 of the rotation of a circle is one degree (°)

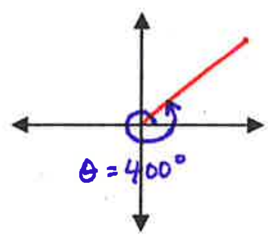
$\theta = 130°$



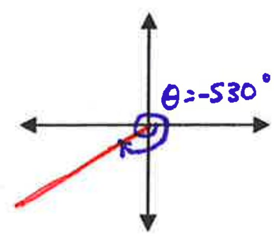
$\theta = -100°$



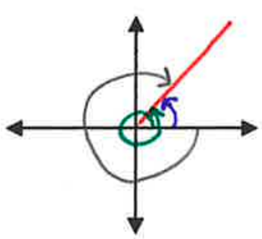
$\theta = 400°$



$\theta = -530°$



Coterminal Angles – share the same terminal arm



➔ Just add or subtract multiples of 360°

Principal Angle - the smallest positive coterminal angle (the angle in standard position)

Example 1: Determine a positive and negative coterminal angle of 70°.

$\theta_1 = 70°$

$\theta_2 = 70° + 360° = 430°$

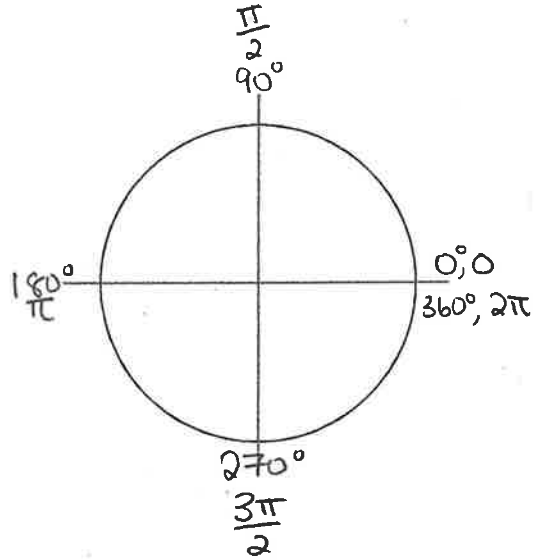
$\theta_3 = 70° - 360° = -290°$

* There are ∞ many more!

Radian Measure – a new unit of measuring an angle that is more useful in science and engineering

Definition of Radian: A unit of angular measure equal to the angle subtended at the center of a circle by an arc equal in length divided by the radius of the circle

$$\text{radian} = \frac{\text{arc length}}{\text{radius}}$$



Consider a circle with a radius of r units.

One complete rotation in degrees is 360°.

The arc length for one complete rotation is $2\pi r$.
which is the circumference of the circle.

The ratio, $\frac{\text{arc length}}{\text{radius}} = \frac{2\pi r}{r} = 2\pi \therefore 360° = 2\pi$ radians.

Example 1: Convert to radians. $\times \frac{\pi}{180°}$

i) $120° \cdot \frac{\pi}{180°}$

$= \frac{120\pi}{180} \begin{matrix} \div 60 \\ \div 60 \end{matrix}$

$= \frac{2\pi}{3}$

ii) $690° \cdot \frac{\pi}{180°}$

$= \frac{69\pi}{18}$

$= \frac{23\pi}{6}$

iii) $-405° \cdot \frac{\pi}{180°}$

$= -\frac{9\pi}{4}$

iv) $\frac{7}{12}$ rotations

"$\frac{7}{12}$ of 2π"

$= \frac{7}{12} \times 2\pi$

$= \frac{7\pi}{6}$

Example 2: Convert to degrees. $\times \frac{180°}{\pi}$

i) $\frac{7\cancel{\pi}}{6} \cdot \frac{180°}{\cancel{\pi}}$

$= \frac{7(180°)}{6}$

$= 210°$

ii) $-\frac{3}{4}\cancel{\pi} \cdot \frac{180°}{\cancel{\pi}}$

$= \frac{-3(180)}{4}$

$= -135°$

iii) $\frac{22\cancel{\pi}}{3} \cdot \frac{180°}{\cancel{\pi}}$

$= \frac{22(180)}{3}$

$= 1320°$

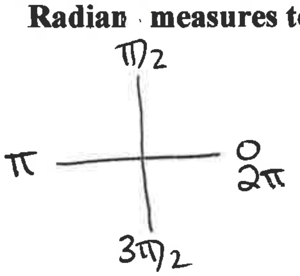
iv) $-\frac{13}{6}$ rotations

"$-\frac{13}{6}$ of 360°"

$= -\frac{13}{6} \times 360°$

$= -780°$

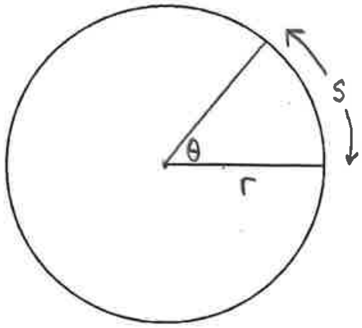
Radian measures to memorize:



$\frac{\pi}{2} = 90°$ $\quad \frac{\pi}{4} = 45°$ $\quad \frac{\pi}{3} = 60°$ $\quad \frac{\pi}{6} = 30°$

Arc Length

The arc length of a circle is proportional to its central angle and the radius of a circle. The equation for the arc length of a circle (s = arc length) can easily be found using our knowledge of the circumference.



For any circle, $C = 2\pi r$

using degrees:

$S = \frac{\theta}{360°} \times C$

$S = \frac{\theta\, 2\pi r}{360°}$

$\boxed{S = \frac{\pi r \theta}{360°}}$

using radians:

$S = \frac{\theta}{2\pi} \times C$

$S = \frac{\theta\, 2\pi r}{2\pi}$

★ $\boxed{S = r\theta}$ ★

Example 3: Determine the arc length of a circle with a radius of 7 cm and a central angle of 130°.

$\theta = 130°$ convert to Rad: $130° \cdot \frac{\pi}{180°}$

$\theta = \frac{13\pi}{18}$

$S = r\theta$

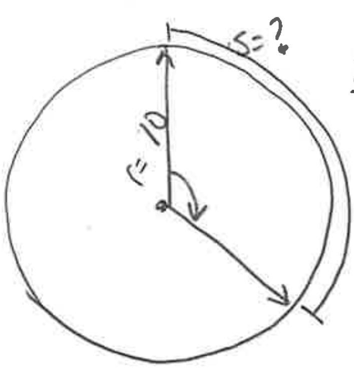
$S = 7\left(\frac{13\pi}{18}\right)$

$\boxed{S = 15.9\text{cm}}$

★ Remember units

★ use $\boxed{\pi}$ button, not an approx.

Example 4: Determine the distance that the tip of the minute hand (length 10 cm) travels in 20 minutes.



$\frac{20\text{min}}{60\text{min}} = \frac{1}{3}$ rotation

$\theta = \frac{1}{3}(2\pi)$

$\theta = \frac{2\pi}{3}$

$S = r\theta$

$S = 10\left(\frac{2\pi}{3}\right)$

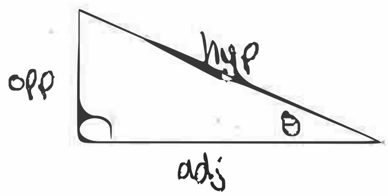
$\boxed{S = 20.9\text{cm}}$

Assignment p. 255 #1-8, 9-16

Trigonometric Functions Day 2: Trigonometric Angles of an Acute Angle

Trigonometric Ratios

Recall from previous math classes, the trig ratios for an acute angled right triangle.

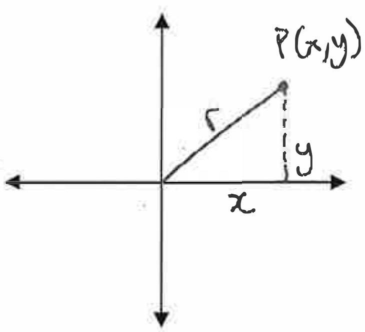


$\sin\theta = \frac{opp}{hyp}$ $\qquad \frac{1}{\sin\theta} = \frac{hyp}{opp} = \csc\theta$ $\qquad$ cosecant

$\cos\theta = \frac{adj}{hyp}$ $\qquad \frac{1}{\cos\theta} = \frac{hyp}{adj} = \sec\theta$ $\qquad$ secant

$\tan\theta = \frac{opp}{adj}$ $\qquad \frac{1}{\tan\theta} = \frac{adj}{opp} = \cot\theta$ $\qquad$ cotangent

We can also define these in terms of an angle in standard position and a point P(x , y)



$\sin\theta = \frac{y}{r}$ $\qquad \csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ $\qquad \sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\qquad \cot\theta = \frac{x}{y}$

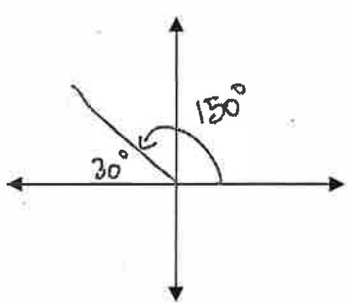
And Pythagoras can help us find the radius.......

$x^2 + y^2 = r^2$

$r = \sqrt{x^2 + y^2}$

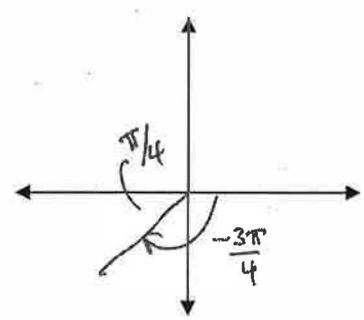
Reference angle - the acute angle formed between the terminal arm and the **nearest** x –axis.

Eg. 150°



Reference angle = $30°$

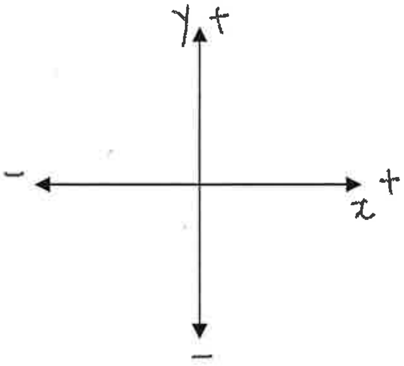
Eg. $-\frac{3\pi}{4}$ rad



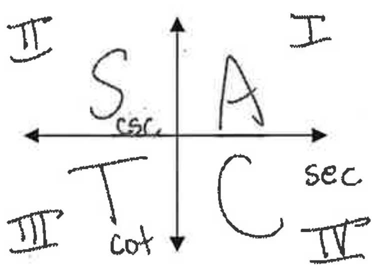
Reference angle = $\pi/4$

Algebraic signs of the Trig Ratios

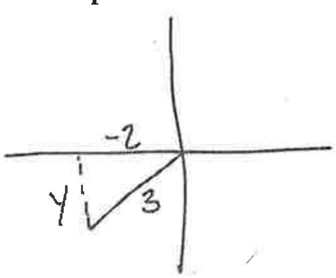
The sign of the trig function depends on the quadrant that the function is in. We can easily determine this using the ratios and we will find an easy way to remember this.



Trig. Ratios	I	II	III	IV
$\sin\theta = \frac{y}{r}$	$\frac{+}{+} = +$	$\frac{+}{+} = +$	$\frac{-}{+} = -$	$\frac{-}{+} = -$
$\cos\theta = \frac{x}{r}$	$\frac{+}{+} = +$	$\frac{-}{+} = -$	$\frac{-}{+} = -$	$\frac{+}{+} = +$
$\tan\theta = \frac{y}{x}$	$\frac{+}{+} = +$	$\frac{+}{-} = -$	$\frac{-}{-} = +$	$\frac{-}{+} = -$
	ALL	sin	tan	cos



Example 1: Determine $\sin\theta$ if $\sec\theta = -\frac{3}{2}$, and $\tan\theta > 0$.



$y^2 + (-2)^2 = 3^2$

$y^2 = 9 - 4$

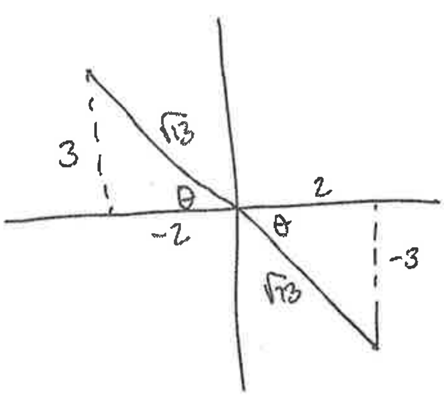
$y = \sqrt{5}$

$\sin\theta = \frac{opp}{hyp}$

$\boxed{\sin\theta = \frac{-\sqrt{5}}{3}}$

Quad III

Example 2: Determine $\sec\theta$ if $\cot\theta = -\frac{2}{3}$ Quad II or IV



$3^2 + (-2)^2 = r^2$

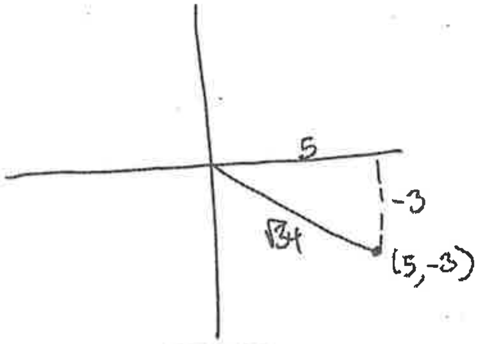
$9 + 4 = r^2$

$r = \sqrt{13}$

$\sec\theta = \frac{hyp}{opp}$

$\sec\theta = \pm\frac{\sqrt{13}}{2}$

Example 3: Find the 6 trig ratios if the terminal arm of angle θ contains the point P (5 ,– 3).



$\sin\theta = \frac{-3}{\sqrt{34}}$ $\qquad \csc\theta = -\frac{\sqrt{34}}{3}$

$\cos\theta = \frac{5}{\sqrt{34}}$ $\qquad \sec\theta = \frac{\sqrt{34}}{5}$

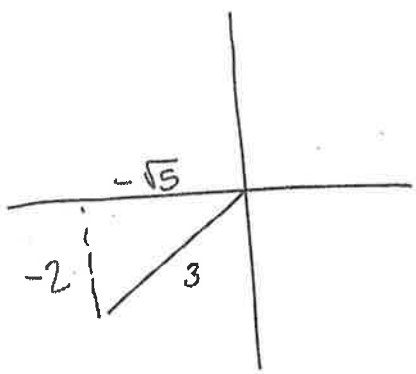
$\tan\theta = \frac{-3}{5}$ $\qquad \cot\theta = -\frac{5}{3}$

$r = \sqrt{5^2 + (-3)^2}$

$= \sqrt{25+9}$

$= \sqrt{34}$

Example 4: Given θ in standard position with its terminal arm in the stated quadrant, find the exact values for each trig ratio: $\csc\theta = -\frac{3}{2}$ in quadrant III.



$x^2 + (-2)^2 = 3^2$

$x^2 = 9 - 4$

$x^2 = 5$

$x = \sqrt{5}$

$\sin\theta = -\frac{2}{3}$

$\cos\theta = -\frac{\sqrt{5}}{3}$

$\tan\theta = \frac{2}{\sqrt{5}}$

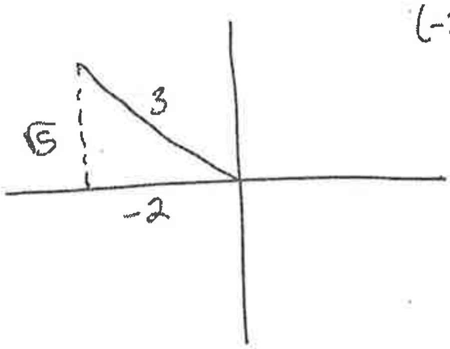
$\sec\theta = -\frac{3}{\sqrt{5}}$

$\cot\theta = \frac{\sqrt{5}}{2}$

Assignment p. 262 #1-6 (as needed)

Trigonometric Functions Day 3: General and Special Angles

Warmup: Given θ in standard position with its terminal arm in the stated quadrant, find the exact values for each trig. ratio. $\sec\theta = -\frac{3}{2}$, *in quadrant II*



$(-2)^2 + y^2 = 3^2$

$y^2 = 9 - 4$

$y^2 = 5$

$y = \sqrt{5}$

$\sin\theta = \frac{\sqrt{5}}{3}$

$\cos\theta = -\frac{2}{3}$

$\tan\theta = -\frac{\sqrt{5}}{2}$

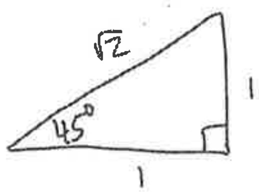
$\csc\theta = \frac{3}{\sqrt{5}}$ or $\frac{3\sqrt{5}}{5}$

$\cot\theta = -\frac{2}{\sqrt{5}}$ or $-\frac{2\sqrt{5}}{5}$

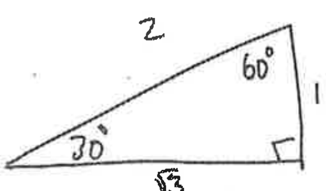
Special Triangles as Reference Angles:

Recall from previous courses the two special triangles that we can use as a reference. This will allow us to find the exact value of any of the special angles.

45° – 45° – 90° Triangles



30° – 60° – 90° Triangles



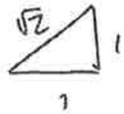
θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
30° $\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
60° $\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{1} = \sqrt{3}$
45° $\frac{\pi}{4}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{1}{1} = 1$

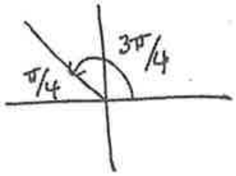
This allows us to find the exact value of any of the special angles or multiples of the special angles. Just follow this technique,

Draw terminal arm(s) in correct quadrant(s) ➡ Draw in reference angle(s)/triangle(s) ➡ Determine unknown angle(s) / sides ➡ Give angle in standard position /give exact value(s) of trig ratio

Example 1: Determine the exact value of

$$\cos\frac{3\pi}{4}$$

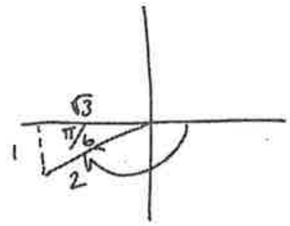




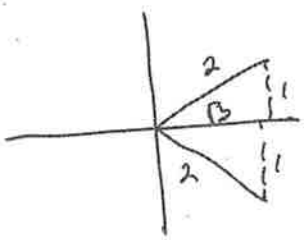
$$\cos\frac{3\pi}{4} = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

Example 2: Determine the exact value of

$$\cot -\frac{5\pi}{6} \qquad \frac{\sqrt{3}}{1} = \sqrt{3}$$



Example 3: Solve. $\cos\theta = \frac{\sqrt{3}}{2}$ $for\ 0 \le \theta < 2\pi$

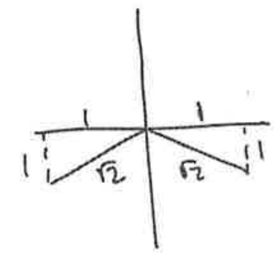


$\theta_{ref} = \frac{\pi}{6}$

$\theta_1 = \frac{\pi}{6}$

$\theta_2 = 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

Example 4: Solve. $\sin\theta = -\frac{1}{\sqrt{2}}$ $for\ 0 \le \theta < 2\pi$



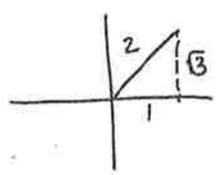
$\theta_{ref} = \pi/4$

$\theta_1 = \pi + \pi/4 = \frac{5\pi}{4}$

$\theta_2 = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

Example 5: Find the exact value of

$$2\sin^2\frac{\pi}{3} - \cos^2\frac{\pi}{3}$$



$2\left(\sin\frac{\pi}{3}\right)^2 - \left(\cos\left(\frac{\pi}{3}\right)\right)^2$

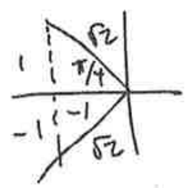
$2\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$

$\frac{2(3)}{4} - \frac{1}{4}$

$\frac{6}{4} - \frac{1}{4} = 5/4$

Example 6: Given $y = \frac{5\pi}{4}$ Find an angle x such that $x \ne y$, $0 \le \theta < 2\pi$ and $\cos x = \cos y$

$\cos y = \cos\frac{5\pi}{4} = \frac{-1}{\sqrt{2}}$

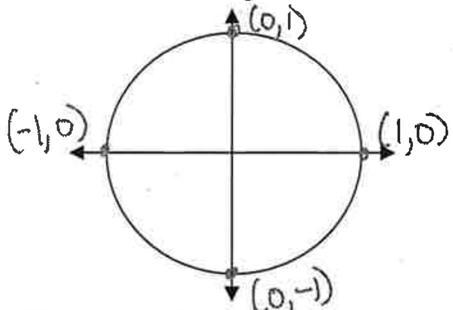


$\cos x = -\frac{1}{\sqrt{2}}$

$x = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

Quadrantal Angles: **0°, 90°, 180°, 270° *or 0,* $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$**

Angles having their terminal side lying along a coordinate axis are quadrantral angles. We can find the exact values of these angles as well.



$sin\ \theta = \frac{y}{r}$ $cos\ \theta = \frac{x}{r}$ $tan\ \theta = \frac{y}{x}$

$csc\ \theta = \frac{r}{y}$ $sec\ \theta = \frac{r}{x}$ $cot\ \theta = \frac{x}{y}$

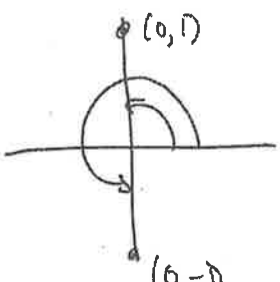
We can use these angles now and add to the angles that we can find the exact values of $r=1$

θ	$sin\ \theta$	$cos\ \theta$	$tan\ \theta$	$csc\ \theta$	$sec\ \theta$	$cot\ \theta$
0 or 2π $x=1$ $y=0$	$\frac{0}{1}=0$	$\frac{1}{1}=1$	$\frac{0}{1}=0$	$\frac{1}{0}$ = undef.	$\frac{1}{1}=1$	$\frac{1}{0}$ = undef
$90°$ or $\frac{\pi}{2}$ $x=0$ $y=1$	$\frac{1}{1}=1$	$\frac{0}{1}=0$	$\frac{1}{0}$ = undef.	$\frac{1}{1}=1$	$\frac{1}{0}$ = undef	$\frac{0}{1}=0$
$180°$ or π $x=-1$ $y=0$	$\frac{0}{1}=0$	$\frac{-1}{1}=-1$	$\frac{0}{-1}=0$	$\frac{1}{0}$ = undef	$\frac{1}{-1}=-1$	$\frac{-1}{0}$ = undef
$270°$ or $\frac{3\pi}{2}$ $x=0$ $y=-1$	$\frac{-1}{1}=-1$	$\frac{0}{1}=0$	$\frac{-1}{0}$ = undef.	$\frac{1}{-1}=-1$	$\frac{1}{0}$ = undef	$\frac{0}{-1}=0$

Example 7: Determine all possible values of x, $0 \le x < 2\pi$.

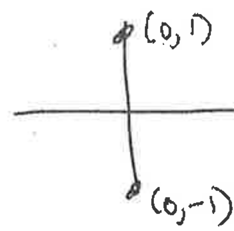
$\cot x = 0 = \frac{0}{\pm 1}$

$\cot \theta = \frac{x}{y} = \frac{0}{\pm 1}$



$x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\sec x = undefined = \frac{1}{0} = \frac{r}{x}$



$x = \pi/2, \frac{3\pi}{2}$

Assignment p. 273 #1-10 (as many as needed)

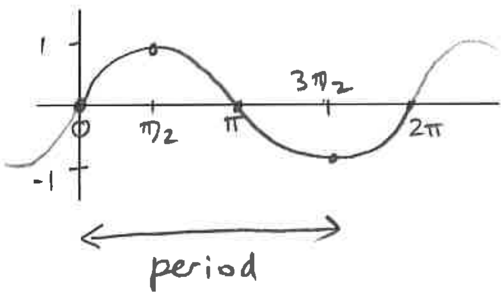
Trigonometric Functions Day 4: Exploration of Trig Functions

Complete the "Exploration of Trig Functions" activity.

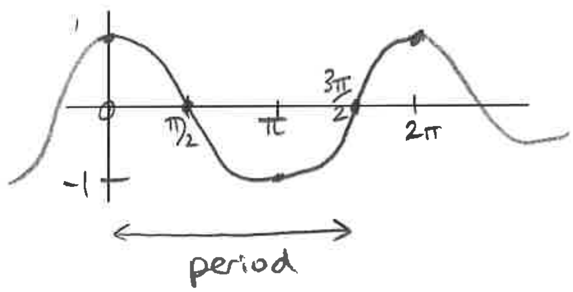
<u>Properties of a sine and cosine graph</u>

Sketch each function:

Sine Function: $y = \sin x$



Cosine Function: $y = \cos x$



- Both have a **period** of 2π (or $360°$)
- Both have an **Amplitude** = 1 Amplitude = $\frac{|\text{max} - \text{min}|}{2}$
- Amplitude is ALWAYS positive
- Sine curve shifted $\frac{\pi}{2}$ units to the left becomes the cos curve

<u>Describe in words the effect of each of a, b, c, d on the sin/cos curves:</u>

$f(x) = a \sin b(x - c) + d$ or $f(x) = a \cos b(x - c) + d$

a vertical stretch (changes amplitude)

b horizontal stretch (changes period)

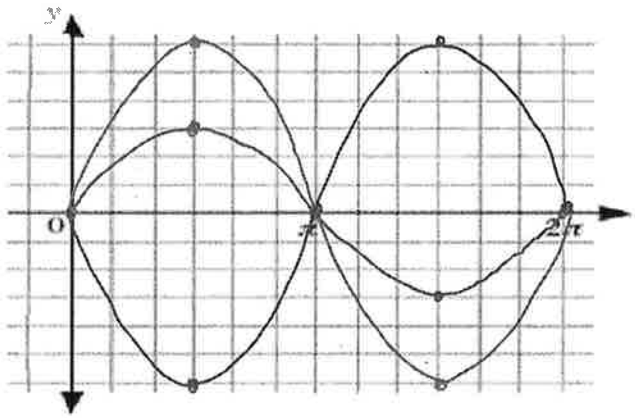
c horizontal shift left/right

d vertical shift up/down

Trigonometric Functions Day 5: Graphing Trig Functions

Comparing: $y = \sin\theta$ **and** $y = a\sin\theta$

Graph $y = \sin\theta$, $y = 2\sin\theta$, and $y = -2\sin\theta$



In general for $y = a \sin bx$, a controls the amplitude:

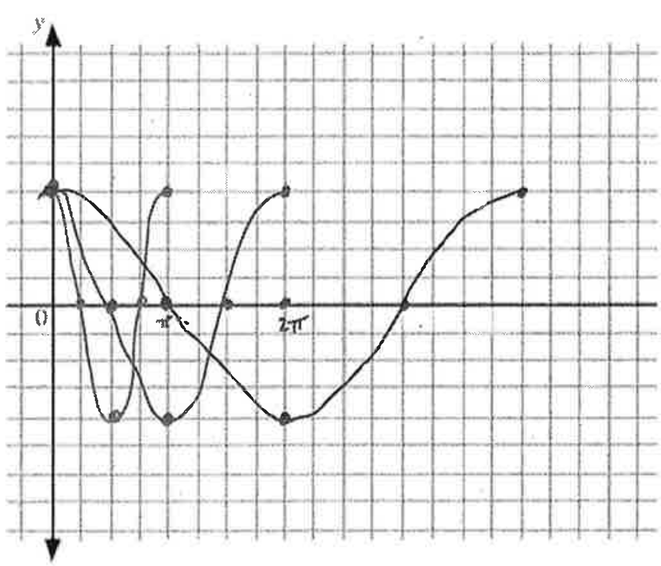
$|a|>1$ → vertical expansion

$0 < |a| < 1$ → vertical compression

$a < 0$ → reflection in the x-axis

Comparing: $y = \cos\theta$ **and** $y = \cos b\theta$

Graph $y = \cos\theta$, $y = \cos 2\theta$, and $y = \cos\frac{1}{2}\theta$



In general, b controls the period:

$|b| > 1$ → horizontal compression

$0 < |b| < 1$ → horizontal expansion

For $y = \cos bx$ the new period is:

$$p = \frac{2\pi}{|b|} \text{ or } \frac{360°}{|b|} \quad \textit{also} \quad b = \frac{2\pi}{p} \text{ or } \frac{360°}{p}$$

Each period of a sine or cosine function can be divided into 4 **'intervals'** $\quad interval = \frac{period}{4}$

Each interval is a maximum, a minimum, or x –intercept.

Graphing: $y = a \cos b\theta$ and $y = a \sin b\theta$

Using the value of the amplitude and the value of the period will determine the necessary scale for the graph. The y-scale must include $\pm a$ and the x scale must be an interval $= \frac{period}{4}$

The graph then follows the pattern for the basic sine or cosine function (max /min/x –int)

Example 1: Sketch the graph over two periods.

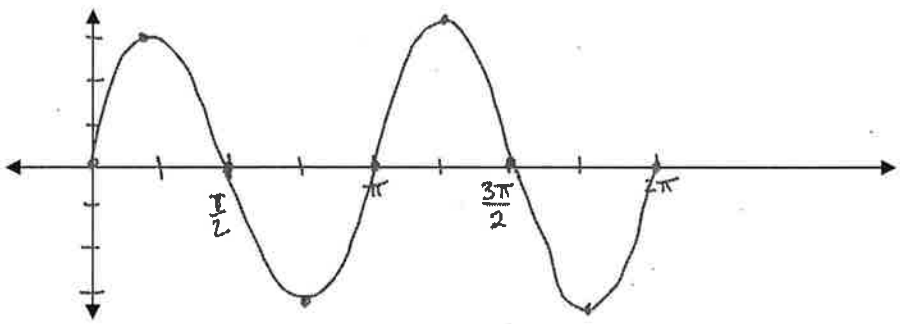
$y = 3 \sin 2x$

$a = 3$

$b = 2$

$\text{period} = \frac{2\pi}{2} = \pi$

$\text{interval} = \frac{\pi}{4}$



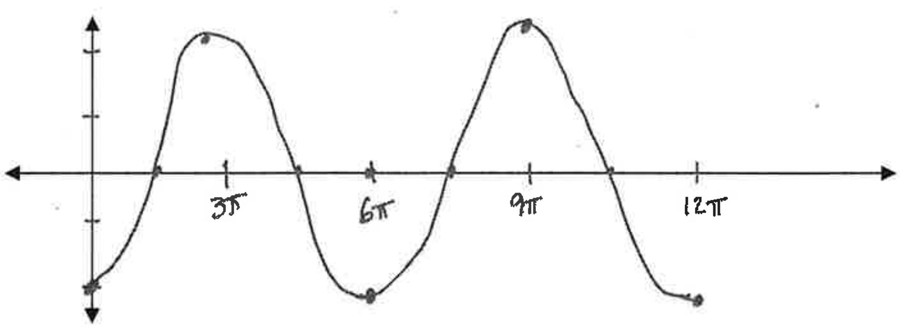
Example 2: Sketch the graph over two periods.

$y = -2 \cos \frac{x}{3} = -2 \cos \frac{1}{3}x$

$a = 2$

$b = \frac{1}{3}$

$\text{period} = \frac{2\pi}{\frac{1}{3}} = 6\pi$



Graphing: $y = a \sin b(x - c) + d$ and $y = a \cos b(x - c) + d$

When transforming $f(x) = a \sin b(x - c) + d$, the "c" value is just the horizontal translation.

When working with periodic functions is called the **phase shift.**

The "d" value is still referred to as the **vertical translation.** This is known as the **midline** of the function. The midline now plays the role that that x-axis did before the transformation.

Note: As with any transformation, the coefficient on "x" must equal 1. If it doesn't, then the coefficient must be factored out of the brackets.

Example 3: Given the function: $y = 6\cos\frac{2\pi}{14}x - 2$, determine the amplitude, period, phase shift, and vertical displacement.

$a = 6$

$b = \frac{2\pi}{14}$

$c = 0$

$d = -2$

Amplitude $= 6$

period $= \dfrac{2\pi}{\frac{2\pi}{14}} = 14$

phase shift $= 0$

vert. disp $= -2$

Example 4: Given the function: $y = -3\sin\left(2x + \frac{\pi}{3}\right) - 4$, determine the amplitude, period, phase shift, and vertical displacement.

$y = -3\sin 2\left(x + \frac{\pi}{6}\right) - 4$

$a = -3$

$b = 2$

$c = -\frac{\pi}{6}$

$d = -4$

Amp $= 3$

period $= \frac{2\pi}{2} = \pi$

PS $= \pi/6$ left

VD $= 4$ down

Example 5: Determine the domain and range of the function: $y = 4\cos 3(x+1) - 3$

-3 ± 4

Domain: $\{x \mid x \in R\}$

Range: $\{y \mid -7 \le y \le 1\}$

Range : $d \pm a$

Assignment p. 284 #1-4

$*$ Note: period of tan function is π.

Trigonometric Functions Day 6: Graphing Trig Functions

Graphing a Sine and Cosine Function

To graph a sine or cosine function it must first be in the form:

$f(x) = a\ sin\ b(x - c) + d$ *or*

$g(x) = a\ cos\ b(x - c) + d$

$|\mathbf{a}| =$ amplitude

b → $period = \frac{2\pi}{|b|} = \frac{360°}{|b|}$ *also* $b = \frac{2\pi}{p} = \frac{360°}{p}$

c: phase shift (horizontal translation)

d: vertical translation/displacement

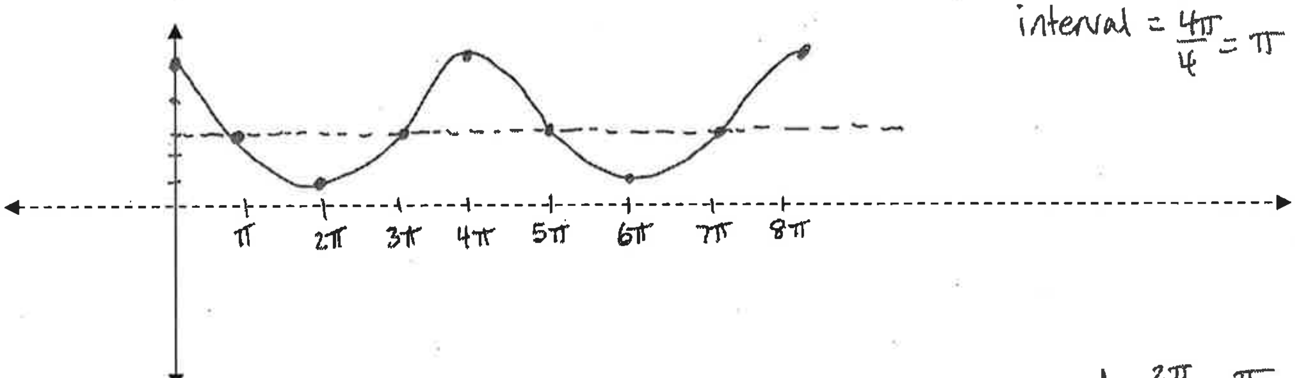
Graphing steps:

1. Write the function in the form: $f(x) = a\ sin\ b(x - c) + d$ *or* $g(x) = a\ cos\ b(x - c) + d$
2. Plot Midline: **d** <-------->
3. Establish min/max from the amplitude: $\mathbf{d \pm a}$ and label on the y-axis
4. Determine period: $p = \frac{2\pi}{|b|}$ and determine the interval value: this is the x-scale
5. Establish the new starting point, after phase shift '**c**', of the basic sin/cos graph
6. Graph key points at each interval: maximum/minimum/midline
7. Continue pattern to begin function at x = 0, showing at least one full period

Example 1: Graph. $y = 2\ cos\frac{x}{2} + 3$ $= 2\cos\frac{1}{2}x + 3$

$period = \frac{2\pi}{\frac{1}{2}} = 4\pi$

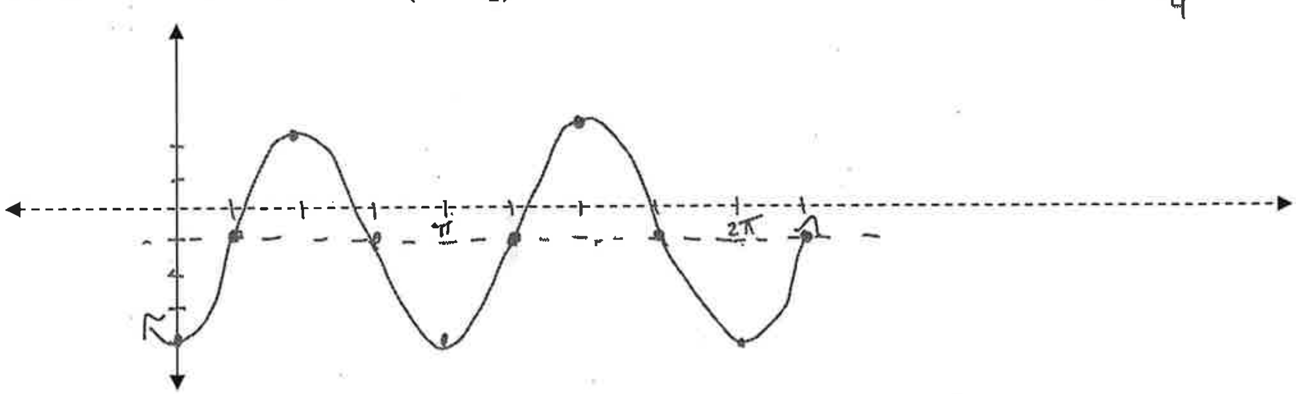
$interval = \frac{4\pi}{4} = \pi$

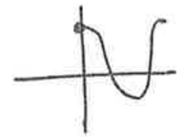


Example 2: Graph. $y = 3\sin\left(2x - \frac{\pi}{2}\right) - 1$ $= 3\sin 2\left(x - \frac{\pi}{4}\right) - 1$

$period = \frac{2\pi}{2} = \pi$

$interval = \frac{\pi}{4}$

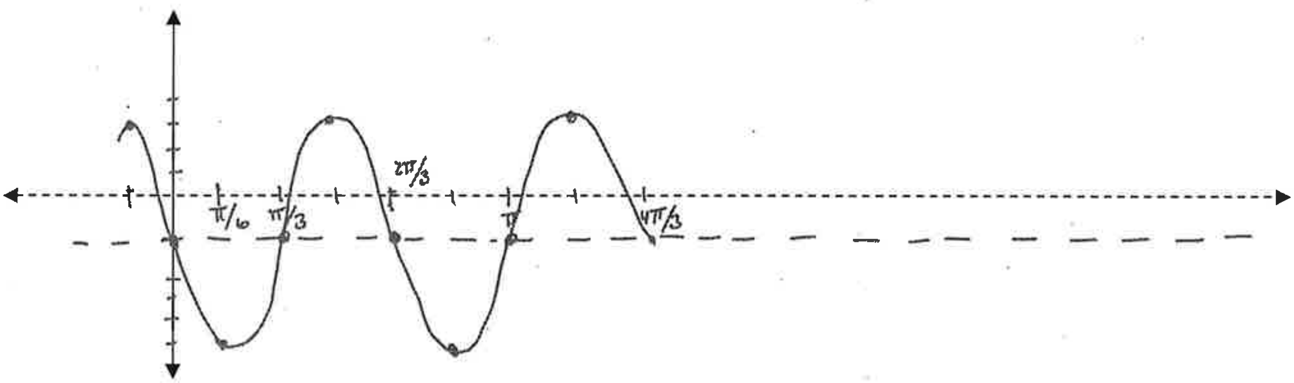




period $= \frac{2\pi}{3}$

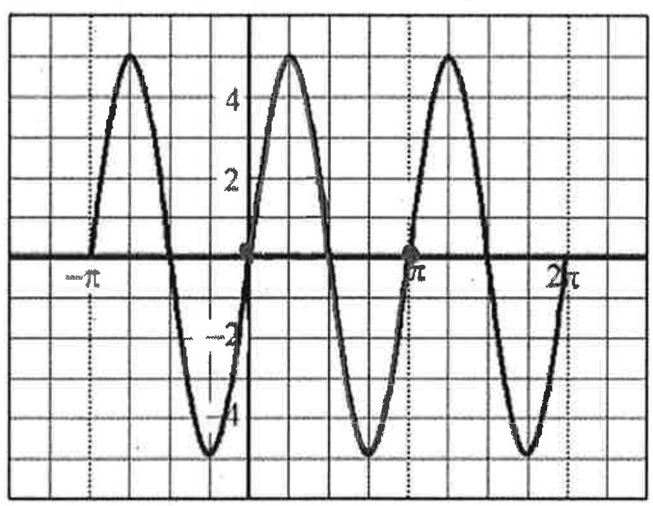
interval $= \frac{2\pi}{12} = \frac{\pi}{6}$

Example 3: Graph. $y = 2.5\cos\left(3\theta + \frac{\pi}{2}\right) - 1$ $= 2.5\cos 3\left(\theta + \frac{\pi}{6}\right) - 1$



Example 4: Find the equation of the sine function graphed below.

cosine



$a = 5$

$b = \frac{2\pi}{P} = \frac{2\pi}{\pi} = 2$

$c = 0$

$d = 0$

$y = 5\sin 2x$

$y = 5\cos 2\left(\pi - \frac{\pi}{4}\right)$

Assignment p. 285 ~~#5-10~~

#5-7, worksheet

Trigonometric Functions Day 7: Applications

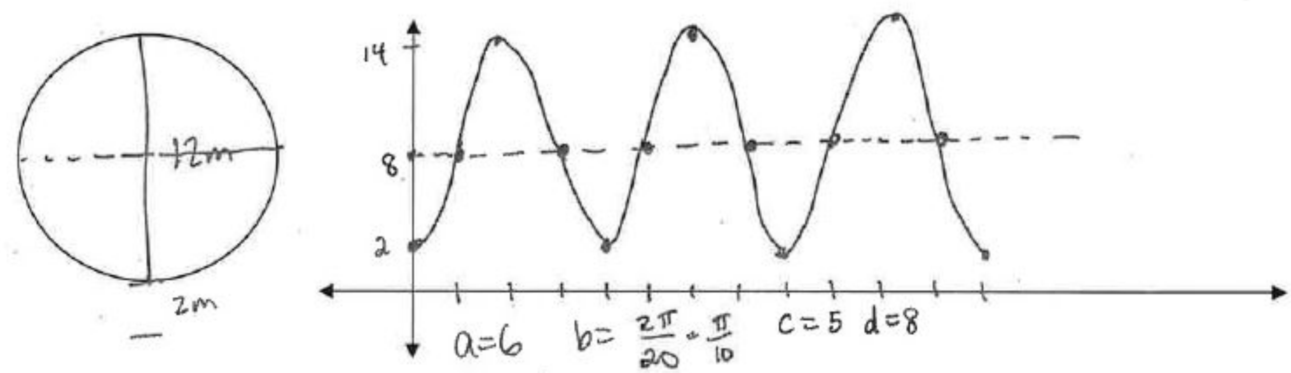
Applications of Trigonometric Functions

Sine or cosine functions can be used to describe **periodic or harmonic motion,** motion that repeats over a fixed time interval. There are many real life types of motion that can be modeled using a periodic function such as pendulums, springs, Ferris wheels, alternating electrical current (AC), tides, heart beats, annual temperatures and rainfall, radio waves, etc.

To solve questions involving periodic motion, we need to first determine the sinusoidal function (either sine or cosine) that models the motion. The easiest way to do this is to first create the graph of the motion.

Example 1: The bottom of a Ferris wheel is 2 m above the ground. The wheel rotates once every 20 seconds and has a diameter of 12m. Brandon gets on the Ferris wheel at the bottom.

a) Graph the sinusoidal motion of Brandon's height as a function of time for his first minute on the ride.



b) Determine sine and cosine equations for the height as a function of time.

$$y = 6\sin\frac{\pi}{10}(x-5)+8$$
$$y = 6\cos\frac{\pi}{10}(x-10)+8$$

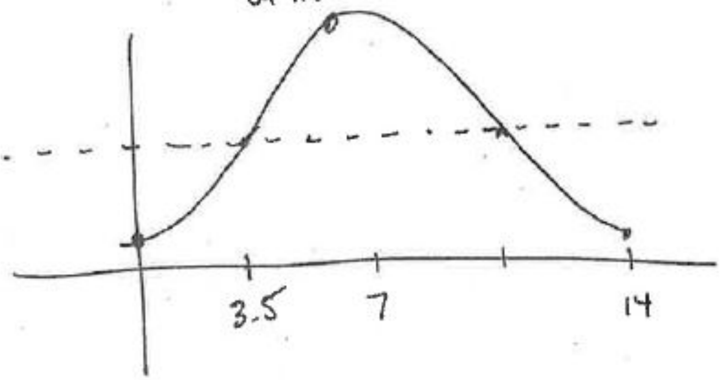
c) Determine his height above the ground at 12 seconds.

$$y = 6\sin\frac{\pi}{10}(12-5)+8$$
$$= 6\sin\frac{7\pi}{10}+8 = 12.8541$$
$$\doteq 12.9\text{m}$$

Example 2: Tides are a periodic rise and fall of water in the ocean. A low tide of 4.2 metres occurs at 4:30 am and a high tide occurs at 11:30 am on the same day. Determine the height of the tide at 1: 30 pm on that same day.

of 11.8m

9 hours later



$$\text{Mid} = \frac{\text{Max}+\text{Min}}{2}$$

$$= \frac{11.8+4.2}{2}$$

$$= \frac{16}{2} = 8$$

$$b = \frac{2\pi}{14} = \frac{\pi}{7}$$

$$y = 3.8\sin\frac{\pi}{7}(x-3.5)+8$$

$$= 3.8\sin\frac{\pi}{7}(9-3.5)+8$$

$$y = 10.4\text{m}$$

Assignment p. 289 #1-12