PC12 Name____________________________

Trigonometric Equations and Identities

Chapter Notes *key*

Assignment List

Date	Lesson	Assignment
	1. Trigonometric Equations part I	Mickelson Page 321 #1,2ab, 5a-e,h,i,m,o
	2. Trigonometric Equations part II	Mickelson Page 323 #4
	3. The Fundamental Identities	Mickelson Page 304 #2-9 (as many as needed*)
	4. Verifying Trigonometric Identities	Mickelson Page 311 #1-25
	5. Sum and Difference Identities	Mickelson Page 332 #1ace,2a-d,3,5
	6. Double Angle Identities	Mickelson Page 340 #1-5 (as many as needed*)
		Practice Test
		Review
		Trigonometric Identities and Equations Test

To repeat every 2π, we add (or subtract) whole numbers of 2π. Use "$n \in I$" (integers)

$n = \ldots -3, -2, -1, 0, 1, 2, 3 \ldots$

Day 1 – Trigonometric Equations part I

Trigonometric Equations

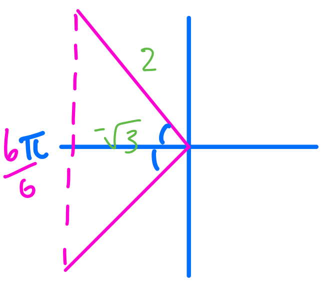
When solving a trig equation, we can solve for a specified domain of values (usually $0 \le \theta < 2\pi$) or in general form (for all potential values)

sin/cos/csc/sec repeat every 2π

$\sec\theta = \frac{r}{x}$ (negative in QII, QIII)

Example 1: Solve: $\sec\theta = -\frac{2}{\sqrt{3}}$ for a) $0 \le \theta < 2\pi$ b) general form

($2 \leftarrow r$, $\sqrt{3} \leftarrow x$)



a) $\boxed{\theta = \frac{5\pi}{6} \text{ or } \theta = \frac{7\pi}{6}}$

b) $\boxed{\theta = \frac{5\pi}{6} + 2\pi n, n \in I \qquad \theta = \frac{7\pi}{6} + 2\pi n, n \in I}$

reference angle: $\frac{\pi}{6}$

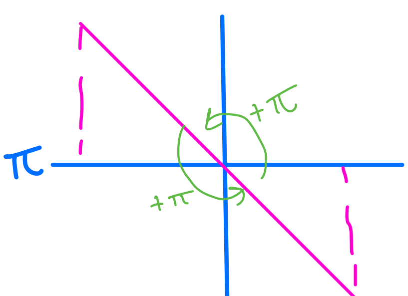
in QII: $\frac{6\pi}{6} - \frac{\pi}{6} = \frac{5\pi}{6}$

in QIII: $\frac{6\pi}{6} + \frac{\pi}{6} = \frac{7\pi}{6}$

Example 2: Solve: $\tan\theta = -0.3124$ for a) $0 \le \theta < 2\pi$ b) general form

tan is neg. in QII, QIV

(now ignore negative)



reference angle: $\tan^{-1}(0.3124) = 0.3028$ (Rad mode)

QII: $\theta = \pi - 0.3028 = 2.8388$

QIV: $\theta = 2\pi - 0.3028 = 5.9804$

a) $\boxed{\theta = 2.8388, 5.9804}$

b) Tan/cot repeat every π, so just use first answer and add $+\pi n, n \in I$

$\boxed{\theta = 2.8388 + \pi n, n \in I}$

(!) Use a substitution

Example 3: Solve: $2\cos^2 x - \cos x - 1 = 0$ for a) $0 \le x < 2\pi$ b) general form

Let $a = \cos x$

$$2(a)^2 - (a) - 1 = 0$$

Solve for "a" $2a^2 - a - 1 = 0$

Factor

$$(2a+1)(a-1) = 0$$

$\underline{-2} \times \underline{+1} = -2$

$\underline{-2} + \underline{1} = -1$

using ac

$\left(a - \frac{2}{2}\right)\left(a + \frac{1}{2}\right)$

$(a-1)(2a+1)$

or decomposition

$2a^2 - a - 1$

$2a^2 - 2a + 1a - 1$

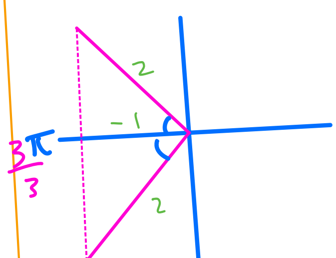
$2a(a-1) + 1(a-1)$

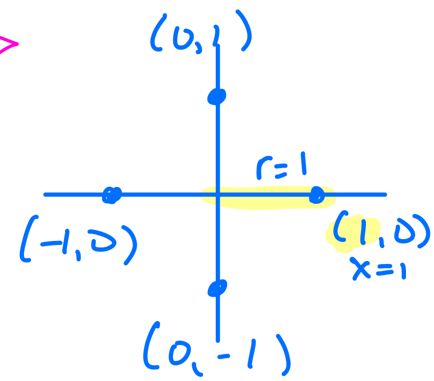
$(a-1)(2a+1)$

$a = -\frac{1}{2}$ or $a = 1$

(sub back in: $a = \cos x$)

$\cos x = \frac{-1}{2} \begin{matrix} \leftarrow x \\ \leftarrow r \end{matrix}$ $\qquad$ $\cos x = \frac{1}{1} \begin{matrix} \leftarrow x \\ \leftarrow r \end{matrix} \Longrightarrow$





angle is 0° or 0 rad

$\boxed{x = 0}$

(cos is neg. in QII, QIII)

reference angle $= \frac{\pi}{3}$

QII: $x = \frac{3\pi}{3} - \frac{\pi}{3} = \boxed{\frac{2\pi}{3}}$

QIII: $x = \frac{3\pi}{3} + \frac{\pi}{3} = \boxed{\frac{4\pi}{3}}$

a) $x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$

b) $x = 2\pi n$ $(0 + 2\pi n)$

$x = \frac{2\pi}{3} + 2\pi n$

$x = \frac{4\pi}{3} + 2\pi n$

$n \in I$

Assignment p. 321 #1,2ab, 5a-e,h,i,m,o

Day 2 – Trigonometric Equations part II

→ e.g. $2x, 3x, \ldots$

Solving trigonometric equations with angles other than θ

Solving a trig equation for angles with coefficients (2θ, 3θ, etc.) follows very similar steps:

1. Isolate the trig function. e.g. $\sin 2\theta$
2. Determine the general solution(s) for angle 2θ. * find general first
3. Determine the general solution(s) for angle θ. ***(divide by the coefficient)***
4. Determine the specific ('conditional') solution(s) for the given interval.

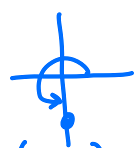
$2\pi \times \frac{4}{4} = \frac{8\pi}{4}$

Example 1: Solve: $\sin 2x = -1$ for a) general form b) $0 \le x < 2\pi$

stop at $\frac{8\pi}{4}$

Let $\theta = 2x$

$\sin(\theta) = -1$



$(0,-1)$
$x \quad y$

$\theta = \frac{3\pi}{2}$

$\theta = \frac{3\pi}{2}$

$\theta = \frac{3\pi}{2} + 2\pi n$

(sub back in)

$\frac{2x}{2} = \frac{3\pi}{2 \cdot 2} + \frac{2\pi n}{2}$

$x = \frac{3\pi}{4} + \pi n$

a) $\boxed{x = \frac{3\pi}{4} + \pi n \quad n \in I}$

b)
$x_1 = \frac{3\pi}{4}$

$x_2 = \frac{3\pi}{4} + \frac{4\pi}{4} = \frac{7\pi}{4}$

(can keep going...)

$x = \frac{7\pi}{4} + \frac{4\pi}{4} = \frac{11\pi}{4}$

too big!

Example 2: Solve: $\sqrt{3}\tan(2\theta) + 1 = 0$ for a) general form b) $0 \le \theta < 2\pi$

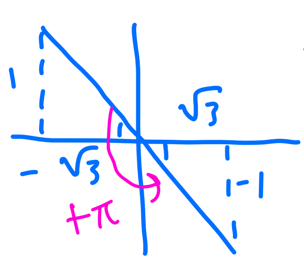
max $\frac{24\pi}{12}$

Let $A = 2\theta$

$\sqrt{3}\tan A + 1 = 0$
$\quad -1 \quad -1$

$\frac{\sqrt{3}\tan A}{\sqrt{3}} = \frac{-1}{\sqrt{3}}$

$\tan A = \frac{-1}{\sqrt{3}}$



ref. angle $= \frac{\pi}{6}$

QII: $A = \frac{6\pi}{6} - \frac{\pi}{6} = \frac{5\pi}{6}$

repeats every π

$A = \frac{5\pi}{6} + \pi n$

(sub back in)

$\frac{2\theta}{2} = \frac{5\pi}{6 \cdot 2} + \frac{\pi n}{2}$

a) $\boxed{\theta = \frac{5\pi}{12} + \frac{\pi}{2}n \quad n \in I}$

b) $\theta_1 = \frac{5\pi}{12}$

$\theta_2 = \frac{5\pi}{12} + \frac{\pi}{2}$
$= \frac{5\pi}{12} + \frac{6\pi}{12}$
$= \frac{11\pi}{12}$

$\theta_3 = \frac{11\pi}{12} + \frac{6\pi}{12}$
$= \frac{17\pi}{12}$

$\theta_4 = \frac{17\pi}{12} + \frac{6\pi}{12}$
$= \frac{23\pi}{12}$

sin has up to 2 answers

$(\ )^2$ quadratic has up to 2 answers
$(2x)$ double angle doubles number of answers
$\}$ up to 8 answers!!!

Example 3: Solve: $4\sin^2 2x + 2\sin 2x - 2 = 0$ for a) $0 \le \theta < 2\pi$ b) general form

① Let $a = \sin 2x$

$$4(a)^2 + 2(a) - 2 = 0$$

$$\frac{4a^2}{2} + \frac{2a}{2} - \frac{2}{2} = \frac{0}{2}$$

$$2a^2 + a - 1 = 0$$

$$(2a-1)(a+1) = 0$$

Factor:
$2 \quad x \quad \underline{-1} = -2$
$2 \quad + \quad \underline{-1} = +1$
ac
$(a - \frac{1}{2})(a + \frac{2}{2})$
$(2a-1)(a+1)$
or
decomp:
$2a^2 + a - 1$
$2a^2 + 2a - a - 1$
$2a(a+1) - 1(a+1)$
$(a+1)(2a-1)$

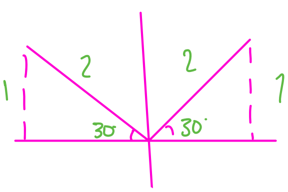
$\therefore a = \frac{1}{2}$ $\quad$ $a = -1$

$\therefore \sin(2x) = \frac{1}{2}$ $\quad$ $\therefore \sin(2x) = -1$

② Let $\theta = 2x$ and solve for θ

$\sin\theta = \frac{1}{2}$ $\boxed{\text{or}}$ $\sin\theta = -1$

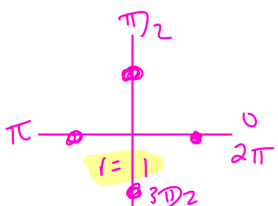
sin is ⊕ in QI, QII



$\theta_R = \frac{\pi}{6}$

QI: $\theta = \frac{\pi}{6}$

QII: $\theta = \frac{6\pi}{6} - \frac{\pi}{6} = \frac{5\pi}{6}$



$(0, -1)$
$x \quad y$

$\sin\theta = \frac{-1}{1} \frac{y}{r}$

when $\theta = \frac{3\pi}{2}$

③ Find general sol'n for θ:

$$\boxed{\theta_1 = \frac{\pi}{6} + 2\pi n \qquad \theta_2 = \frac{5\pi}{6} + 2\pi n \qquad \theta_3 = \frac{3\pi}{2} + 2\pi n}$$

④ Find general solution for x: $(\theta = 2x)$

$\theta_1 = \frac{\pi}{6} + 2\pi n$ $\qquad$ $\theta_2 = \frac{5\pi}{6} + 2\pi n$

$\frac{2x_1}{2} = \frac{\pi}{2 \cdot 6} + \frac{2\pi n}{2}$ $\qquad$ $\frac{2x_2}{2} = \frac{5\pi}{2 \cdot 6} + \frac{2\pi n}{2}$

$\boxed{x_1 = \frac{\pi}{12} + \pi n}$ $\qquad$ $\boxed{x_2 = \frac{5\pi}{12} + \pi n}$

$\theta_3 = \frac{3\pi}{2} + 2\pi n$

$\frac{2x_3}{2} = \frac{3\pi}{2 \cdot 2} + \frac{2\pi n}{2}$

$\boxed{x_3 = \frac{3\pi}{4} + \pi n}$

b) general form solutions $(n \in I)$

⑤ Find solutions from $0 \le \theta < 2\pi$

$+\frac{12\pi}{12}$ $\quad$ $+\frac{12\pi}{12}$ (add $\pi = \frac{12\pi}{12}$, but not past $\frac{24\pi}{12}$)

x_1: $\frac{\pi}{12}$, $\frac{13\pi}{12}$, $\cancel{\frac{25\pi}{12}}$ too big

$+12\pi/12$ $\quad$ $+12\pi/12$

x_2: $\frac{5\pi}{12}$, $\frac{17\pi}{12}$, $\cancel{\frac{29\pi}{12}}$ too big

$+\frac{4}{4}\pi$ $\quad$ $+\frac{4}{4}\pi$ (add $\pi = \frac{4\pi}{4}$ but not past $\frac{8\pi}{4}$)

x_3: $\frac{3\pi}{4}$, $\frac{7\pi}{4}$, $\cancel{\frac{11\pi}{4}}$ too big

a)

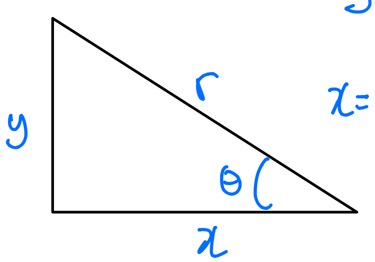
$$\boxed{x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{3\pi}{4}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{7\pi}{4}}$$

Assignment p. 323 #4

Trigonometric Identities and Equations Day 3: The Fundamental Identities

In trigonometry, there are expressions and equations that are true for any given angle. These are called identities. An infinite number of trigonometric identities exist, and we are going to prove many of these identities, but we are going to need some basic identities first.

The six basic trig ratios will lead to our first identities



$y = r\sin\theta$

$x = r\cos\theta$

$\sin\theta = \frac{y}{r}$ $\quad$ $\csc\theta = \frac{r}{y} = \frac{1}{\sin\theta}$

$\cos\theta = \frac{x}{r}$ $\quad$ $\sec\theta = \frac{r}{x} = \frac{1}{\cos\theta}$

$\tan\theta = \frac{y}{x}$ $\quad$ $\cot\theta = \frac{x}{y} = \frac{1}{\tan\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\cot\theta = \frac{\cos\theta}{\sin\theta}$

$\tan\theta = \frac{r\sin\theta}{r\cos\theta} = \frac{\sin\theta}{\cos\theta}$

And our knowledge of Pythagoras will determine the remaining Fundamental Trigonometric Identities

$a^2 + b^2 = c^2$

$x^2 + y^2 = c^2$

$(r\cos\theta)^2 + (r\sin\theta)^2 = r^2$

$\frac{r^2\cos^2\theta}{r^2} + \frac{r^2\sin^2\theta}{r^2} = \frac{r^2}{r^2}$

$\cos^2\theta + \sin^2\theta = 1$

$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

$1 + \left(\frac{\sin\theta}{\cos\theta}\right)^2 = \left(\frac{1}{\cos\theta}\right)^2$

$1 + \tan^2\theta = \sec^2\theta$

$\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$

$\frac{\cos^2\theta}{\sin^2\theta} + 1 = \left(\frac{1}{\sin\theta}\right)^2$

$\left(\frac{\cos\theta}{\sin\theta}\right)^2 + 1 = \csc^2\theta$

$\boxed{\cot^2\theta + 1 = \csc^2\theta}$

Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

Reciprocal and Quotient Identities:

$$\sec\theta = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta} \qquad \cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Corollary Identities (a statement that follows readily from a previous statement)

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\sin^2\theta = 1 - \cos^2\theta$

$1 = \sin^2\theta + \cos^2\theta$

$1 + \tan^2\theta = \sec^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

$\tan^2\theta - \sec^2\theta = -1$

$\sec^2\theta - \tan^2\theta = 1$

$1 + \cot^2\theta = \csc^2\theta$

$\cot^2\theta = \csc^2\theta - 1$

$\csc^2\theta - \cot^2\theta = 1$

Simplifying a Trigonometric Expression

There are many different strategies to simplifying a trigonometric expression. The following examples will look at the most common types of strategies.

Write as a fraction with a common denominator

$$\cos\theta + \frac{\sin\theta}{\cos\theta} = \frac{\cos^2\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta + \sin\theta}{\cos\theta}$$

Factor as a difference of squares

$1 - \cos^2\theta$ $\qquad$ $\sec^2\theta - \tan^2\theta$

$(1 - \cos\theta)(1 + \cos\theta)$ $\qquad$ $(\sec\theta - \tan\theta)(\sec\theta + \tan\theta)$

Change everything to sine and cosine.

$$\frac{\tan\theta}{\sec\theta} = \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}$$

$$= \frac{\sin\theta}{\cancel{\cos\theta}} \cdot \frac{\cancel{\cos\theta}}{1}$$

$$= \sin\theta$$

Multiply by the conjugate

$$\frac{1}{1 - \sin\theta} \cdot \frac{(1 + \sin\theta)}{(1 + \sin\theta)} = \frac{1 + \sin\theta}{1 - \sin^2\theta}$$

$$= \frac{1 + \sin\theta}{\cos^2\theta}$$

Now we can use these strategies along with the eight fundamental identities to simplify expressions

Example 1: Simplify

$$\sin\theta + \frac{cos^2\theta}{\sin\theta} = \frac{\sin^2\theta}{\sin\theta} + \frac{\cos^2\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} = \csc\theta$$

Example 2: Simplify

$$\frac{1}{\sin^2\theta} - 1$$

$$\frac{1}{\sin^2\theta} - \frac{\sin^2\theta}{\sin^2\theta}$$

$$\frac{1-\sin^2\theta}{\sin^2\theta}$$

$$= \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \cot^2\theta$$

Example 3: Simplify

$$\frac{1+csc\,\theta}{cos\,\theta + cot\,\theta} = \frac{1 + \frac{1}{\sin\theta}}{\cos\theta + \frac{\cos\theta}{\sin\theta}}$$

$$= \frac{\cancel{1+\frac{1}{\sin\theta}}}{\cos\theta\left(\cancel{1+\frac{1}{\sin\theta}}\right)}$$

$$= \frac{1}{\cos\theta} = \sec\theta$$

Example 4: Simplify

let $a = \cos x$

$$cos^4x - 2cos^2x + 1$$

$$a^4 - 2a^2 + 1$$

$$(a^2-1)(a^2-1)$$

$$(\cos^2\theta - 1)(\cos^2\theta - 1)$$

$$(-\sin^2 x)(-\sin^2 x)$$

$$\sin^4 x$$

Example 5: Simplify $\frac{\cos\theta - 1}{\sin\theta - \sin\theta\cos\theta}$

$$= \frac{\cos\theta - 1}{\sin\theta(1-\cos\theta)}$$

$$= \frac{-1(\cancel{1-\cos\theta})}{\sin\theta(\cancel{1-\cos\theta})}$$

$$= \frac{-1}{\sin\theta} = -\csc\theta$$

Example 6: Simplify $\frac{\sin x}{(1-\cos x)} + \frac{\sin x}{(1+\cos x)}$

$$\frac{\sin x(1+\cos x)}{(1-\cos x)(1+\cos x)} + \frac{\sin x(1-\cos x)}{(1+\cos x)(1-\cos x)}$$

$$\frac{\sin x + \sin x\cos x}{1-\cos^2 x} + \frac{\sin x - \sin x \cos x}{1-\cos^2 x}$$

$$\frac{\sin x + \sin x}{1-\cos^2 x} = \frac{2\cancel{\sin x}}{\sin^{\cancel{2}} x}$$

$$= \frac{2}{\sin x} = 2\csc x$$

Restrictions

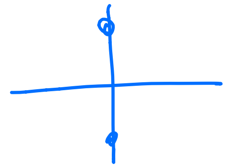
Just like any algebraic expression, a trigonometric expression cannot have zero in the denominator. We must consider the exact values that would result in a denominator of zero.

Example 7: Determine the restrictions on $\tan x - \csc x$ for $0 \leq x < 2\pi$

$\tan x - \csc x$

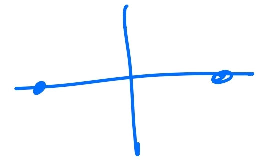
$\dfrac{\sin x}{\cos x} - \dfrac{1}{\sin x}$

$\cos x \neq 0$



$x \neq \frac{\pi}{2}, \frac{3\pi}{2}$

$\sin x \neq 0$



$x \neq 0, \pi$

$x \neq 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

Assignment p; 304 #2-9 (as many as needed)

Trigonometric Identities and Equations Day 4: Verifying Trigonometric Identities

Trigonometric Identities

When verifying trigonometric identities, the key is using the rules for algebra as well as the fundamental trigonometric identities to rewrite and simplify expressions. An identity has been proven when the right side of the equal sign is the same as the left side of the equal sign.

Example 1: Prove the identity: $\frac{1-\cos^2\theta}{\cos^2\theta} = \tan^2\theta$

$$\frac{\sin^2\theta}{\cos^2\theta}$$

$\tan^2\theta$ | $\tan^2\theta$

LHS = RHS

Example 2: Prove the identity: $\frac{1+\tan\theta}{\sec\theta} = \sin\theta + \cos\theta$

$$\frac{1}{\sec\theta} + \frac{\tan\theta}{\sec\theta}$$

$$\cos\theta + \frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}$$

$$\cos\theta + \frac{\sin\theta}{\cancel{\cos\theta}} \cdot \frac{\cancel{\cos\theta}}{1}$$

$\cos\theta + \sin\theta$ | $\sin\theta + \cos\theta$

LHS = RHS

LS RS

Example 3: Prove the identity: $\dfrac{1}{1-\sin\theta} = \dfrac{1+\sin\theta}{\cos^2\theta}$

to create "squared" trig expressions (which have identities) try using the conjugate.

$$\frac{1}{1-\sin\theta}\cdot\frac{(1+\sin\theta)}{(1+\sin\theta)}$$

The conjugate of $1-\sin\theta$ is $1+\sin\theta$

$$\frac{1+\sin\theta}{1-\sin^2\theta}$$

identity: $\sin^2\theta+\cos^2\theta=1$

$\therefore \cos^2\theta = 1-\sin^2\theta$

$$\frac{1+\sin\theta}{\cos^2\theta}$$

LS = RS

Example 4: Prove the identity: $\sec\theta\csc\theta\tan\theta = \sec^2\theta$

$$\frac{1}{\cos\theta}\cdot\frac{1}{\cancel{\sin\theta}}\cdot\frac{\cancel{\sin\theta}}{\cos\theta}$$

$$\frac{1}{\cos^2\theta}$$

$$\sec^2\theta$$

LS = RS

Assignment p. 311 #1-25

Trigonometric Identities and Equations Day 5: Sum and Difference Identities

α β $\rightarrow$ different angles

Sum and Difference Identities

Identities are not limited to the fundamental identities and single angles. We can also use identities involving sums and differences. The derivations of these identities are shown on page 327 of your textbook. We will be looking not at proving these identities but using these identities.

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta \qquad \sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta \qquad \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} \qquad \tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Example 1: Find the exact value: $\sin(\alpha+\beta)$

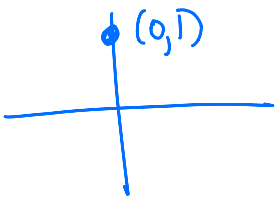
$$\sin\frac{\pi}{3}\cos\frac{\pi}{6} + \cos\frac{\pi}{3}\sin\frac{\pi}{6} = \sin\left(\frac{\pi}{3}+\frac{\pi}{6}\right)$$

$$= \sin\left(\frac{2\pi}{6}+\frac{\pi}{6}\right)$$

$$= \sin\left(\frac{3\pi}{6}\right)$$

$$= \sin\left(\frac{\pi}{2}\right)$$

$$= 1$$



Example 2: Find the exact value: $\cos\ 345°$

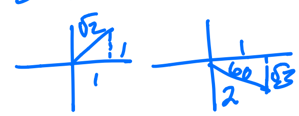
$$\cos(\alpha \pm \beta) = \cos(45+300)$$

$$= \cos 45 \cos 300 - \sin 45 - \sin 300$$

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{-\sqrt{3}}{2}\right)$$

$$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$$

$$= \frac{1+\sqrt{3}}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}+\sqrt{6}}{4}$$



Example 3: Express as a single function, then evaluate. $\cos^2\frac{\pi}{4} - \sin^2\frac{\pi}{4}$

$$\cos\frac{\pi}{4}\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\sin\frac{\pi}{4}$$

$$\cos\left(\frac{\pi}{4}+\frac{\pi}{4}\right)$$

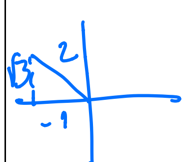
$$\cos\left(\frac{\pi}{2}\right) = 0$$

Example 4: Express as a single function, then evaluate. $\frac{\tan\pi - \tan\frac{\pi}{3}}{1 + \tan\pi\ \tan\frac{\pi}{3}} = \tan\left(\pi - \frac{\pi}{3}\right)$

$$= \tan\left(\frac{2\pi}{3}\right)$$

$$= \frac{\sqrt{3}}{-1}$$

$$= -\sqrt{3}$$



Example 5: Given angle A in quadrant I and angle B in quadrant II, such that $sin\,A = \frac{4}{5}$ $and\ \cos B = -\frac{12}{13}$, find $tan\,(A - B)$.

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{\frac{4}{3} - \left(\frac{-5}{12}\right)}{1 + \left(\frac{4}{3}\right)\left(\frac{-5}{12}\right)}$$

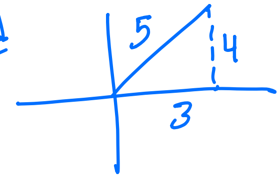
$$= \frac{\frac{16}{12} + \frac{5}{12}}{\frac{36}{36} - \frac{20}{36}}$$

$$= \frac{\frac{21}{12}}{\frac{16}{36}}$$

$$= \frac{21}{\cancel{12}} \cdot \frac{\cancel{36}\,3}{16}$$

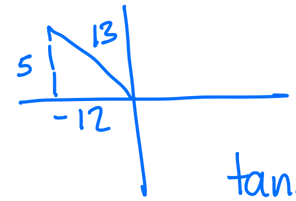
$$= \frac{63}{16}$$

A



$\tan A = \frac{4}{3}$

B



$\tan B = \frac{-5}{12}$

Example 6: Prove:

$$\frac{sin\,2x}{cos\,2x + 1} = tan\,x$$

$$\frac{\sin(x+x)}{\cos(x+x)+1} \quad \Bigg| \quad \frac{\sin x}{\cos x}$$

$$\frac{\sin x\cos x + \sin x \cos x}{\cos x \cos x - \sin x \sin x + 1}$$

$$\frac{2\sin x \cos x}{\cos^2 x - \sin^2 x + 1}$$

$$\frac{2\sin x \cos x}{\cos^2 x + \cos^2 x}$$

$$= \frac{\cancel{2}\sin x\,\cancel{\cos x}}{\cancel{2}\cos^{\cancel{2}} x} = \frac{\sin x}{\cos x}$$

LHS = RHS

Assignment p. 332 #1ace,2a-d,3,5

Trigonometric Identities and Equations Day 6: Double Angle Identities

Double Angle Identities

Using the sum and difference identities, we can determine other trigonometric identities

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\alpha+\alpha) = \sin\alpha\cos\alpha + \cos\alpha\sin\alpha$

$= 2\sin\alpha\cos\alpha$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos(\alpha+\alpha) = \cos\alpha\cos\alpha - \sin\alpha\sin\alpha$

$= \cos^2\alpha - \sin^2\alpha$

$= \cos^2\alpha - (1-\cos^2\alpha)$

$= 2\cos^2\alpha - 1$

$= 1 - \sin^2\alpha - \sin^2\alpha$

$= 1 - 2\sin^2\alpha$

$\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\tan(\alpha+\alpha) = \dfrac{\tan\alpha + \tan\alpha}{1 - \tan\alpha\tan\alpha}$

$= \dfrac{2\tan\alpha}{1-\tan^2\alpha}$

Double Angle Identities (same angle)

$\sin 2\theta = 2\sin\theta\cos\theta$

$$\begin{aligned}\cos 2\theta &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta\end{aligned}$$

$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Example 1: Simplify $A = 2x$

$$\frac{2}{1-\cos 4x}$$

$$\frac{2}{1-\cos 2(2x)}$$

$$\frac{2}{1-\cos 2A}$$

$$\frac{2}{1-(1-2\sin^2 A)}$$

$$= \frac{2}{2\sin^2 A}$$

$$= \frac{1}{\sin^2 A}$$

$$= \frac{1}{\sin^2(2x)}$$

Example 2: Solve, $0 \leq x \leq 2\pi$

$$\csc^2 x = 2\sec 2x$$

$$\frac{1}{\sin^2 x} = \frac{2}{\cos 2x}$$

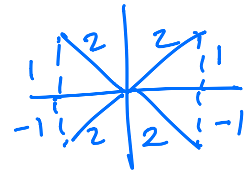
$$\cos 2x = 2\sin^2 x$$

$$1 - 2\sin^2 x = 2\sin^2 x$$

$$1 = 4\sin^2 x$$

$$\sin^2 x = \frac{1}{4}$$

$$\sin x = \pm\frac{1}{2}$$



$$\theta_r = \frac{\pi}{6}$$

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

Example 3: Prove: $\frac{1-\cos 2x}{\sin 2x} = \frac{1-\sin x}{\cot x - \cos x}$

$\frac{1-\cos 2x}{2\sin x\cos x}$	$\frac{1-\sin x}{\frac{\cos x}{\sin x} - \cos x}$
$\frac{1-(1-2\sin^2 x)}{2\sin x\cos x}$	$\frac{\sin x\left(\frac{1}{\cancel{\sin x}} - 1\right)}{\cos x\left(\frac{1}{\cancel{\sin x}} - 1\right)}$
$\frac{\cancel{2}\sin^{\cancel{2}} x}{\cancel{2\sin x}\cos x}$	$\frac{\sin x}{\cos x}$
$\frac{\sin x}{\cos x}$	
$\tan x$	$\tan x$

LHS = RHS

Assignment p. 340 #1-5 (as many as needed)